Technische Universität Braunschweig

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Large Scale Invasion of New Species and of Genetic Information

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Contents

Introduction

An "invasive species" is defined as a species that is

- non-native (or alien) to the ecosystem under consideration
- and whose introduction causes or is likely to cause economic or environmental harm or harm to human health. (USDA)

- Changing environmental conditions and human activities have triggered species migration at a global scale causing biosecurity problems

- Understanding the mechanisms of dispersal by means of mathematical modelsl is important for the development of control strategies of invasive species

„Invading alien species in the United States cause major environmental damages and losses adding up to almost $120 billion per year. There are approximately 50,000 foreign species and the number is increasing. About 42% of the species on the Threatened or Endangered species lists are at risk primarily because of alien-invasive species."
Pimentel 2004

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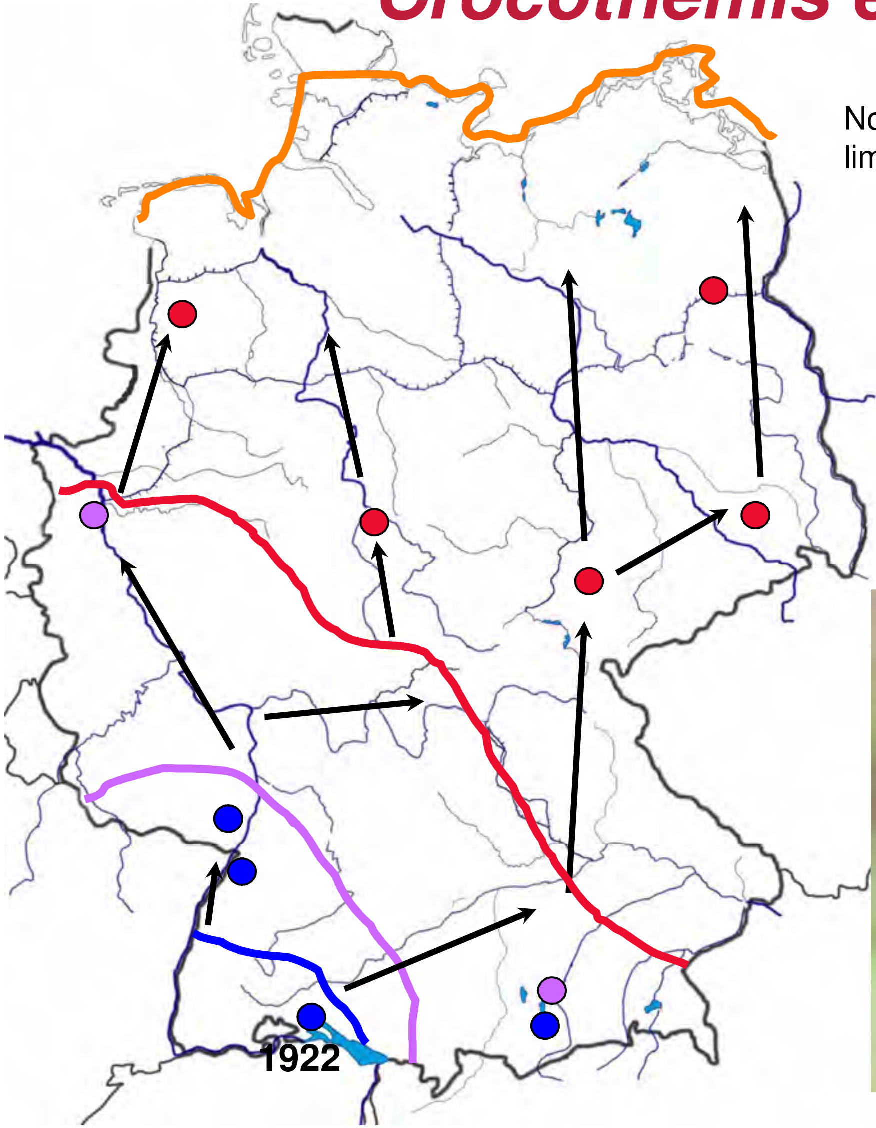
Northern migration of *Crocothemis erythraea*



Northerrn limit

Stray finds

unitl 1979

until 1989

until 1999

until 2009

Recontruction of the invasion of the Odonata speciest *Crocothemis erythraea* (verändert und ergänzt nach Ott 2007).

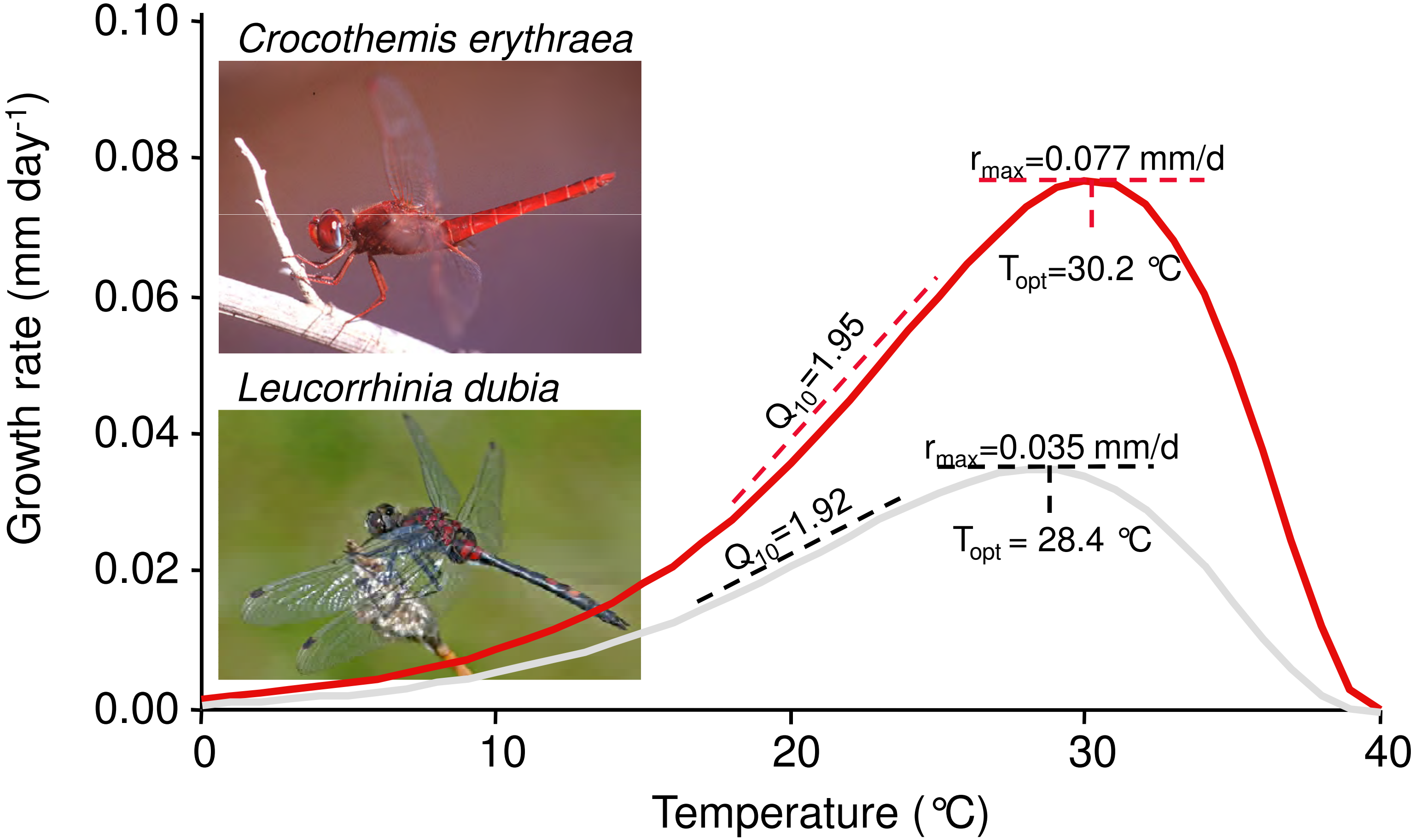
Temperature response functions of larvae of two Odonata species

ONeill function

$$\Phi(T) = k_{\max}\left[\frac{T_{\max} - T}{T_{\max} - T_{opt}}\right]^X Exp\left[X\,\frac{T_{\max} - T}{T_{\max} - T_{opt}}\right]$$

$$w = (Q_{10} - 1)(T_{\max} - T_{opt})$$

$$X = \frac{w^2}{400}\left(1 + \sqrt{1 + \frac{40}{w}}\right)^2$$



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Reaction-diffusion equations in biology

$$\frac{\partial N_i}{\partial t} = L[N_i] + f_i(N_1, N_2 \ldots N_n) \qquad i = 1 \cdots n$$

space operator — **reaction terms: population dynamics, genetics**

$$L[N] = \nabla D(N) \nabla N - \nabla \vec{v} N$$

Diffusion — **Convection**

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$$

Interacting populations with different temperature response

$$\frac{\partial u_i}{\partial t} = \nabla \cdot D_i \nabla + \beta_i(T) u_i \frac{u_i}{u_i + K_i} - \mu_i u_i \left(1 + \sum_{j=1}^{n} a_{ij} u_j\right)$$

Dispersion Reproduction Mortalilty and interaction

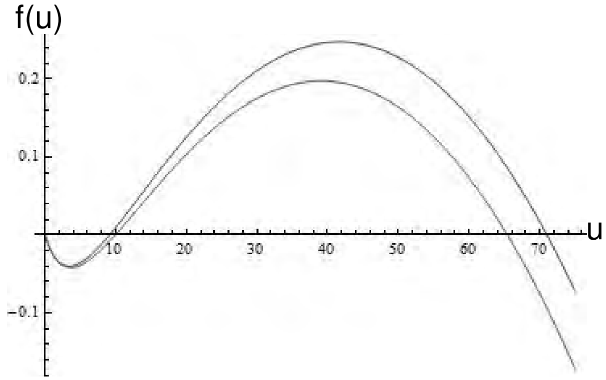
$$\beta(T) = \beta_{i\max} \left(\frac{T_{\max} - T}{T_{\max} - T_{opt}} \right)^{p} Exp\left(\frac{p(T - T_{opt})}{T_{\max} - T_{opt}} \right)$$

Temperature response of reproduction

Travelling wave solutions in one dimension

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial}{\partial x} u + f(u)$$

$$f(u) = \Phi(T) u \frac{u}{u+K} - \mu u (1 + \alpha u)$$



Travelling wave solutions have the form

$$u = u(z) \quad z = x - ct$$

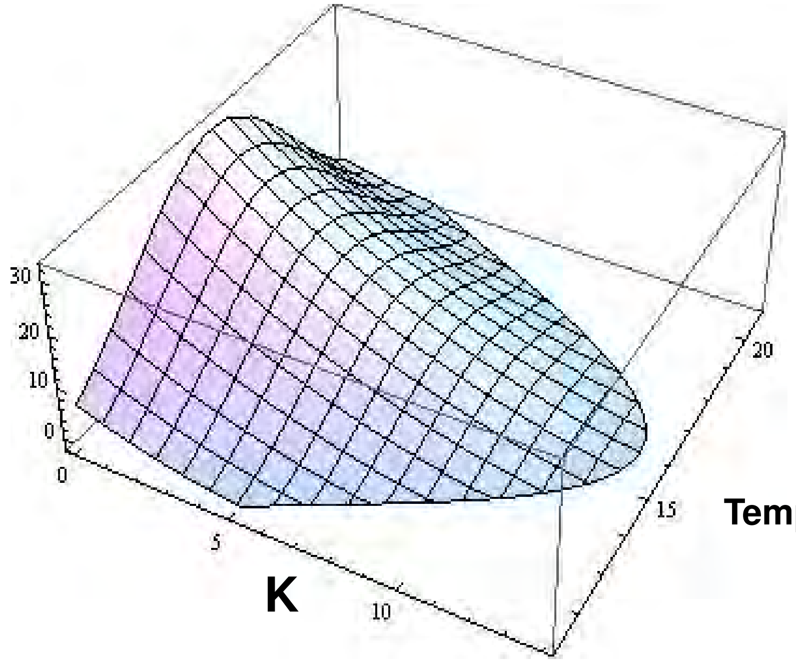
and lead to the ordinary differential equation

$$DU''(z) + cU'(z) + f(U(z)) = 0$$

Some mathematics

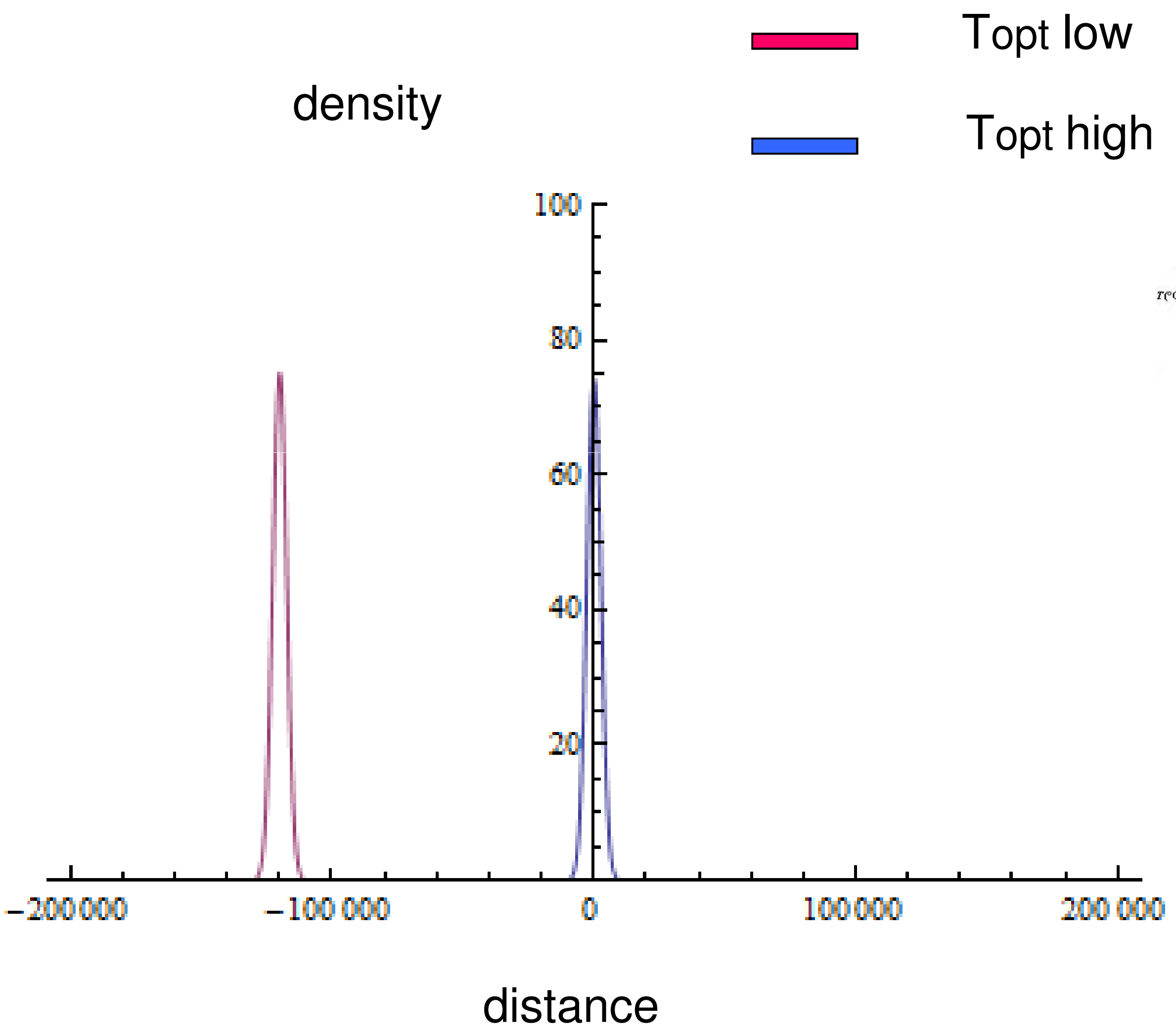
Theorem: let f(u) be Lipschitz-continuous. There exist positive constants a and U_{max} with a< U_{max}, such that f(a)=0, f(U_{max}) = 0, f(u)<0 for u<a and f(u)>0 for a<u<U_{max} and f(u)<0 for u>U_{max}. Then travelling waves exist if

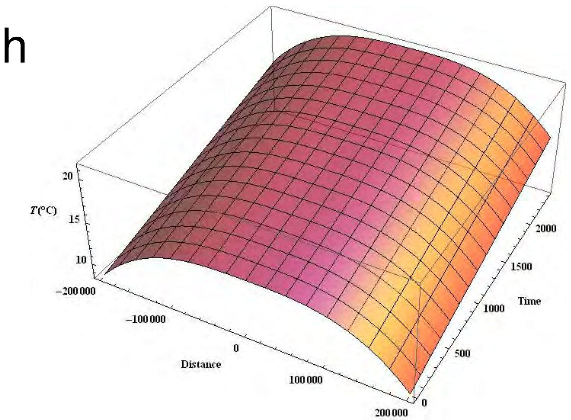
$$C = \int_0^{U_{max}} f(u)du > 0$$



(Hadeler 1984)

Interaction of two populations under rising temperature

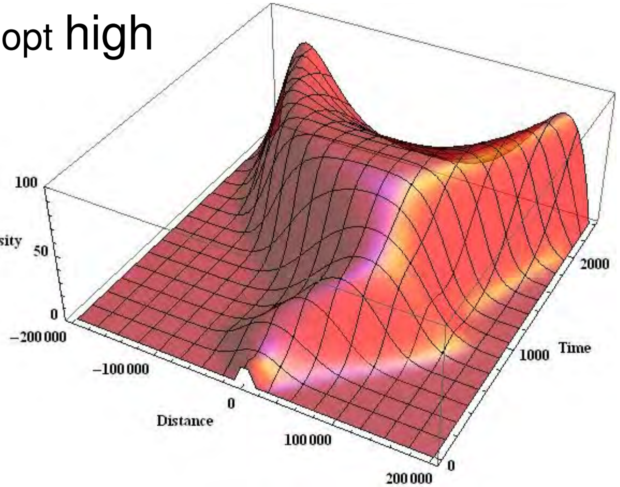




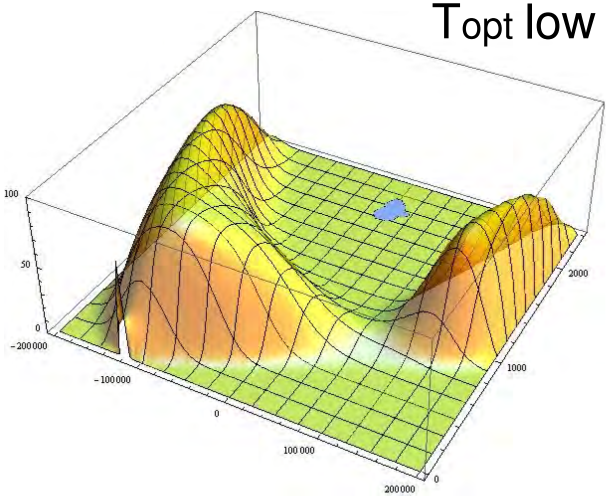
temperature profile with a positive trend

Interaction of two populations under rising temperature

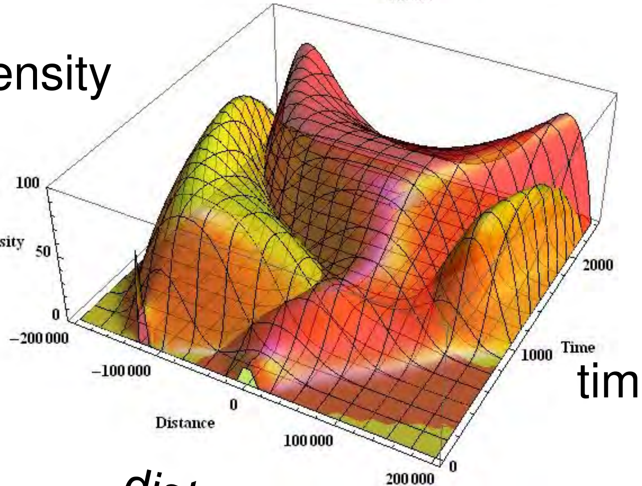
T_{opt} high



T_{opt} low

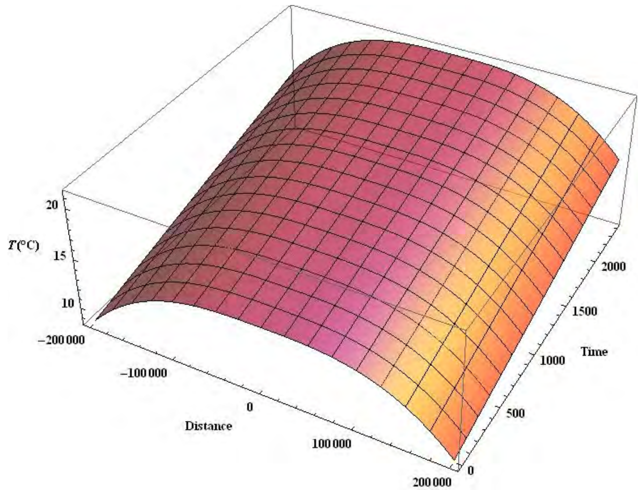


density



time

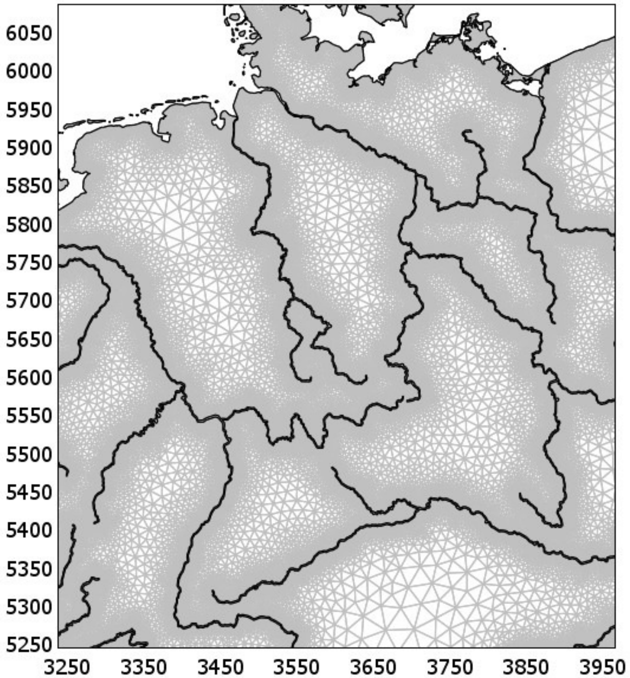
distance



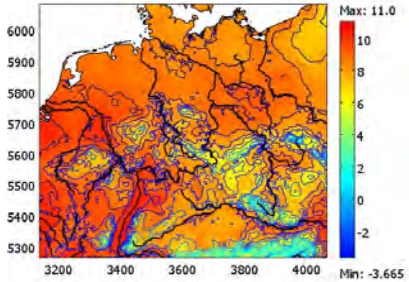
Temperature profile with a positive trend

Coupling GIS and finite element methods

COMSOL Multiphysics environment



Finite element net
Petrov Galerkin method



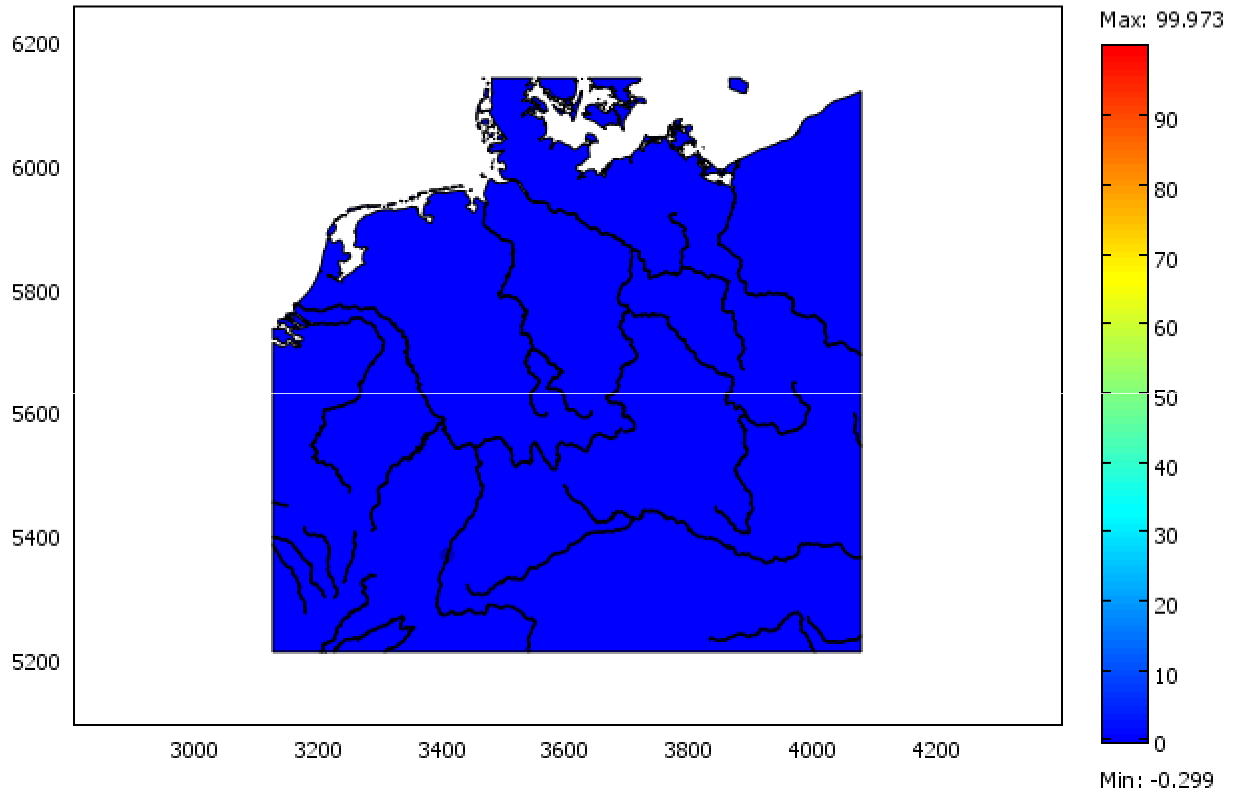
Temperature map

$$\frac{\partial u_i}{\partial t} = \nabla \cdot D_i \nabla + \beta_i(T) u_i \frac{u_i}{u_i + K_i} - \mu_i u_i (1 + \sum_{j=1}^{n} a_{ij} u_j)$$

Model equations

Partial differential equation solver

Dispersal of a species with high temperature demand



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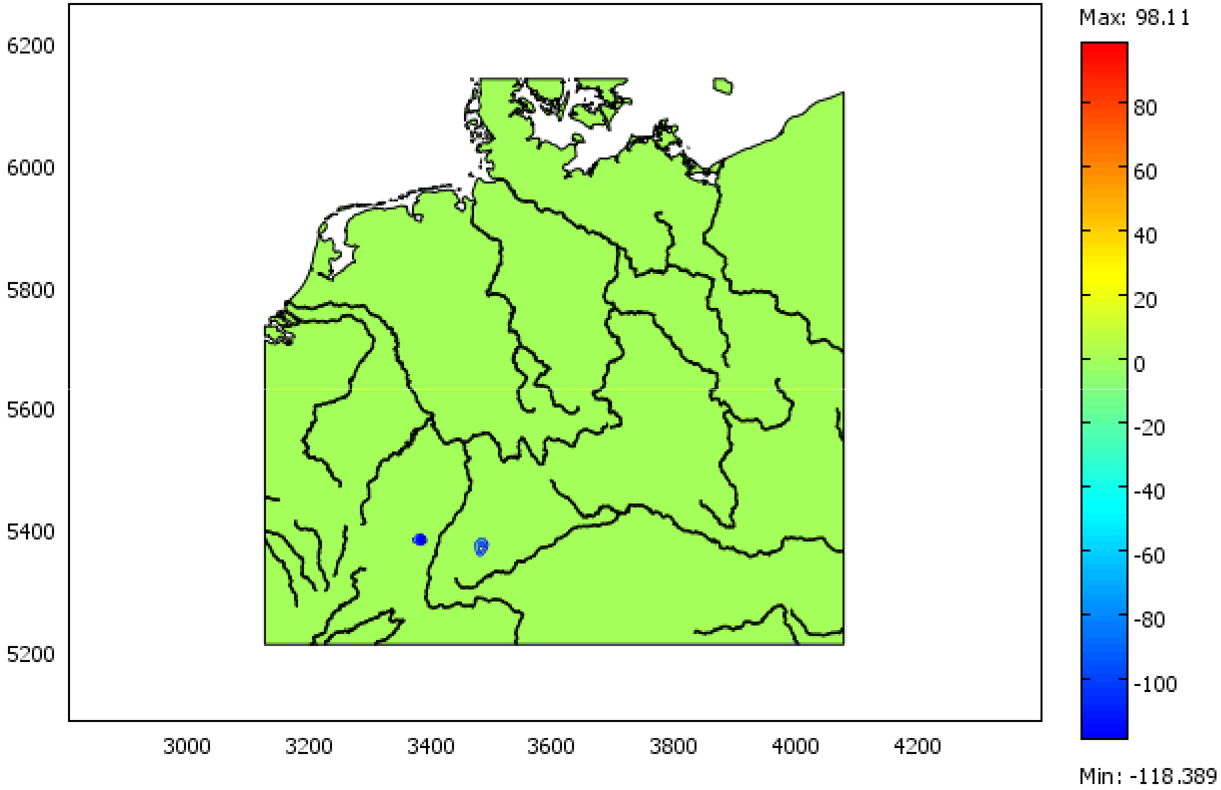
Dispersal of a species with low temperature demand



Dispersal, genetics and population dynamics

$$\frac{\partial N_i}{\partial t} = L_i[N_i] + \underbrace{f_i\left(\vec{N},T\right)}_{(a)} - \underbrace{\mu_i N_i \left(1 + \sum_{j=1}^{n} \alpha_{ij} N_j\right)}_{(b)}$$

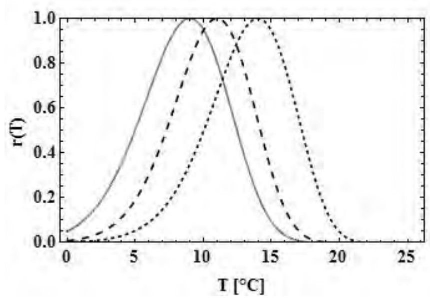
fertility rates,
genetic exchange

mortality and competition

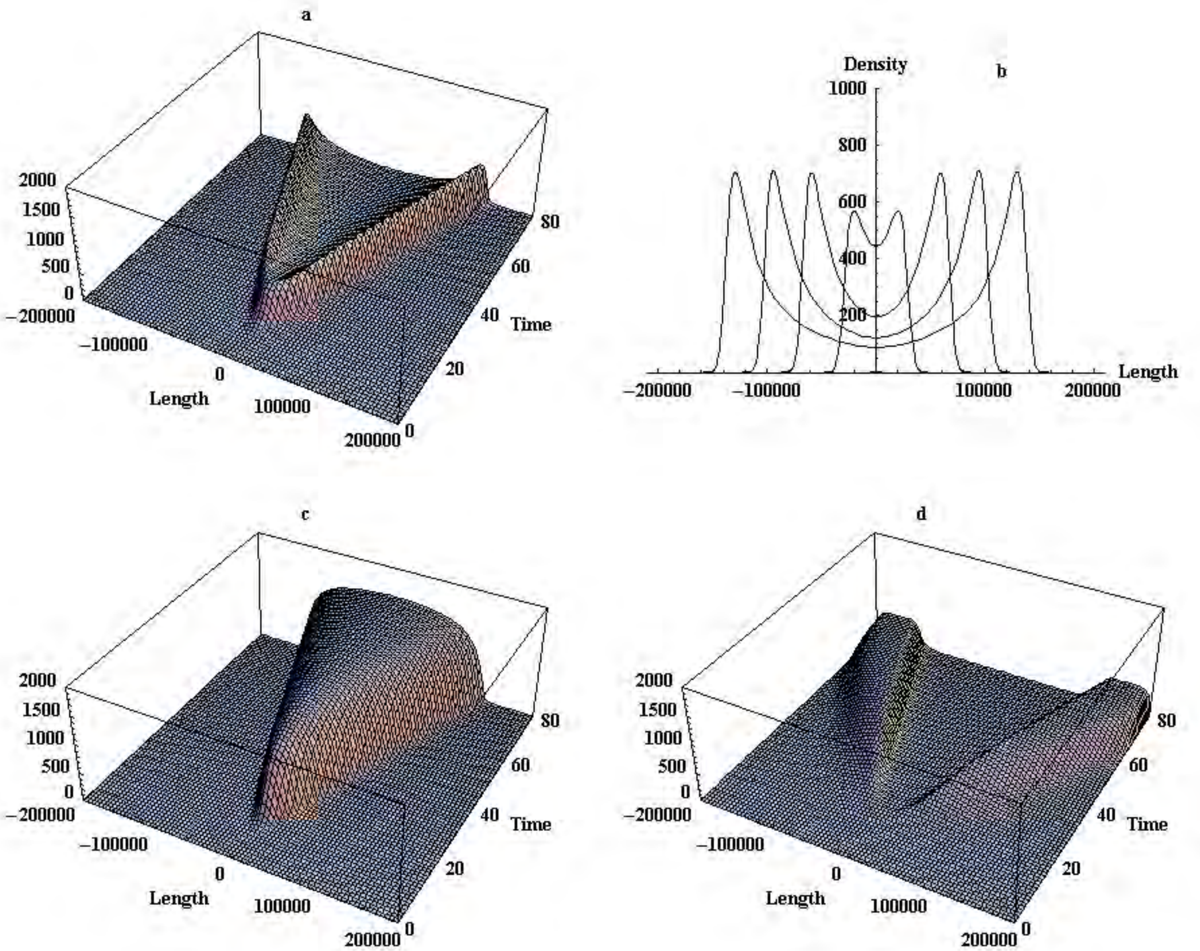
$$f_1\left(\vec{N},T\right) = r_1(T)\frac{1}{N}\left(N_1 + \frac{1}{2}N_2\right)\left(A_1 N_1 + \frac{1}{2}A_2 N_2\right)$$

$$f_2\left(\vec{N},T\right) = r_2(T)\frac{1}{N}\left(N_3 + \frac{1}{2}N_2\right)\left(A_1 N_1 + \frac{1}{2}A_2 N_2\right) + \frac{1}{N}\left(N_1 + \frac{1}{2}N_2\right)A_3 N_3$$

$$f_3\left(\vec{N},T\right) = r_3(T)\frac{1}{N}\left(N_3 + \frac{1}{2}N_2\right)\left(A_3 N_3 + \frac{1}{2}A_2 N_2\right)$$



Propagation of biotypes triggered by temperature rise



Final remarks

Reaction diffusion equations are capable of modelling dispersal of interacting populations or biotypes in dependencs of temperature

A temperature rise is able to trigger invasion of a new species or of genetic information in form of travelling waves

By importing landscape covers from a GIS into a finite element solver environment, simulation of dispersal at large scales is feasible

Thank you for your attention!

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Aquashift

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