An Investigation of Violin Sound Quality through Resonant Modes

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Abstract: This paper describes a simulation study performed with the goal of quantifying the correlation between violin design and sound quality. The investigation first explored the resonant modes of the front plate of a violin by examining its predicted eigenfrequencies and normal modes. Chladni patterns of increasingly complex plate shapes were modeled using COMSOL® Structural Mechanics - Plate Module, version 5.4. The plate had a thickness of 10 mm with no initial velocity or displacement, and all edges were unconstrained with no loads. The materials used were AISI 4340 steel and a generic soft wood. The study examined eigenfrequencies between 175 Hz and 2700 Hz, a range chosen based on the pitch range of the violin.

Three geometries were studied, and changes in geometry demonstrated visible differences in the predicted Chladni patterns. By matching specific modes to experimental results, a refinement of the geometry was possible. If differences in resonant modes between diverse violin geometries can be properly modeled, they can create another metric which could be utilized in improving violin sound quality. For mass-produced violins, this could allow for a reduction in manufacturing costs while increasing sound quality.

Keywords: acoustics, Chladni patterns, instrument design, violin

Introduction

Violins have been made and played for around 450 years. During that time, they have gone through some very drastic changes, both in body geometry and material composition. In more recent years, luthiers, or violin makers, have also changed how violin plates are tuned. Hutchins[1] popularized the use of Chladni patterns for violin plate tuning in 1983 and luthiers have been utilizing this technique ever since to optimize the vibration of violin front and back plates.

Unfortunately, not all violins are made by hand. Student violins are often mass-produced and sold for cheaper prices, which often results in lower-quality violins. In order to help bring a better sound quality to these manufactured violins, the normal modes of a rendered violin were simulated and studied.

A normal mode is a physical vibration pattern that occurs at a resonant frequency of a structure. Resonant modes are present in all structures, including a violin. Ernst Chladni[2] was the first to discover a way to visualize these modes in a more macroscopic setting through what is now known as Chladni patterns. Chladni patterns were initially studied on simple geometries such as a square or a circle with a fixed constraint at its center. They were excited by running a violin bow along the edge of a thin plate covered with flour. As technology has progressed, they are more commonly excited by constraining the center of a thin plate to a loudspeaker which can stimulate the body more precisely[1]. When using simulation software to perform this experiment, the model has a fixed constraint at the center and is excited over a specific range of frequencies.

The Chladni patterns of violins studied by Hutchins[1] have been used as a baseline for luthiers and researchers alike when studying the violin. Gough[3] investigated how each part of the violin affects its resonant modes, utilizing Chladni pattern simulations to analyze the front and back plates. Lu[4] employed both simulation and experimental testing to study the composition of the material used for violin-making. Lu considered a more realistic violin plate and focused on the effect of material differences rather than geometry changes. The COMSOL Blog[5] also details an application that models the Chladni patterns of various geometries and materials.

This paper's research focused on replicating physical patterns found by Hutchins as a first step towards improving the quality of manufactured violins. This was achieved through the refinement of several geometries and meshes to attain the most accurate representation of a physical experiment. Through the use of simulation software, manufacturers can utilize an affordable and computationally efficient way to check how a violin design will perform.

Theory

The model reported in this paper uses one physics interface: the 2D Structural Mechanics – Plate Module, version 5.4. Within this module, equations from the Linear Elastic Material are used to solve this simulation and are shown below:

$$\sigma_m = \sigma_{ad} + \underset{\dots}{C}[\gamma - \gamma_{inel}] \tag{1}$$

$$\gamma_{inel} = \gamma_0 + \gamma_{th} + \gamma_{hs} \tag{2}$$

$$\sigma_b = \sigma_{ad} + \frac{\underset{\dots}{C}d}{2}[\chi - \chi_{inel}] \tag{3}$$

$$\chi_{inel} = \chi_0 + \chi_{th} + \chi_h \tag{4}$$

$$\sigma_s = \sigma_{ad} + \frac{5}{6}2\underset{\dots}{G}(\zeta - \zeta_0) \tag{5}$$

$$\sigma_{inplane} = \sigma_m + z\sigma_b \tag{6}$$

$$\sigma_{ad} = \sigma_0 + \sigma_{ext} + \sigma_d \tag{7}$$

$$\mathbf{C} = \mathbf{C}(E, \nu) \tag{8}$$

$$G = G(E, \nu) \tag{9}$$

where
- d = plate thickness (m)
- z = relative thickness coordinate
- **C** = plane stress constitutive matrix
- E = Young's modulus (Pa)
- G = transverse shear modulus (Pa)
- γ = membrane strain (m/m)
- ζ = transverse shear strain (m/m)
- ν = Poisson's ratio
- σ = stress (Pa)
- χ = bending strain (m/m)

Three geometries were studied using the Plate Module. The boundary conditions stipulated no initial velocity and no initial displacement. The edges were unconstrained with no loads. The first two simulated geometries were made of steel as this was the material used in real-world experiments for these geometries. The violin's material was a generic soft wood, chosen for its similarity to materials used in violin-making. The study examined eigenfrequencies between 175 Hz and 2700 Hz, a range chosen based on the pitch range of the violin, i.e. G_3 = 196 Hz through ~E_7 = 2637 Hz.

Simulation Design

There were three distinct geometries used during the course of the research: a composite square, a full square, and a traditional violin geometry. The creation of the initial two geometries provided an effective framework for the violin geometry to be confidently based on. All three geometries held a constant thickness of 10 mm.

The COMSOL blog references a Chladni pattern application[5] that simulates four different geometries. The simulation this app draws from was rebuilt in order to provide a benchmark for other geometries. The first geometry, the composite square, was modeled after an example in this application. It was comprised of a triangle whose results were mirrored three times: the first to create a smaller square, then a rectangle, and lastly to create a larger composite square of length 0.24 m as shown in Fig. 1. This geometry was modeled using AISI Steel 4340, which has a density of 7850 kg/m^3, a Young's modulus of 205 x 10^9 Pa, and a Poisson's ratio of 0.28.

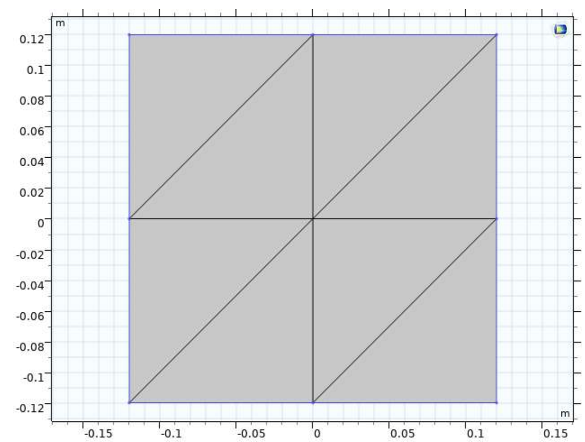


Figure 1. Composite Square Geometry

The mesh for this geometry shown in Fig. 2 consisted of triangular elements and was physics-controlled with a setting of 'finer'. It contained a total of 1043 elements with an average quality of 0.98.

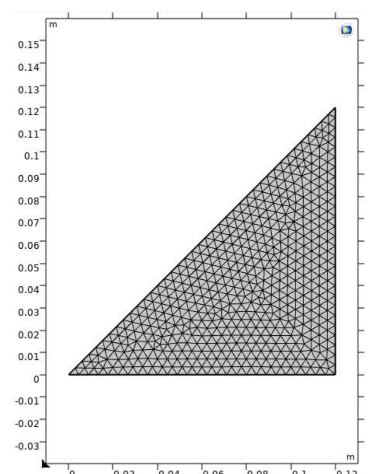


Figure 2. Composite Square Mesh

The second geometry, the full square, was a single square of length 0.24 m also modeled using AISI Steel 4340. The mesh for this geometry consisted of triangular elements and was physics-controlled with a setting of 'extremely fine'. It contained a total of 25316 elements with an average quality of 0.99.

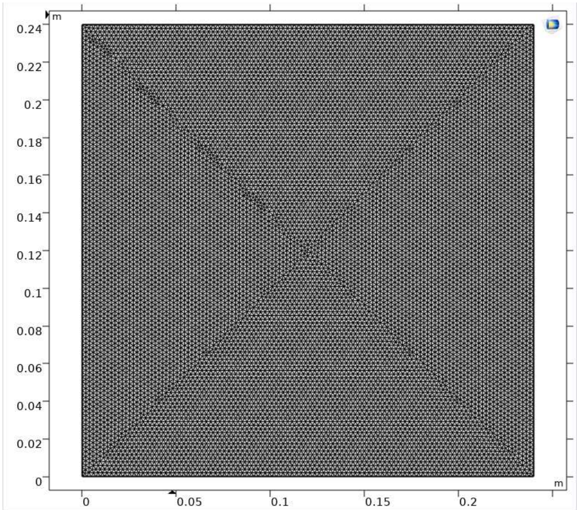


Figure 3. Full Square Geometry and Mesh

Figure 4 shows the final geometry, the violin, which was first created in SOLIDWORKS 2019 Modeling Software. Its geometry was modeled after a physical violin, detailed in Table 1. A number of simplifications were imposed on the geometry: the top plate was rendered as a flat plate, discounting the curvature in the *z*-direction. The sound post and bass bar of the violin, parts of the violin used to emphasize shell modes, were also discounted. Lastly, the plate was held at a uniform thickness, whereas actual violin plates vary thickness throughout the body.

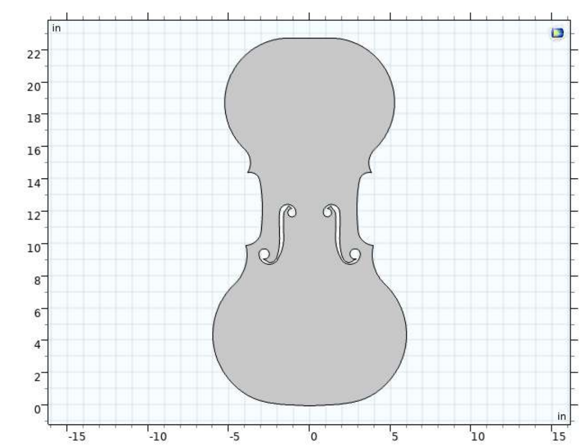


Figure 4. Violin Geometry

Table 1. Width and Length of Violin Geometry:

Position of the Measured Points	Dimension (mm)
Total Length	579
Lower Bout Width	305
Upper Bout Width	267
Upper Corner Width	195
Lower Corner Width	200
Middle Line Width	150
f-hole inner notches distance	78

Figure 5 shows the mesh for this geometry, which consisted primarily of tetrahedral elements and was physics-controlled with a setting of ‘extra fine’. It contained a total of 213397 elements with an average quality of 0.63. This geometry was modeled using a generic softwood with a density of 420 kg/m^3, a Poisson’s ratio of 0.30, and a Young’s modulus of 15.13 x 10^9 in the *x*–direction and 1.2 x 10^9 in the *y*–direction.

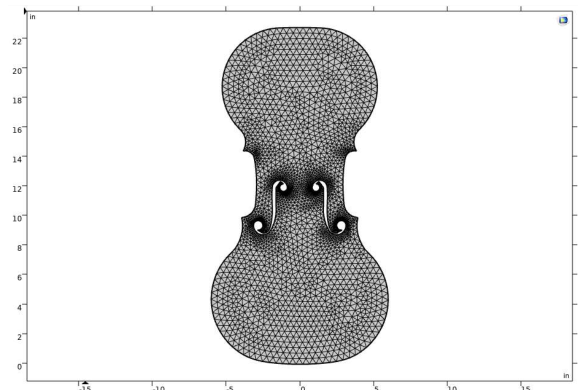


Figure 5. Violin Mesh

Results

The results from the first design, the composite square, matched the results from the COMSOL simulation exactly. However, the results shown in Fig. 6 did not match published experimental results of Chladni patterns of squares (Fig. 7).

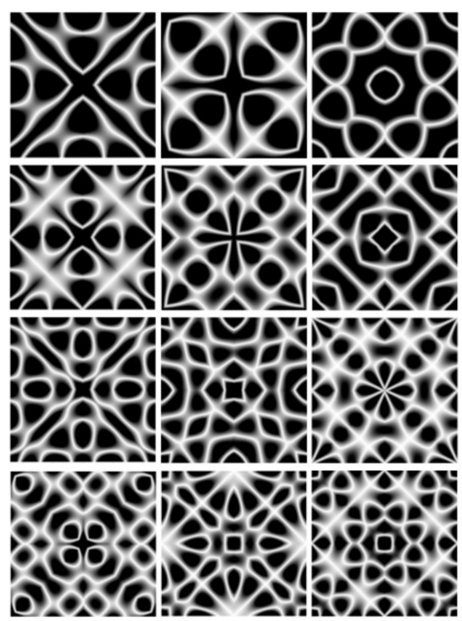


Figure 6. Composite Square Geometry Simulation Results

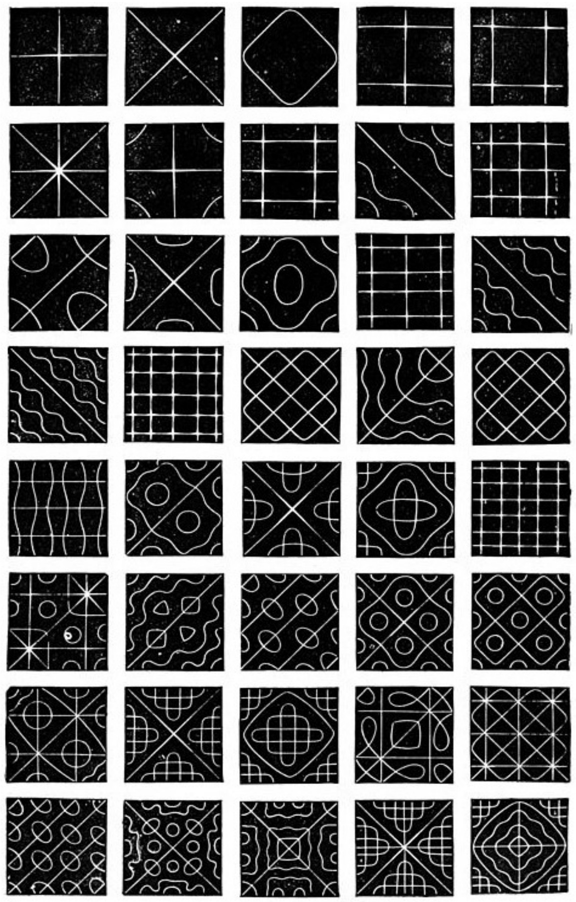


Figure 7. Experimental Chladni Patterns of a Square Steel Plate[6]

Two issues were identified regarding the symmetry in the geometry. First, the design defaulted to locating antinodes along the hypotenuse of the original triangle. This appeared to interfere with the results and made it difficult to determine if the simulation was operating under the correct boundary conditions. Second, the patterns were all completely symmetrical. All previously compiled experimental results had some order of asymmetry in higher modes that was not present in the composite square results due to the constraints of its geometry. With these complications in mind, it appears that the particular geometry used in the COMSOL application was insufficient for more detailed experiments. In order to validate the simulation, the full square was studied.

The results from the full square were found to have matched over half the experimentally-found modes[6]. Four of these are shown in Figs. 8a-d.

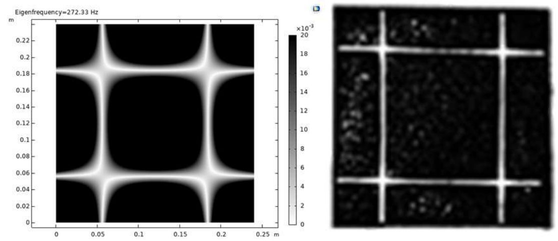


Figure 8a. Comparison of simulation results (left) to experimentally found results (right) for f = 272Hz

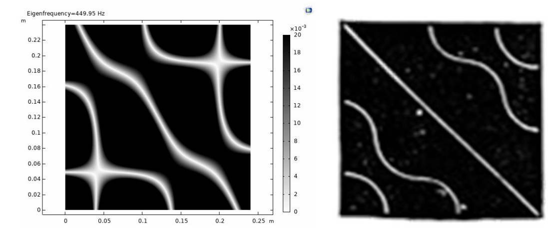


Figure 8b. Comparison of simulation results (left) to experimentally found results (right) for f = 450Hz

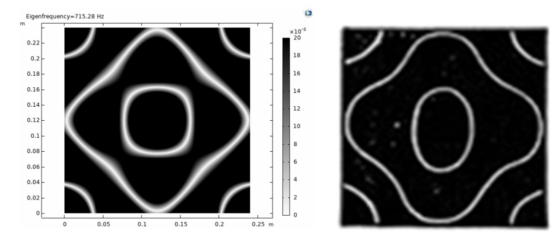


Figure 8c. Comparison of simulation results (left) to experimentally found results (right) for f = 715Hz

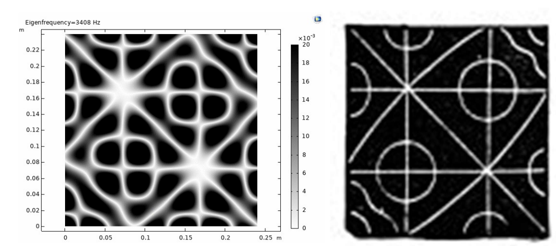
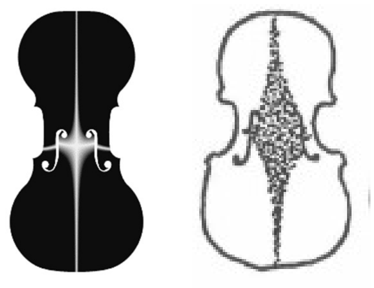


Figure 8d. Comparison of simulation results (left) to experimentally found results (right) for f = 3408Hz

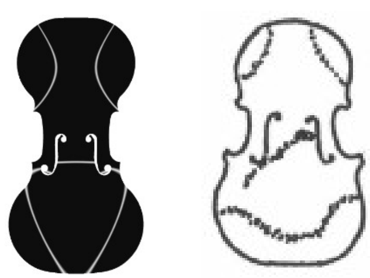
Notably, many modes exhibited an asymmetry (Fig. 8b,d) not found in the composite square results. These similarities provided confidence in the simulation results.

With the validation of the second geometry, the third geometry, the rendered violin, was studied. The results from the simulation and the physical experiment[1,3,7] visually matched the first breathing mode of the violin, with obvious similarities in compared results of higher modes as well. Figure 9a-c shows these comparisons. However, the correlating modes did not occur at the expected frequencies.



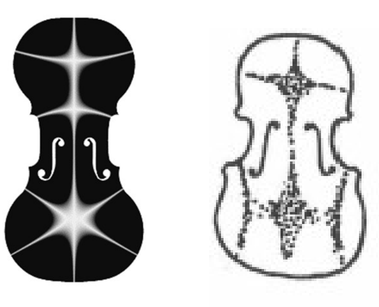
176.13 Hz 45Hz

Figure 9a. Comparison of simulation results (left) to experimentally found results (right) for mode 1



420.4 Hz 315

Figure 9b. Comparison of simulation results (left) to experimentally found results (right) for mode 12



1272.7 Hz 232

Figure 9c. Comparison of simulation results (left) to experimentally found results (right) for mode 11

In Figures 9b and 9c, the simulated Chladni patterns exhibit many of the same components present in the experimentally-found results, suggesting a correlation between the results of the two methods. Figure 9b shows both results having arches in the top bout with a nodal line in the center across the *f*-holes and a *u*-shaped nodal line in the lower bout. The angle of the nodal line across the *f*-holes exhibited in the experimentally-found result may be due to the presence of the bass bar, which was not modelled in the simulation. Additionally, the shift in position of the *u*-shaped node may be due to the arched displacement in the *z*-direction.

Both results in Figure 9c have the node down the center with a cross in the top bout and five nodal lines in the bottom bout. Again, the simplification of the simulated violin geometry may be the cause of this incongruence. With the inclusion of the arch of the top plate, the node found across the *f*-holes may be absorbed into the cross in the top bout and the star-shaped node in the bottom bout may be separated into the arches seen in the experimental result.

The presence of the correct mode shapes confirms the simulation is valid. The inconsistencies in the placement of the nodes and in the frequencies of the results however show that it still requires improvement. This development will be seen in the refinement of the geometry. The current geometry does not include the displacement in the *z*-direction that all violins have, nor are the varying thicknesses of the plate considered.

Conclusions

This paper presents the results of a modeling effort for violin design. Three different geometries were investigated to render a valid simulation. Correct mode shapes were found, but at somewhat different frequencies. Due to these discrepancies, further research is required. Changes in geometry show visible alterations in the Chladni patters, and by matching specific modes to experimental results, a refinement of the geometry is possible. The second phase of the project will focus on incorporating the curvature of the front plate in the *z*-direction as well as the modeling of different part interactions, including the back plate of the violin and the interaction of front and back plates with air cavity modes. If differences in resonant modes between diverse violin geometries can be properly modeled, then they can create another metric to be utilized in improving violin sound quality. For mass-produced violins, this could allow for a reduction in manufacturing costs while increasing sound quality.

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Acknowledgements

This work was funded by the Dorothy Goodwin Scholars Program provided by the Women's Advancement Initiative at the University of Hartford. The purpose of these annual grants is to enable and empower female students in research areas of their choosing.

Appendix

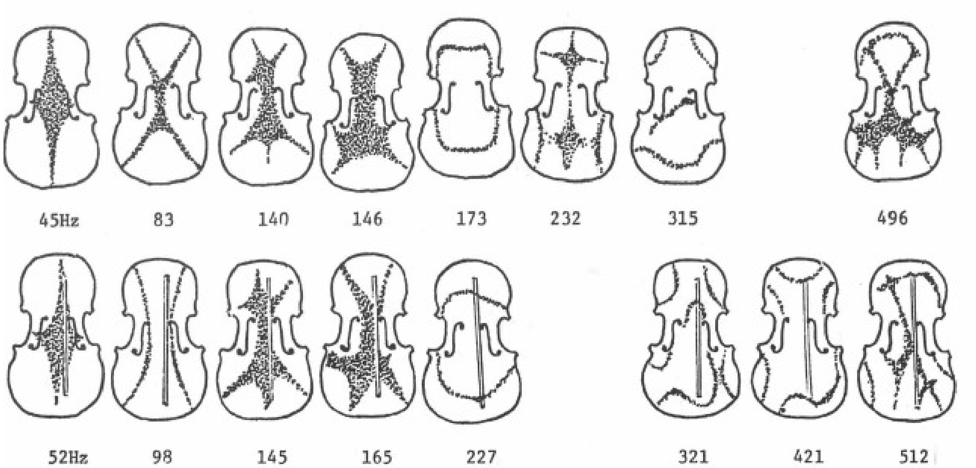

Figure A1. Chladni Patterns of Violin Plates[1]