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Experimental And Theoretical Investigation Of Acoustic Metamaterial With Negative Bulk-Modulus

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Abstract: This paper compares the results of the experimental investigations of different acoustic metamaterials with negative bulk-modulus to their COMSOL modeling. Fixed and tunable negative bulk modulus materials have been realized using daisy chained variable volume Helmholtz resonators. The propagation characteristics of these metamaterials obtained from COMSOL Multiphysics show reasonable agreement with the analytical and experimental results.

Keywords: Acoustic metamaterials, Acoustic structures, Acoustic array systems, Helmholtz resonators.

1. Introduction

Acoustic metamaterials are structured materials of negative mass density or negative dynamic bulk-modulus or both of them [1]. Because of these negative parameters metamaterials show some unconventional properties such as reversal of Doppler effect, flat lens beam shaping, sub-wavelength diffraction, cloaking of objects etc. [2]. The metamaterials are tailored in sub-wavelength dimensions so as to behave as homogenous material around the resonant frequency of the locally resonant elements of the materials. The negative parameters of metamaterials have been demonstrated using local monoplole and dipole resonances of the materials or using concatenated Helmholtz resonators. [1, 3]. In the latter type, metamaterial properties are independent of the material used and are dependent only on the geometry of the structure and the medium that fills it. Therefore these structures are ideal for the realization of tunable negative bulk modulus. This work, therefore concentrates on metamaterials with negative bulk-modulus achieved by concatenating Helmholtz resonators of sub-wavelength dimensions with sub-wavelength periodicity.

2. THEORY

In a one-dimensional array of Helmholtz resonators connected to a common duct through a narrow neck; any perturbation that affects the pressure inside the duct will be propagated to the cavity of the Helmholtz resonator through the neck. This leads to the oscillation of fluid inside the system. This is analogous to a inductor capacitor resonant circuit, where cavity acts as capacitor and neck as inductor. The resonant frequency of the Helmholtz resonator determines the first bandgap of the system . This frequency of the resonator can be expressed by the equation $\omega_0 = c\sqrt{\left(a_{neck}/\left(Vl'_{neck}\right)\right)}$ in the limit where the cavity is much larger than the neck. a_{neck} is the cross-sectional area of the neck of effective length l'_{neck} which connects to the cavity of volume V and 'c' is the velocity of sound in the medium inside the cavity. Since the system is effectively homogenous, effective medium theory can be applied for the system. Then effective modulus of the system can be expressed as [1]

$$E_{eff}^{-1} = E_0^{-1}\left(1 - \frac{F\omega_0^2}{\omega^2 - \omega_0^2 + i\Gamma\omega}\right) \quad (1)$$

where E_o is the bulk modulus of fluid filled inside the metamaterial, F is the geometrical factor [4], ω_o is the resonant angular frequency, ω is the angular frequency of the signal and Γ is the damping factor, that is found out empirically [4].The real part of the effective bulk-modulus will be negative in the vicinity of resonant frequency and is usually referred to as the anomalous dispersion. The region over which anomalous dispersion occurs will show a sharp absorption. Since the absorption has a reciprocal relationship to the imaginary part of the bulk modulus, the imaginary part of bulk modulus will be a minimum in the anomalous dispersion region [1]. The propagation of longitudinal waves through the system depends upon the effective bulk-modulus of the system. Then propagation vector can be expressed as [1]

$$k = (-\alpha + i\beta)^{1/2}\omega\sqrt{\rho/(\alpha^2+\beta^2)} \qquad (2)$$

where $\alpha = |\mathrm{Re}\,(E)|$ and β = - Im (E). From equation (2), it is clear that in the vicinity of anomalous dispersion region the imaginary part of the propagation vector will be positive. This implies that the system in this region introduce a stop-band. The transmission through the resonator array structure has been derived by Wang et al using interface response theory and is expressed as [5]

$$T = \left| \frac{2\sin(\alpha d_1)(e^{i2kd_1}-1)e^{iNkd_1}}{(1-e^{i(\alpha+k)d_1})^2 - e^{i2Nkd_1}(e^{ikd_1}-e^{i\alpha d_1})^2} \right|^2 \qquad (3)$$

Where $\alpha = \omega/c$, c is the velocity of sound through air, N is number of resonators, d_1 is the separation between resonators and k is the wave-vector of an infinite system which can be calculated using equation (2).

3. Experiment

Negative bulk modulus (NBM) materials have been fabricated with and without the provision of tunability. In the first sample, a one-dimensional array of eight identical air filled Helmholtz resonators connected to a common channel as shown in Fig 1(a) has been fabricated using perspex. The resonator cavity is cylindrical in shape and is having a radius 2 mm and height 1mm. Length of the neck is 1 mm and radius is 0.96 mm. Necessary length correction has been made to get the effective length of the neck [1]. Each resonator is separated by 9 mm and the channel has a cross-section of 4 mm^2. Here we used air as fluid. In the range of the frequency from 5 kHz to 6.7 kHz, the periodicity is less than one fifth of the wavelength and hence in this range the structure acts as a homogenous medium. For the construction of second sample, with tunable negative bulk modulus region, commercially available 2 ml plastic syringes have been used. The length and diameter of the neck of the resonator are 1.17 cm and 2.36 mm respectively. Five such Helmholtz resonators were attached by their necks to a 7 mm diameter plastic tube as illustrated in Fig 1(b). The pistons of the independent syringes were connected together so that cavities could be tuned simultaneously to the same volume. Using this arrangement the cavity volumes were tuned between 240 mm^3 to 840 mm^3. Transmission loss measurements were carried out at discrete volumes of 240 mm^3, 440 mm^3, 640 mm^3 and 840mm^3.Two provisions, one upstream after the first resonator and one downstream after the last resonator were made to insert microphones to measure the signals within the channel. A composite audio frequency signal covering the interested frequency region was launched into the channel The signals picked up by the downstream microphones were amplified and captured using a Tektronix Digital Storage Oscilloscope (TDS 1012B) which gives an average of 1G samples/s. The data was then subsequently transferred to a computer for further analysis.

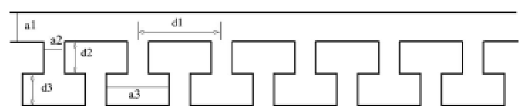

Fig 1(a) Schematic of a daisy chained Helmholtz resonators with a spacing of d_1.

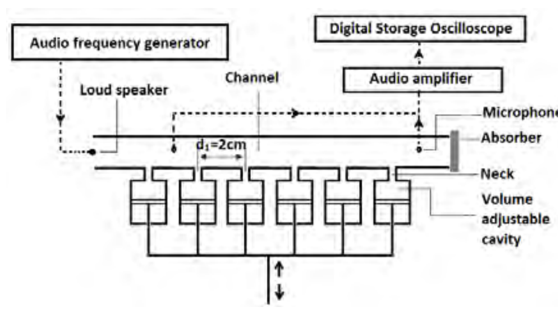

Figure 1(b). Schematic diagram of tunable one dimensional NBM formed by the array of Helmholtz resonators

4. Use of COMSOL Multiphysics

The acoustic module of the COMSOL Multiphysics has been used for modeling these acoustic metamaterials. Time harmonic, scattered wave analysis has been made for the metamaterial structure and a parametric sweep has been carried out over he frequencies of interest.. Since all the elements used in the metamaterial is acoustically rigid, sound hard boundary condition has been applied to all the boundary surfaces except at the both openings of the duct of the metamaterial. Matched boundary condition has been imposed at the ends of the duct of metamaterial. The cross-sectional plots of pressure variations in the metamaterials at their resonant frequencies are represented in Fig 2. The pressure difference across the neck indicates the oscillation of the air column inside the neck. Figure 2(a) represents the cross-sectional plot of fixed metamaterial and Fig 2(b) to fig 2(e) represents that of the tunable structure with cavity volumes of 240 mm^3, 440 mm^3, 640 mm^3 and 840mm^3 respectively.

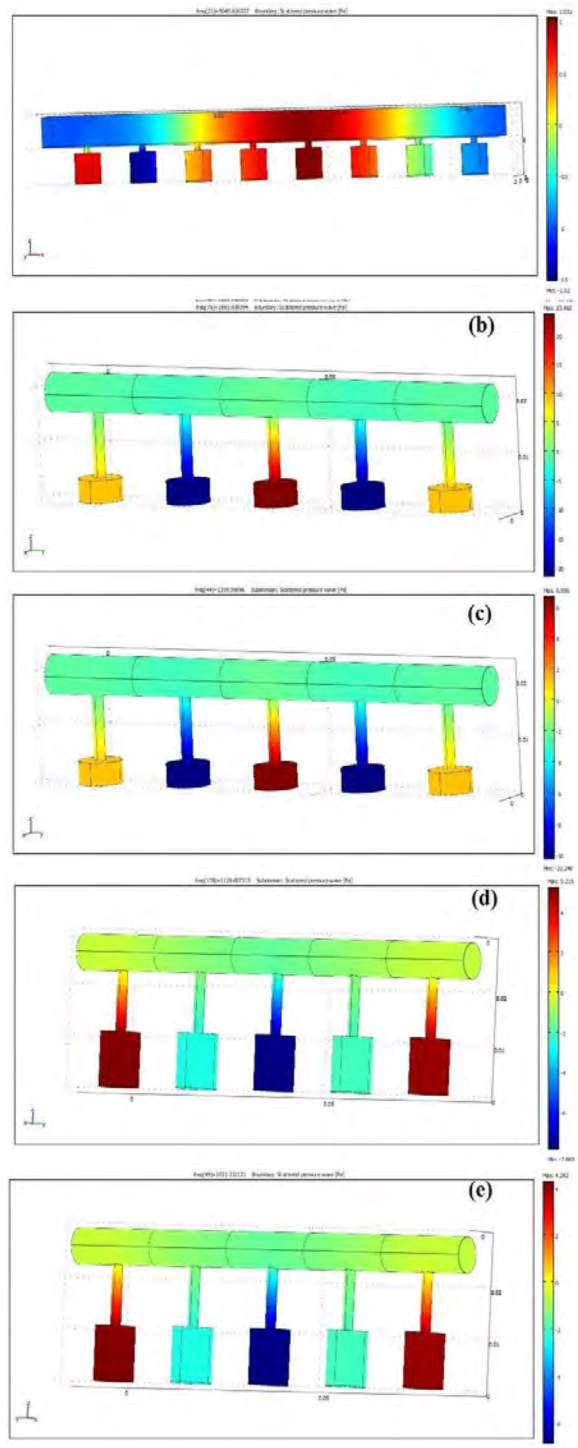

Figure 2. Cross-sectional plot of pressure variations in **(a)** fixed NBM with cavity volume 12.57mm^3 and tunable NBM with cavity volume of **(b)** 240 mm^3. **(c)** 440 mm^3. **(d)** 640 mm^3. **(e)** 840 mm^3.

5. Results and Discussion

Fig 3 to 6 shows the transmission data obtained from the analytical expression and experiments and COMSOL Multiphysics simulation Fig 2 represents the transmission characteristics of the fixed metamaterial. From the plot it can be observed that results from COMSOL show reasonable agreement with theoretical and experimental results. Convergence accuracies in the COMSOL simulation were limited by system capabilities and is probably the reason for the observed shift in the resonant frequency.

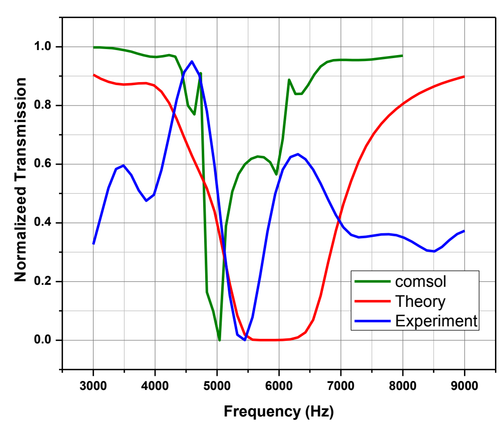

Figure 3. Transmission characteristics of fixed NBM

Figures from 4 to 7 represent the propagation characteristics of tunable metamaterials with cavity volumes from 240 mm^3, 440 mm^3, 640 mm^3 and 840 mm^3 respectively. The results obtained from COMSOL shows some discrepancy with the theoretical and experimental results of the tunable metamaterials of different volumes. The larger dimensions of the metamaterials required the larger computational domain. Mesh refinements though in this case was restricted due to the restraint imposed by this large computational domain and could be the reason for the discrepancy of COMSOL Multiphysics results with theory and experiment.

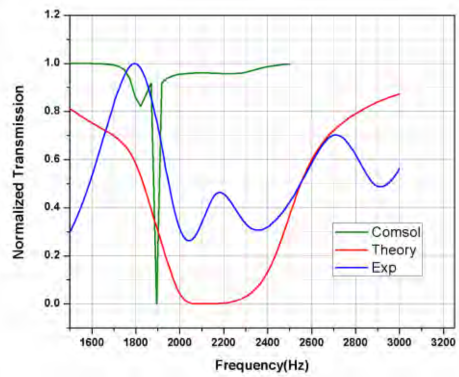

Figure 4. Transmission characteristics of tunable NBM with cavity volume 240 mm^3.

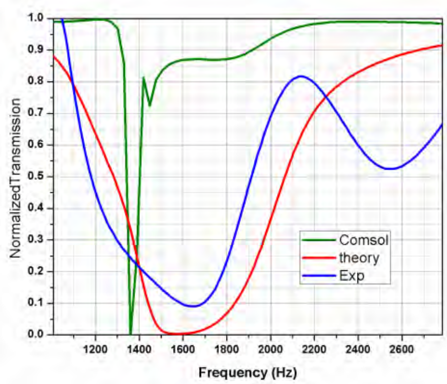

Figure 5. Transmission characteristics of tunable NBM with cavity volume 440 mm^3.

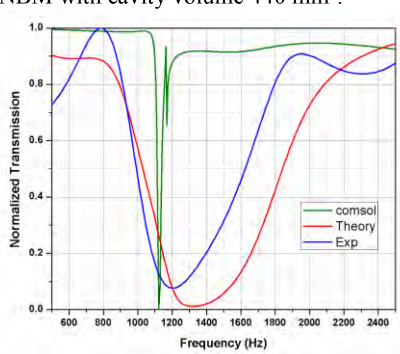

Fig 6. Transmission characteristics of tunable NBM with cavity volume 640 mm^3.

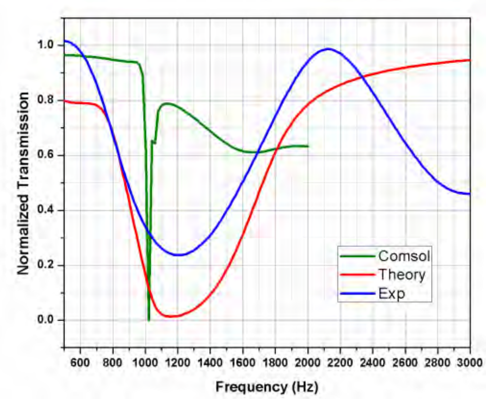

Fig.7. Transmission characteristics of tunable NBM with cavity volume 840 mm^3.

Figure 8(a) represents the characteristics of real component of k vector for the tunable NBM with cavity volume 840 mm^3 obtained from theory and experiments. The negative slope can be observed in the vicinity of anomalous dispersion region in the theory and experiments. This confirms the negative group velocity in the region which is one of the properties of a metamaterial. The real k vector response obtained from COMSOL Multiphysics is shown in figure 8(b) and it shows some discrepancy as in the transmission response.

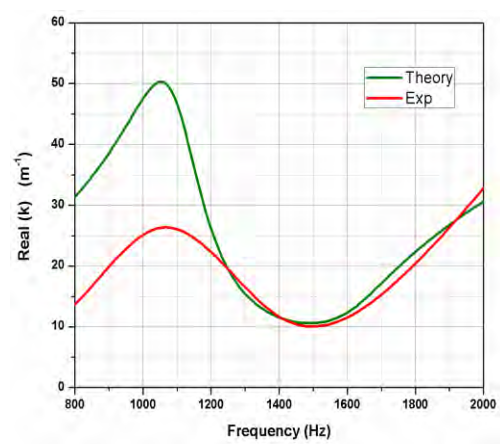

Figure.8 (a). Calculated and measured frequency dependence of k-vector of tunable NBM with cavity volume 840 mm^3

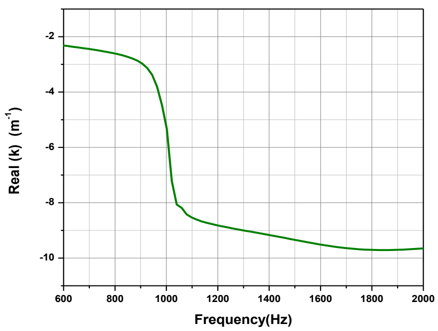

Figure.8 (b). Frequency dependence of k-vector of tunable NBM with cavity volume 840 mm^3 obtained from COMSOL Multiphysics .

6. Conclusions

Single negative metamaterials which exhibit negative bulk modulus and hence transmission loss has been fabricated. The analytical and experimental analysis of the stop bands of these metamaterials are compared with the simulation results obtained from COMSOL Multiphysics. The results shows reasonable agreements with the theory and experiment. The accuracies of FEM based simulation has been restricted by the computational capabilities available at our end. Frequency dependence of wavevectors exhibit negative slopes near the resonant frequencies indicating existence of negative group velocity in this region.

7. References

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