Solving PDEs With Spatial & Time Varying Coefficients: Dirac Wave Function Thru EM Wave

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Introduction: Find the relativistic quantum mechanics steady state wave function $\Psi_m(x,y,z,t)$ as a solution to the Dirac equations with a pre-existing EM traveling wave via magnetic and electric potentials $\bar{A}$, ϕ. The probability density, ρ, of a particle's location is given by $\rho=\Sigma|\Psi_m|^2$ m=1..4 .

Computational Method: The EM Dirac equations [1] for the behavior of a particle of mass *m* with $M=mc/\hbar$, c=light speed, $\hbar$=Planck's constant, $\bar{\mathbf{A}}=\bar{A}e/\hbar$, $\Phi=e\phi/c\hbar$, e=charge, β=v/c, $\alpha_E\equiv \mathcal{E}'_o/(k'_D)^2$:

$$\frac{1}{c}\frac{\partial\Psi_1}{\partial t}+\frac{\partial\Psi_4}{\partial x}-i\frac{\partial\Psi_4}{\partial y}+\frac{\partial\Psi_3}{\partial z}+i\Psi_1(\Phi+M)+i(i\mathbf{A}_y\Psi_4-\mathbf{A}_z\Psi_3-\mathbf{A}_x\Psi_4)=0$$

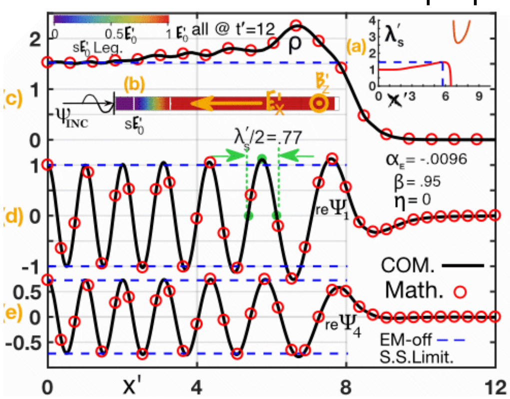
$$\frac{1}{c}\frac{\partial\Psi_2}{\partial t}+\frac{\partial\Psi_3}{\partial x}+i\frac{\partial\Psi_3}{\partial y}-\frac{\partial\Psi_4}{\partial z}+i\Psi_2(\Phi+M)+i(A_z\Psi_4-\mathbf{A}_x\Psi_3-i\mathbf{A}_y\Psi_3)=0 \qquad (1)$$

$$\frac{1}{c}\frac{\partial\Psi_3}{\partial t}+\frac{\partial\Psi_2}{\partial x}-i\frac{\partial\Psi_2}{\partial y}+\frac{\partial\Psi_1}{\partial z}+i\Psi_3(\Phi-M)+i(i\mathbf{A}_y\Psi_2-\mathbf{A}_z\Psi_1-\mathbf{A}_x\Psi_2)=0$$

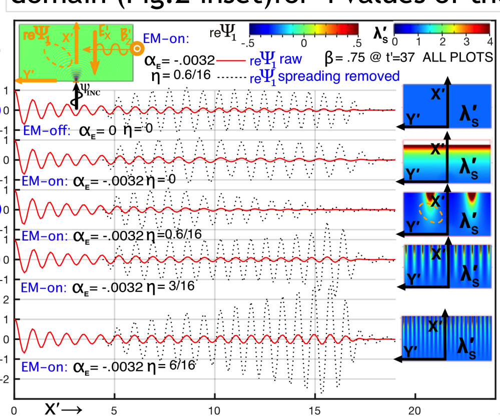
$$\frac{1}{c}\frac{\partial\Psi_4}{\partial t}+\frac{\partial\Psi_1}{\partial x}+i\frac{\partial\Psi_1}{\partial y}-\frac{\partial\Psi_2}{\partial z}+i\Psi_4(\Phi-M)+i(\mathbf{A}_z\Psi_2-\mathbf{A}_x\Psi_1-i\mathbf{A}_y\Psi_1)=0$$

are solved with COMSOL'S "*General-Form PDE*". When the wave vector $\bar{\mathbf{k}}$ is in the xy plane, $\partial\Psi_m/\partial z$ terms drop out and the 1st & 4th eqs. decouple, where Ψ_1,Ψ_4 are solved alone.

Results: • **Fig.1** **PW in Long wavelength EM Field** below validates the $\Psi_n=\Psi_{on}\,e^{-i\omega' t'}$ end driven *Wave Guide* COMSOL FEM↔Mathematica propagation vs $x'=x/\lambda_D$ and is shown for $\eta=\tilde{\omega}'/\omega' \to \varepsilon$ long wavelength limit. The effect of the $\mathcal{E}'$, $\mathcal{B}'$ field @ freq. $\tilde{\omega}'$, gradually increases the λ'_s spatial wave length and ρ probability density vs +x'. The local s.s. wavelength approx. λ'_s is shown in the inset.



• **Fig.2** **PW Thru Slit into EM Field** examines the slit driven $\Psi_n=\Psi_{on}\,e^{-i\omega' t'}$ wave propagation into the spatial domain (Fig.2 inset) for 4 values of the freq. parameter η={0,0.6,3,6}/16. Re Ψ_1 is plotted normal to slit and acts like $1/\sqrt{r}$ cylindrical spreading. The local s.s. wavelength approx. λ'_s (e.g. Figs.(d-f) insets with big deviations from 1.0 zones) indicates where Ψ_n distortions are expected.



• **Fig.3** **CYL.Wave in EM Field** upper right validates the $\Psi_n=\Psi_{on}(\theta)\,e^{-i\omega' t'}$ inner radius driven *cylindrical wave* COMSOL FEM↔S.S. EXACT wave propagation vs x',y' and is shown for 2 values of electric field strength parameter α_E={.0, -.0032}. Figs.(3a-b) compares Exact reΨ_4 S.S. limit vs transient FEM @ $t'=t/T_D$=18 for EM field off (i.e. α_E=0).

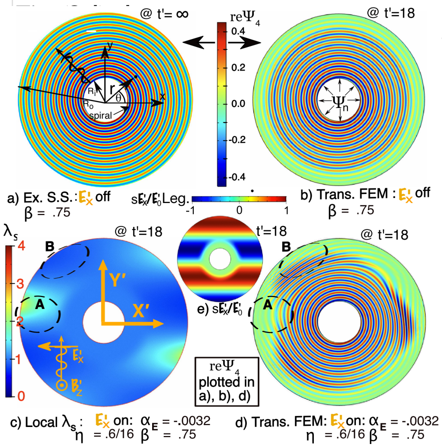
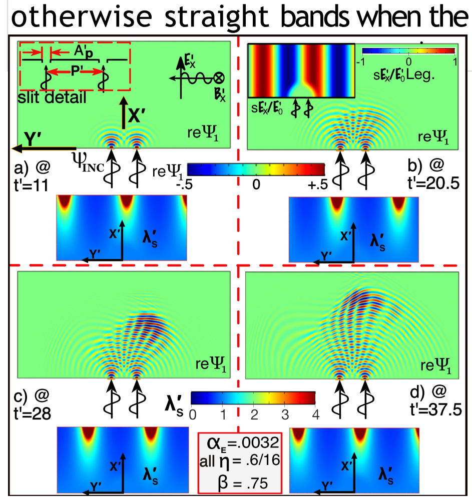


Figure (3d) shows the effect of EM turned on(i.e. α_E=-.0032). Zones in Fig.(3c) (where local S.S. wavelength λ'_s deviate from 1), line up with Fig.(3d) re Ψ_4 distortions (compare encircled "A" markers). Inset Fig.(3e) shows the pre-existing EM wave.

• **Fig.4** **2 Slit Demo; Electric $\mathcal{E}'$ Field On** Particles fired at 2 slits, is a classic quantum mechanics demo, represented by a *free field* $\Psi_n=\Psi_{on}\,e^{-i(x'k'_D-\omega' t')}$ PW wave function incident upon the slits. Figs.(4a-d) show a time snapshot growth of the reΨ_1 component. Bands of constructive & destructive interference form where the effect of the EM field (with electric field strength parameter α_E=-0.02) is to curve the blades like Fig.(4c) as compared to otherwise straight bands when the EM field is turned off.



The pre-existing traveling EM wave field is shown in upper Fig.(4b) inset. The local s.s. wavelength approx. λ'_s is shown in the lower Fig.(4a-d) insets where big deviations from 1.0 zones indicates where Ψ_n distortions are expected. The λ'_s field changes in each frame.

Conclusions: The *General-Form PDE* option successfully solved the EM transient Dirac equations. The classic 2 slit model produced EM influenced curved constructive interference bands (compared to EM off straight ones). The λ'_s local S.S. wavelength gives a-prori estimates where "EM on" effects the solutions and guides mesh selection).

References:1. P. Strange, Relativistic Quantum Mech., Camb. Univ. Press 1998