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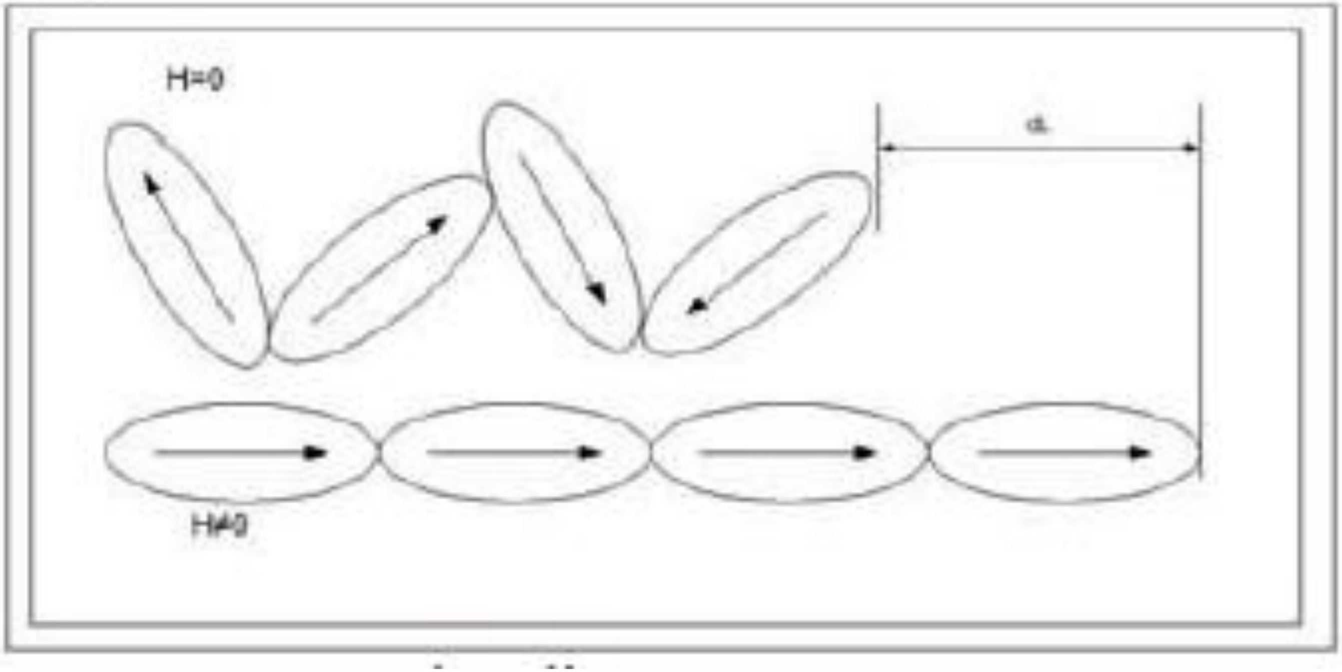
Fracture Toughness Evaluation for magnetostrictive problem using COMSOL-Multiphysics

Presented by
Awani Bhushan
IIT(BHU) Varanasi

Introduction

- **Magnetostriction**: Changes in geometrical shape due to alignment of dipole in a magnetic environment



Introduction

- **Applications:**

- ferromagnetic and magneto-ceramic materials
- Nuclear reactors
- Aerospace
- Electrical motors,
- Transducers etc,
- Electro-magnetic devices

Introduction

Presence of crack

- The state of stress near the crack tip become complex
- Needs to be characterize properly, it will require

  (1) A new integral

  (2) The numerical validation (COMSOL Multiphysics)

Formulation new integral

- Concept of Conservation of Energy is used to derive the integral

Introduction

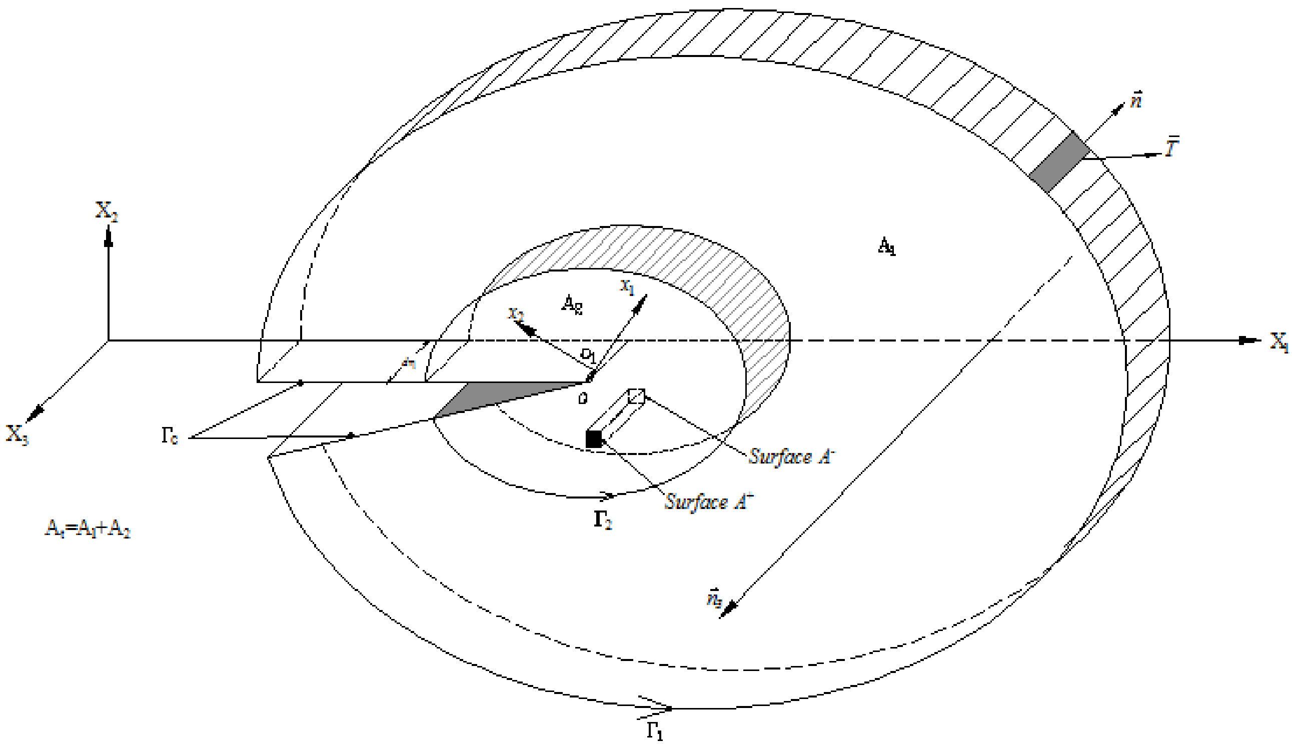
- The Rice's Integral has been modified for magneto-thermo-elastic environment.
- The integral is derived for generalized 3D geometry.

Numerical Validation

- Two physics is used: AC/DC and structural physics
- Rectangular cracked beam is used under magnetic environment
- Integral has been caculated

Integral Formulation

Equation of motion $$\sigma_{ij,j} + F_i = \rho \ddot{u}_i$$



Integral Formulation

Using divergence theorem, re-arranging the terms we get

$$\int_{\Gamma_1+\Gamma_c} T_i \frac{du_i}{dl} d\Gamma + \iint_{A_1} F_i \frac{du_i}{dl} dA = \iint_{A_1} \rho \ddot{u}_i \frac{du_i}{dl} dA + \iint_{A_1} \sigma_{ij} \frac{d\varepsilon_{ij}}{dl} dA + J^u$$

Adding and subtracting the term $\int_{\Gamma_2} T_i \frac{du_i}{dl} d\Gamma$ we get

$$\int_{\Gamma_2} T_i \frac{du_i}{dl} d\Gamma + \int_{-\Gamma_2} T_i \frac{du_i}{dl} d\Gamma + \iint_{A_1} F_i \frac{du_i}{dl} dA = \iint_{A_1} \rho \ddot{u}_i \frac{du_i}{dl} dA + \iint_{A_1} \sigma_{ij} \frac{d\varepsilon_{ij}}{dl} dA + J^u - \int_{\Gamma_1+\Gamma_c} T_i \frac{du_i}{dl} d\Gamma$$

Integral Formulation

$$J^u = \int_{\Gamma_1+\Gamma_c-\Gamma_2} T_i \frac{du_i}{dl} d\Gamma + \int_{\Gamma_2} T_i \frac{du_i}{dl} + \iint_{A_1} \left[\left(F_i - \rho \ddot{u}_i\right) \frac{du_i}{dl} - \left(\sigma_{ij} \frac{d\varepsilon_{ij}}{dl} \right) \right] dA$$

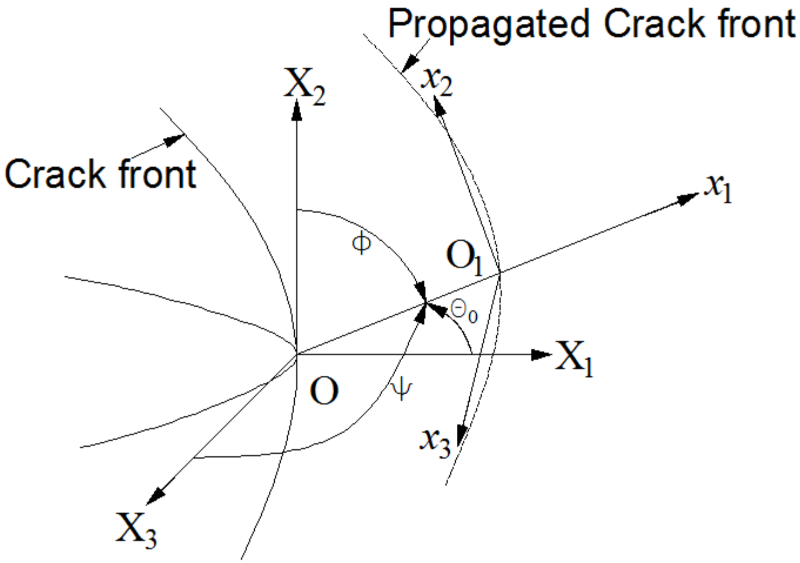
$$J^u = \iint_{A_1} \left(\sigma_{ij} \frac{du_i}{dl}\right)_{,j} dA + \int_{\Gamma_2} T_i \frac{du_i}{dl} d\Gamma + \iint_{A_1} [(F_i - \rho \ddot{u}_i) \frac{du_i}{dl} - (\sigma_{ij} \frac{d\varepsilon_{ij}}{dl})] dA$$

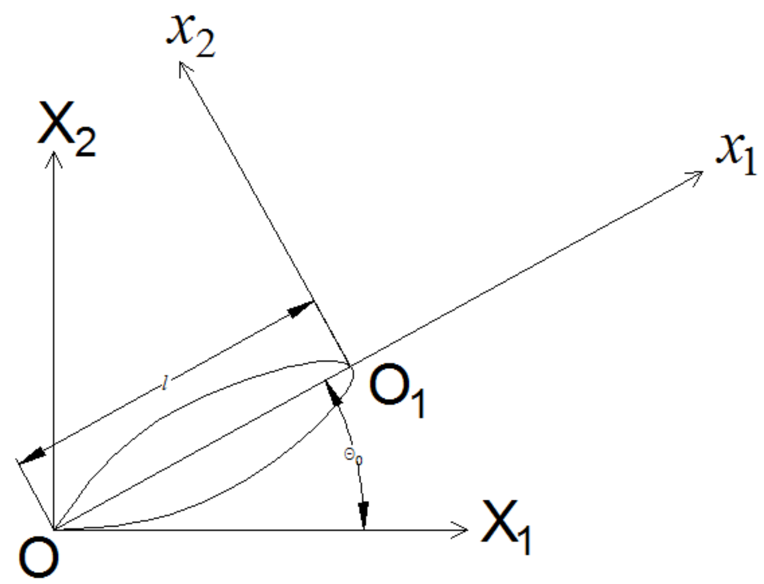
$$J^u = \iint_{A_1} [(\sigma_{ij} \frac{du_i}{dl})_{,j} + (F_i - \rho \ddot{u}_i) \frac{du_i}{dl} - \sigma_{ij} \frac{d\varepsilon_{ij}}{dl}] dA + \int_{\Gamma_2} T_i \frac{du_i}{dl} d\Gamma$$

Terms under parenthesis vanishes and the integral may be written as

Integral Formulation

$$J^u = \int_{\Gamma_2} T_i \frac{du_i}{dl} d\Gamma$$





$$x_1 = X_1 \cos\theta_0 + X_2 \sin\theta_0 - l$$

$$x_2 = -X_1 \sin\theta_0 + X_2 \cos\theta_0$$

Integral Formulation

$$X_1 = x_1 \cos\theta_0 - x_2 \cos\theta_0 + l\cos\theta_0$$

$$X_2 = x_1 \sin\theta_0 + x_2 \cos\theta_0 + l\sin\theta_0$$

$$u_i(X_1, X_2, l) = u_i(x_1\cos\theta_0 + x_2\sin\theta_0 - l, -x_1\sin\theta_0 + x_2\cos\theta_0, l)$$

$$\frac{du_i}{dl} = \frac{\partial u_i}{\partial l} + \frac{\partial u_i}{\partial x_i}\frac{\partial x_i}{\partial l} = \frac{\partial u_i}{\partial l} + \frac{\partial u_i}{\partial x_1}\frac{\partial x_1}{\delta l} + \frac{\partial u_i}{\partial x_2}\frac{\partial x_2}{\partial l}$$

$$\frac{\partial u_i}{\partial x_1}\frac{\partial x_1}{\partial l} = -\frac{\partial u_i}{\partial x_1} \quad ; \quad \frac{\partial x_2}{\partial l} = 0$$

$$\frac{du_i}{dl} = \frac{\partial u_i}{\partial l} - \frac{\partial u_i}{\partial x_1}$$

Integral Formulation

$$J^{u} = \int_{\Gamma_2} T_i \left(\frac{\partial u_i}{\partial l} - \frac{\partial u_i}{\partial x_1}\right) d\Gamma$$

Hence, the fracture process region is assumed to be constant in dimensions and moving along with the same speed as the crack tip, and hence, $\frac{\partial u_i}{\partial l} = 0$ holds in Γ_2

$$J^{u} = -\int_{\Gamma_2} T_i \left(\frac{\partial u_i}{\partial x_1}\right) d\Gamma$$

Integral Formulation

$$\frac{\partial u_i}{\partial x_1} = \cos\theta_0 \frac{\partial u_i}{\partial X_1} + \sin\theta_0 \frac{\partial u_i}{\partial X_2}$$

$$J^u = -\int_{\Gamma_2} T_i \left(\cos\theta_0 \frac{\partial u_i}{\partial X_1} + \sin\theta_0 \frac{\partial u_i}{\partial X_2}\right) d\Gamma$$

Further simplifying we get

$$J_k^u = -\int_{\Gamma_2} T_i \frac{\partial u_i}{\partial X_k} \, d\Gamma$$

Path Independence of Integral

$$\bar{J}_k^u = -\int_{\Gamma_1+\Gamma_c} T_i \frac{\partial u_i}{\partial X_k} d\Gamma + M_k(A)$$

$$\bar{J}_k^u - J_k^u = -\int_{\Gamma_1+\Gamma_c} T_i \frac{\partial u_i}{\partial X_k} d\Gamma + M_k(A) + \int_{\Gamma_2} T_i \frac{\partial u_i}{\partial X_k} d\Gamma$$

$$\bar{J}_k^u - J_k^u = -\int_{\Gamma_1+\Gamma_c-\Gamma_2} T_i \frac{\partial u_i}{\partial X_k} d\Gamma + M_k(A)$$

$$\bar{J}_k^u - J_k^u = -\int_{\Gamma_t} T_i \frac{\partial u_i}{\partial X_k} d\Gamma + M_k(A)$$

Path Independence of Integral

For path independence of integral

$$M_k(A) = \int_{\Gamma_t} T_i \frac{\partial u_i}{\partial X_k} d\Gamma = \iint_A (\sigma_{ij} \frac{\partial u_i}{\partial X_k})_{,j} \, dA$$

Therefore,

$$\bar{J}_k^u = J_k^u = \iint_{A_1} (\sigma_{ij} \frac{\partial u_i}{\partial X_k})_{,j} \, dA - \int_{\Gamma_1 + \Gamma_c} T_i \frac{\partial u_i}{\partial X_k} d\Gamma$$

$$J_k^u = \iint_{A_1} (\rho \ddot{u}_i - F_i) \frac{\partial u_i}{\partial X_k} dA + \iint_{A_1} (\sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial X_k}) \, dA - \int_{\Gamma_1 + \Gamma_c} T_i \frac{\partial u_i}{\partial X_k} \, d\Gamma$$

Path Independence of Integral

$$\iint_{A_1} (\sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial X_k})\, dA = \iint_{A_1} \sigma_{ij} (\frac{\partial \varepsilon_{ij}^{e}}{\partial X_k} + \frac{\partial \varepsilon_{ij}^{m}}{\partial X_k})\, dA$$

Magnetostriction is defined empirically

$$\varepsilon_{ij}^{m} = \frac{3}{2} \frac{\lambda_s}{M_s^2} M^2(t,x)$$

Simplified expression is

$$J_k^u = \int_{\Gamma_1+\Gamma_c} \{ W^e n_k - T_i \frac{\partial u_i}{\partial X_k} \}\, d\Gamma + \iint_{A_1} (\rho \ddot{u}_i - F_i) \frac{\partial u_i}{\partial X_k}\, dA + \iint_{A_1} \frac{3\lambda_s \sigma_{ij}}{M_s^2} M \frac{\partial M}{\partial X_k}\, dA$$

$$J_k^u = (J_k^u)_{2D}$$ for (k = 1, 2)

Path Independence of Integral

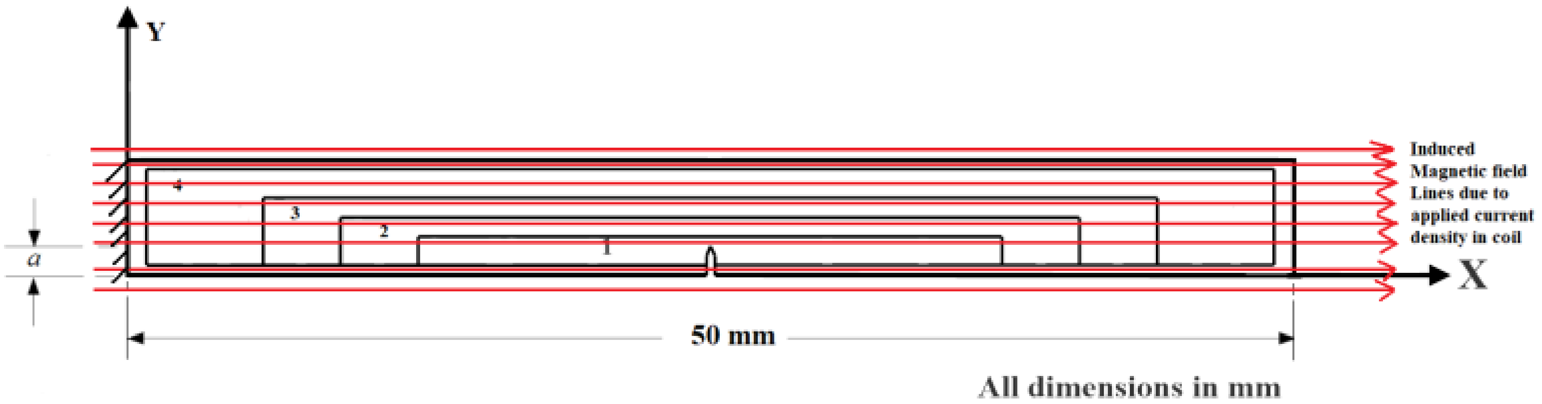
Using contour integral method, the three dimensional integral has been simplified as

$$\left(J_k^u\right)_{3D} = \int_{\Gamma_1+\Gamma_c} \{W^e n_1 - T_i \frac{\partial u_i}{\partial X_1}\} d\Gamma - \iint_{A_t} (\sigma_{i3} u_{i,3})_{,3} \; dA_1 + \iint_{A_t} (\rho \ddot{u}_i - F_i) \frac{\partial u_i}{\partial X_1} dA_1 + \iint_{A_1} \sigma_{ij} \frac{3\lambda_s}{M_s^2} M \frac{\partial M}{\partial X_k} \; dA$$

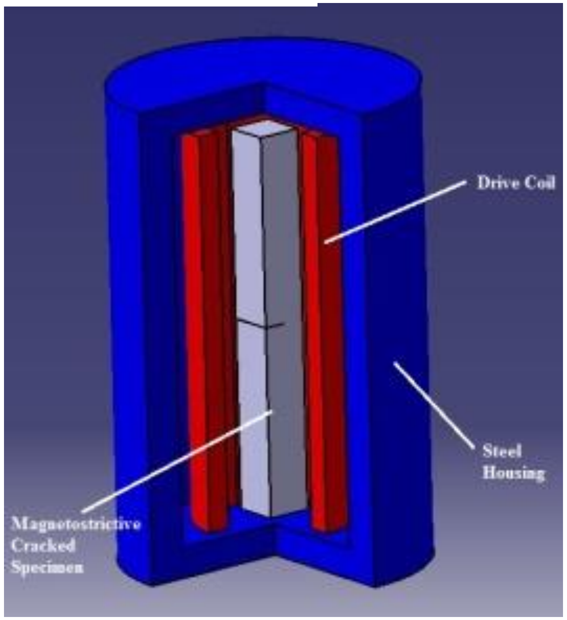
In the absence of body forces and material inertia

$$\left(J_k^u\right)_{3D} = \int_{\Gamma_1+\Gamma_c} \{W^e n_1 - T_i \frac{\partial u_i}{\partial X_1}\} d\Gamma - \iint_{A_t} (\sigma_{i3} u_{i,3})_{,3} \; dA_1 + \iint_{A_1} \sigma_{ij} \frac{3\lambda_s}{M_s^2} M \frac{\partial M}{\partial X_k} \; dA$$

Numerical validation



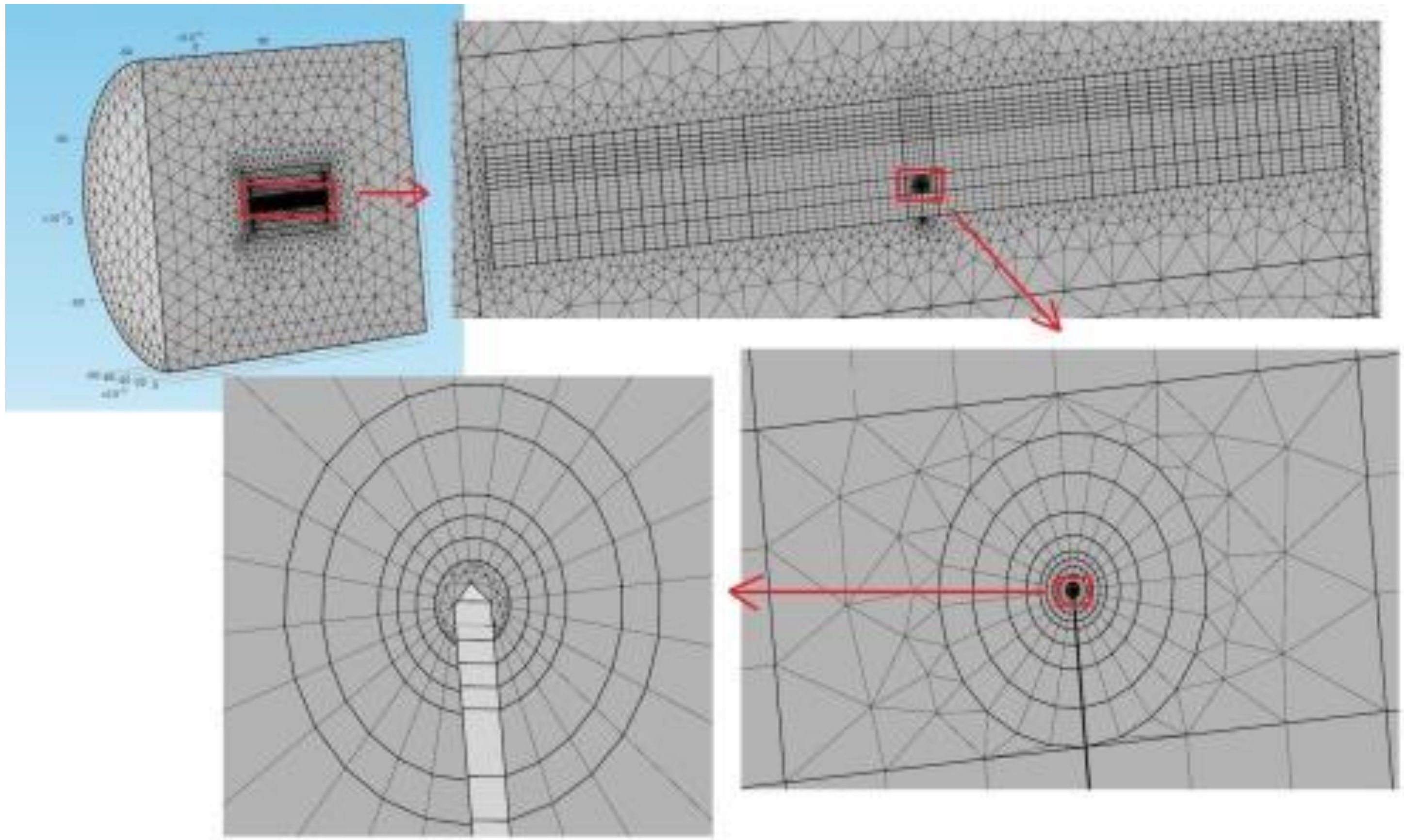
Integration contours



The arrangement of steel housing, helical coil and the cracked specimen (sectional view)

Numerical validation

Model contains 92210 tetrahedral elements and 13296 hexahedral elements



Convergent Meshed model

Numerical validation

Description	Value	Description	Value
Young's modulus (*E*)	60 x 10^9 Pa	Saturated Magnetostriction (λ_s)	2 x 10^{-4}
Density (ρ)	7870 kg/m3	Saturation magnetization (M_s)	15 x 10^5 A/m
Poisson's ratio (ν)	0.3	Effective domain density (a)	7000 A/m

From Magnetostriction Model [1]

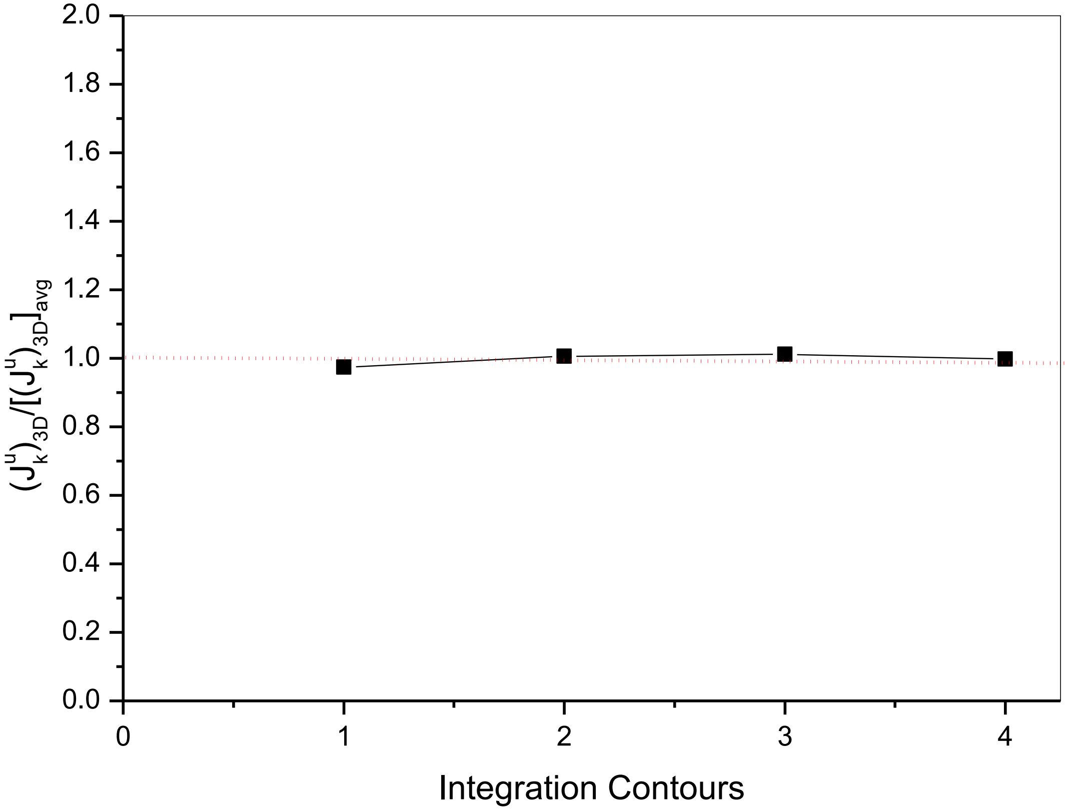
$$\lambda(t,x) = \sum_{i=0}^{\infty} \gamma_i M^{2i}(t,x)$$

The Langevin function [2,3]

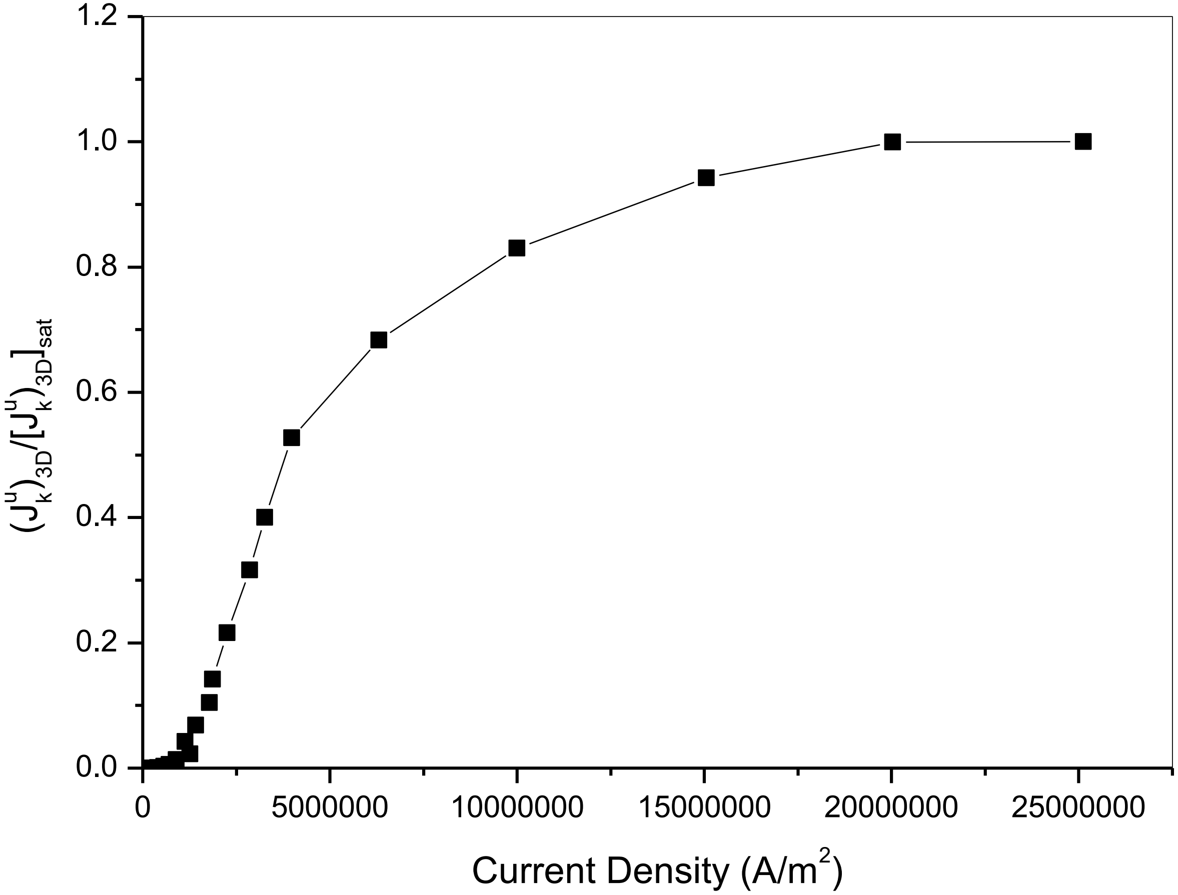
$$\frac{M}{M_s} = \left[\coth\left(\frac{H_e}{a}\right) - \left(\frac{a}{H_e}\right)\right]$$

H_e is the effective magnetic field and a is domain density constant with dimensions of magnetic field.

Numerical validation



Numerical Results



Fracture Toughness

As per ASTM E1921

$$K_{Ic} = \sqrt{\frac{\left(J_{kc}^{u}\right)_{3D} E}{\left(1-\nu^{2}\right)}}$$

- Only magnetic load is not sufficient
- Integral value saturate at saturated Magnetization
- Peak load applied to attain fracture toughness in magnetic field

Conclusion

- A conservation path independent integral $\left(J_k^u\right)_{3D}$ for a straight crack has been proposed to have the physical meaning of energy release rate

- The magnetization saturates after increasing current density.

- The integral $\left(J_k^u\right)_{3D}$ value saturates at saturated magnetization.

References

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[2]F. Liorzou, B. Phelps, D.L. Atherton, Macroscopic models of magnetization, IEEE Transactions on Magnetics. 36 (2000) 418–428. doi:10.1109/20.825802.

[3]M.J. Dapino, R.C. Smith, L.E. Faidley, A.B. Flatau, Coupled structural-magnetic strain and stress model for magnetostrictive transducers, Journal of Intelligent Material Systems and Structures. 11 (2000) 135–152. doi:10.1106/MJ6A-FBP9-9M61-0E1F

[4] Bhushan, A., Panda, S. K., Singh, P. K., Kartheek, P., Kumar, R., and Mittal, Y., 2018, “3D Path Independent Integral for Thermoelastic and Magnetostriction Problem,” Mechanics Research Communications.

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[6]COMSOL-Multiphysics, T., 2017, Nonlinear Magnetostrictive Transducer (Tutorial

Thank you