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Optimum Insulation Thickness Distribution For Heat Loss Uniformity From Heated Corrugated Pipes

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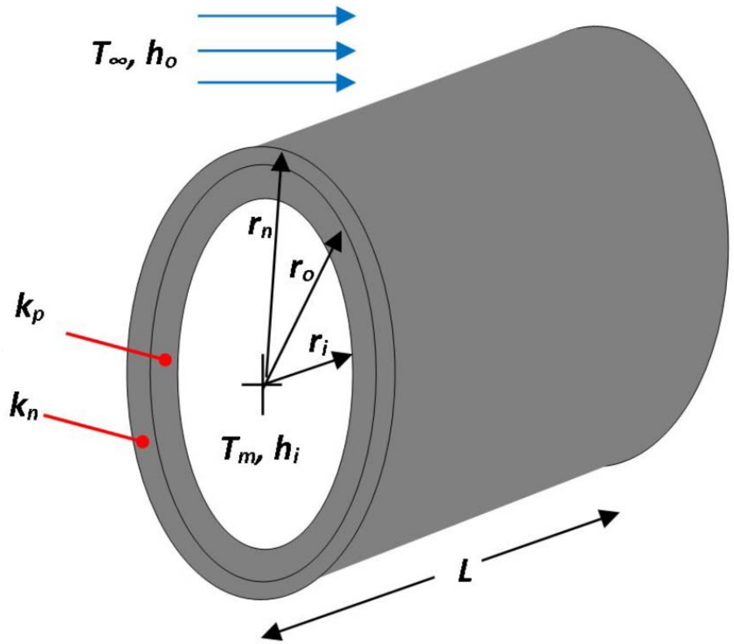
Motivation

Fluid transport in corrugated pipes:

- Underground oil/natural gas
- Buried pipelines
- Water and sewage networks
- Refinery products pipelines
- Power plant steam lines
- Heat exchangers in soil (thermal energy storage)

Problem Definition

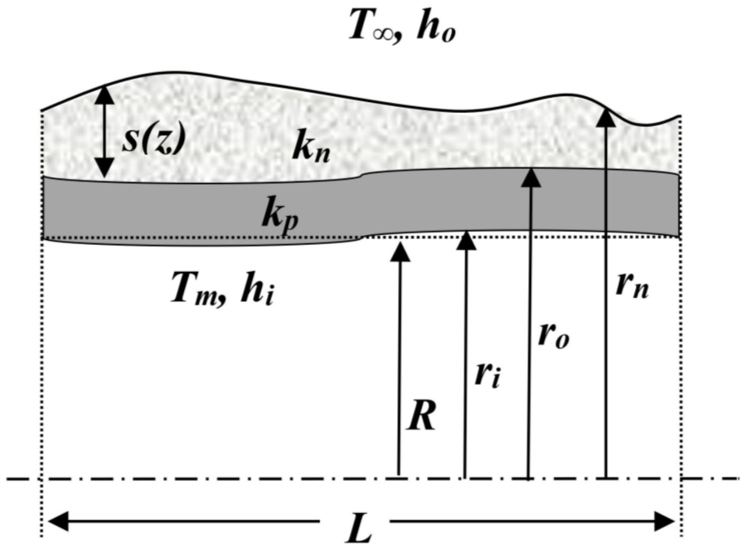


- Pipe… known thickness and length… mean inner radius... Thermal conductivity.
- Placed in environment with known ambient temperature.
- Fluid flowing inside enters with bulk temperature.
- Thermal conductivities of pipe and insulation as well as internal and external heat transfer coefficients are known.
- Radii of the inner pipe surface, outer pipe surface and insulation are given by r_i, r_o and r_n, respectively.
- Insulation thickness is taken to be a function of the pipe axial distance, $s = s(z)$. 🡐 *Subject of the optimization.*

Governing Equations

Heat conduction through the pipe/insulation material is governed by the 2-D, steady, constant-properties, axisymmetric heat equation in cylindrical coordinates,

$$\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial r^2} = 0$$



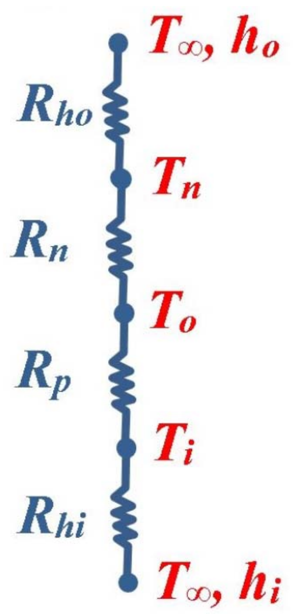
$$r_i \longrightarrow r_i(z) = R + a\sin(2\pi z/L) := P(z)$$

$$r_o \longrightarrow r_o(z) = r_i(z) + t = t + R + a\sin(2\pi z/L) := Q(z)$$

$$r_n \longrightarrow r_n(z) = r_o(z) + s(z) = t + R + a\sin(2\pi z/L) + s(z) = Q(z) + s(z)$$

Numerical Model I

- ▶ **Use thermal resistance paradigm**
- ▶ **Force the radial heat loss to be uniform along the axis**



Radial heat transfer,

$$q_r = \frac{\Delta T}{R_{tot}}$$

Total thermal resistance,

$$R_{tot} = \frac{1}{2\pi L h_i\, r_i(z)} + \frac{\ln\left(\frac{r_o(z)}{r_i(z)}\right)}{2\pi L k_p} + \frac{\ln\left(\frac{r_n(z)}{r_o(z)}\right)}{2\pi L k_n} + \frac{1}{2\pi L h_o\, r_n(z)}$$

Require that,

$$\frac{dq_r}{dz} = \frac{-\Delta T}{R_{tot}^2}\frac{dR_{tot}}{dz}$$

Numerical Model I

Nonlinear 1st order ODE

$$\frac{dq_r}{dz} = \frac{-\Delta T}{R_{tot}^2}\frac{dR_{tot}}{dz}$$

Differentiate and set equal to zero …

$$\frac{dR_{tot}}{dz} = \left[\frac{-F(z)}{H_i P^2(z)}\right] + \left[\frac{-F(z)}{K_p P(z) Q(z)}\right] + \left[\frac{1}{K_n[Q(z)+s(z)]}\frac{ds(z)}{dz} - \frac{F(z)s(z)}{K_n Q(z)[Q(z)+s(z)]}\right] + \left[\frac{-F(z)}{H_o[Q(z)+s(z)]^2} - \frac{1}{H_o[Q(z)+s(z)]^2}\frac{ds(z)}{dz}\right]$$

Each term …

$$\frac{d}{dz}\left[\frac{1}{2\pi L h_i}\frac{1}{r_i(z)}\right] = \frac{1}{2\pi L h_i}\left[\frac{-1}{r_i^2(z)}\right]\frac{d}{dz}\left[r_i(z)\right] = \frac{1}{2\pi L h_i}\left[\frac{-F(z)}{r_i^2(z)}\right]$$

$$\frac{d}{dz}\left[\frac{1}{2\pi L k_p}\ln\left(\frac{r_o(z)}{r_i(z)}\right)\right] = \frac{t}{2\pi L k_p}\left[\frac{-F(z)}{r_i(z) r_o(z)}\right]$$

$$\frac{d}{dz}\left[\frac{1}{2\pi L k_n}\ln\left(\frac{r_n(z)}{r_o(z)}\right)\right] = \frac{1}{2\pi L k_n}\left[\frac{1}{r_n(z)}\frac{ds(z)}{dz} - \frac{F(z)s(z)}{r_o(z) r_n(z)}\right]$$

$$\frac{d}{dz}\left[\frac{1}{2\pi L h_o}\frac{1}{r_n(z)}\right] = \frac{1}{2\pi L h_o}\left[\frac{-F(z)}{r_n^2(z)} - \frac{1}{r_n^2(z)}\frac{ds(z)}{dz}\right]$$

Numerical Model I

Nonlinear 1st order ODE

The resulting ODE … $Ass' + Bs' + Cs^2 + Ds + E = 0$

where …

$$A = H_i K_p P^3 Q$$

$$B = H_i K_p P^3 Q^2 - H_i K_p K_n P^3 Q$$

$$C = -H_o K_p K_n FPQ - H_i H_o K_n FP^2 - H_i K_p FP^3$$

$$D = -2H_o K_p K_n FPQ^2 - 2H_i H_o K_n FP^2 Q - H_i K_p FP^3 Q$$

$$E = -H_i K_p K_n FP^3 Q - H_i H_o K_n FP^2 Q^2 - H_o K_p K_n FPQ^3$$

Rewrite as …

$$\begin{aligned}&(H_o BC - K_n BC + K_n H_o AC^2)\\ &\quad + (K_n H_o AC + 2K_n H_o AC^2 + 2H_o BC - K_n B)s\\ &\qquad + (3K_n H_o AC + H_o B)s^2 + (K_n H_o A)s^3 - K_n Cs' - K_n ss' = 0\end{aligned}$$

Numerical Model I

Nonlinear 1st order ODE

Runge-Kutta … $$s' = f(z, s), \quad s(z_0) = s_0$$

isolating slope … $$s' = \frac{-C(z)s^2 - D(z)s - E(z)}{A(z)s + B(z)}$$

iterate … $$s_{n+1} = s_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

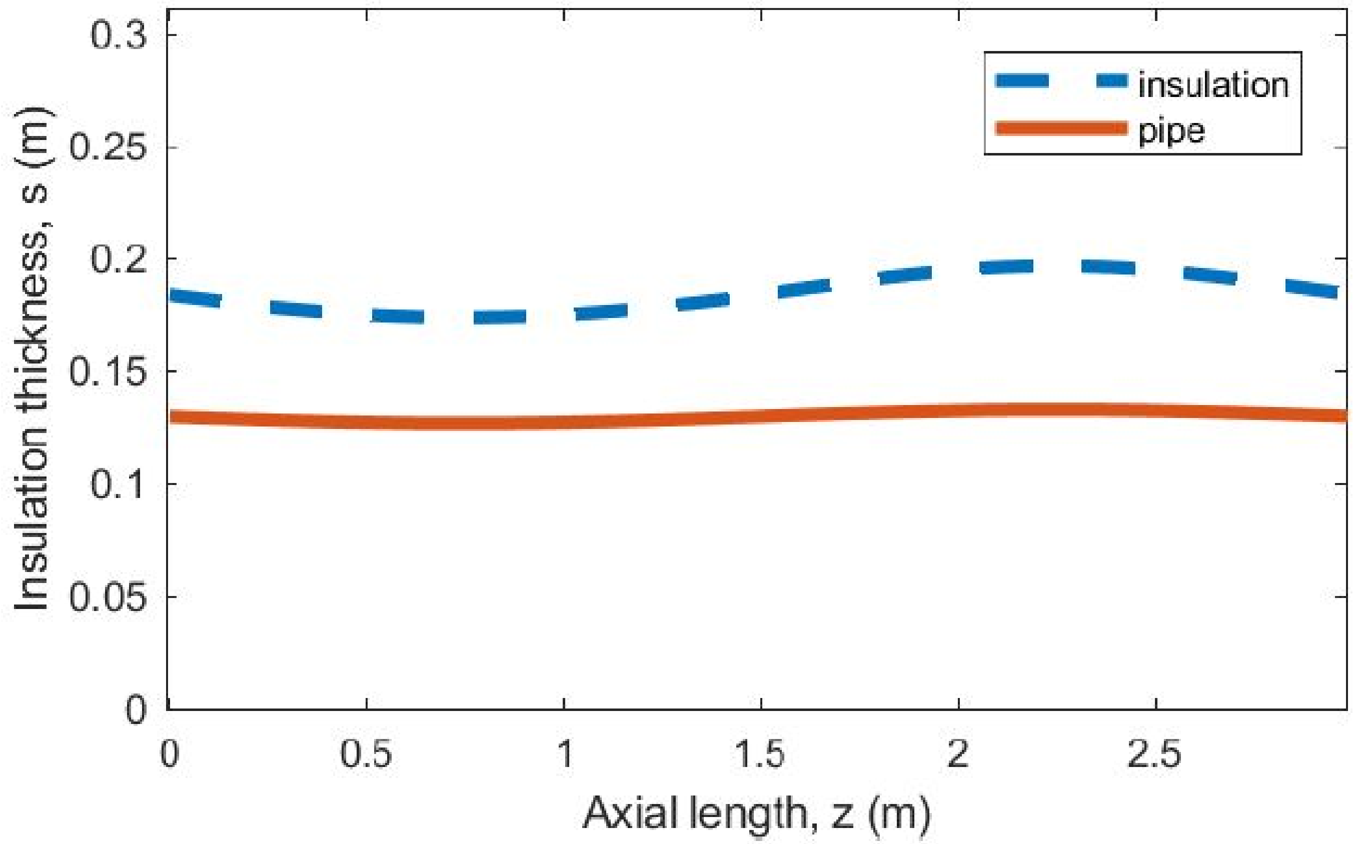
$$z_{n+1} = z_n + dz$$

where k_1, k_2, … , are defined in the usual manner.

Numerical Model Ⅰ Results

Nonlinear 1st order ODE

RK4 Solution

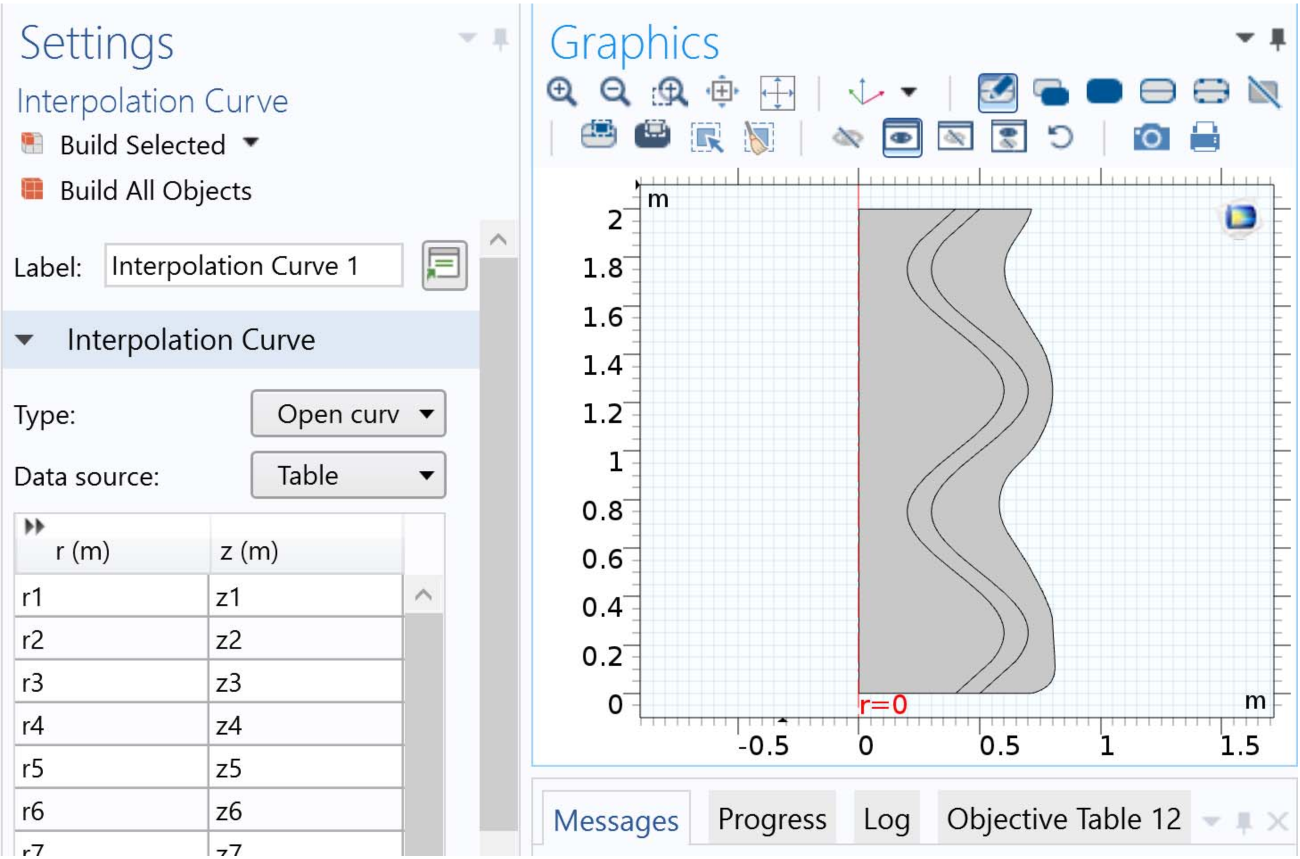


Numerical Model II

COMSOL Optimization Module

a) Series of point connected by straight edges

Parametrize the points representing the radii at east z along the axis.

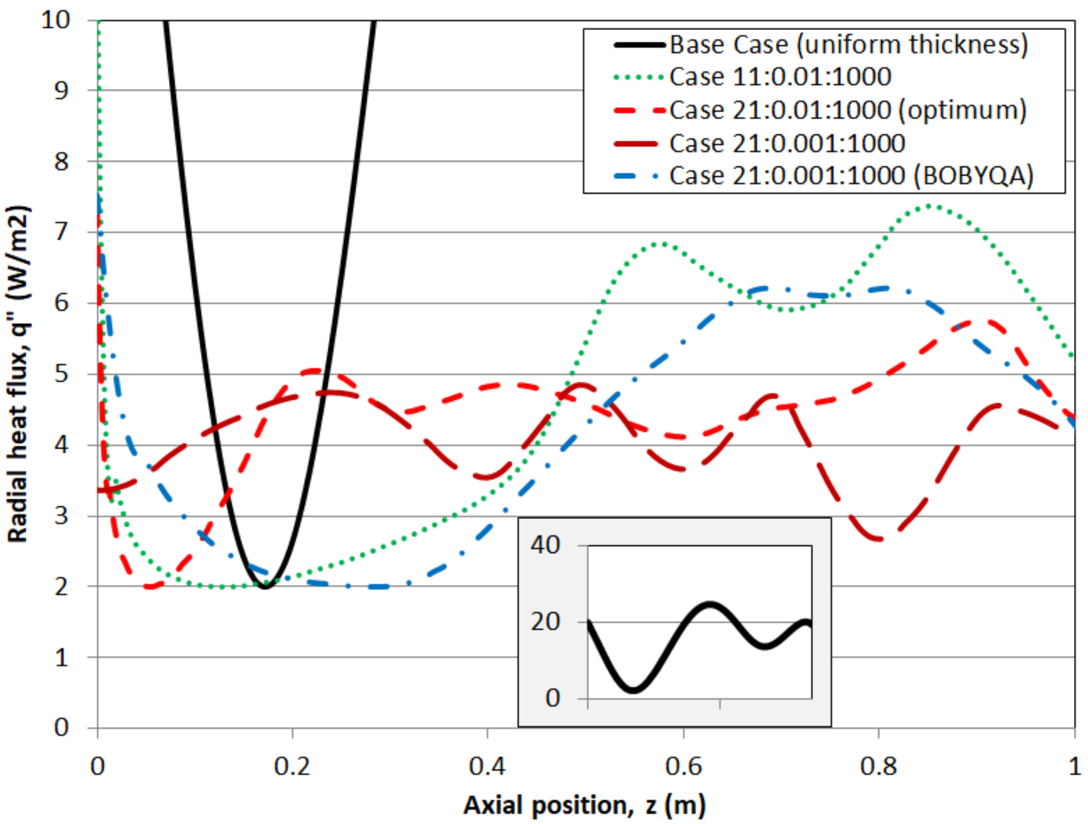


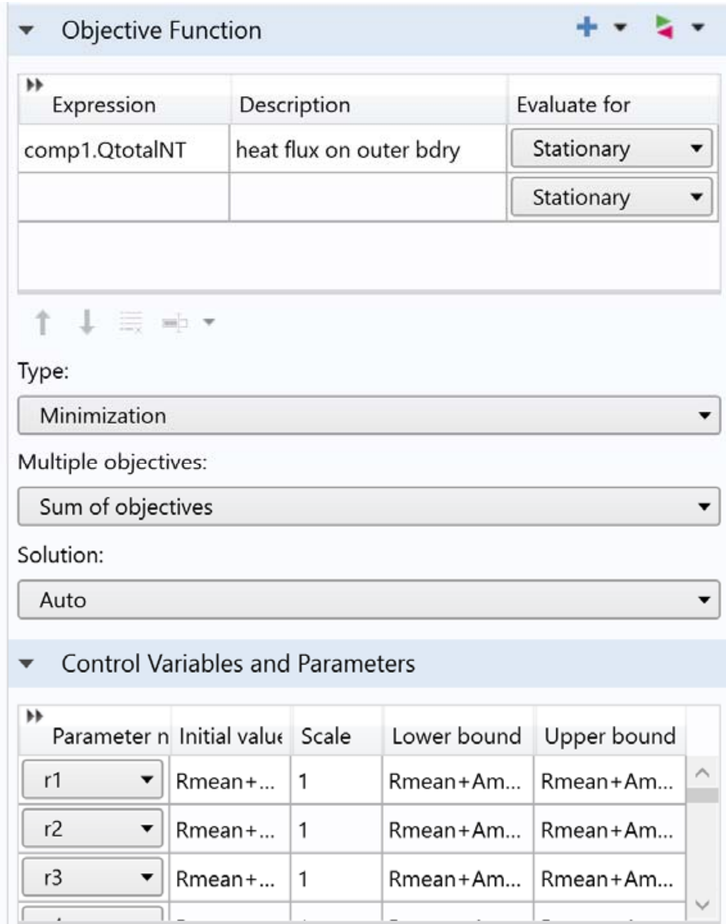
Numerical Model II Results

COMSOL Optimization Module

a) Series of point connected by straight edges

Have the module optimize them.



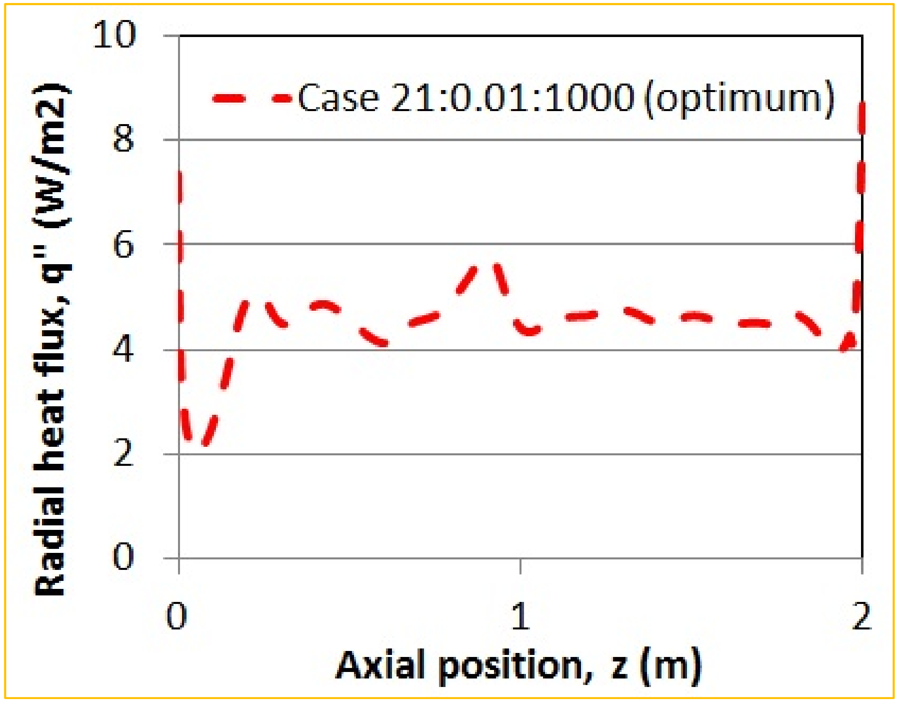


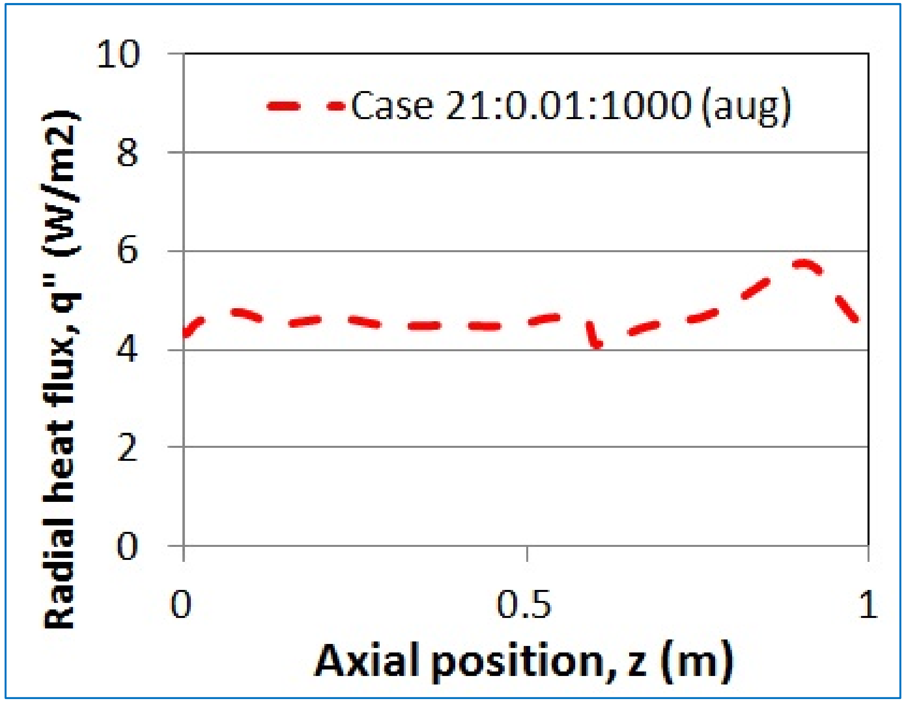
Numerical Model II Results

COMSOL Optimization Module

a) Series of point connected by straight edges

Superimpose two profiles (to avoid end effects!)





Numerical Model II

COMSOL Optimization Module

b) Function to describe insulation outer surface

The outer surface of the pipe was described by ...

$$r_o(z) = r_i(z) + t = t + R + a\sin(2\pi z/L)$$

With some insight, one you propose (for the outer insulation surface) ...

$$R + 2t + t\sin\left(\frac{2\pi z}{L}\right) f_1$$

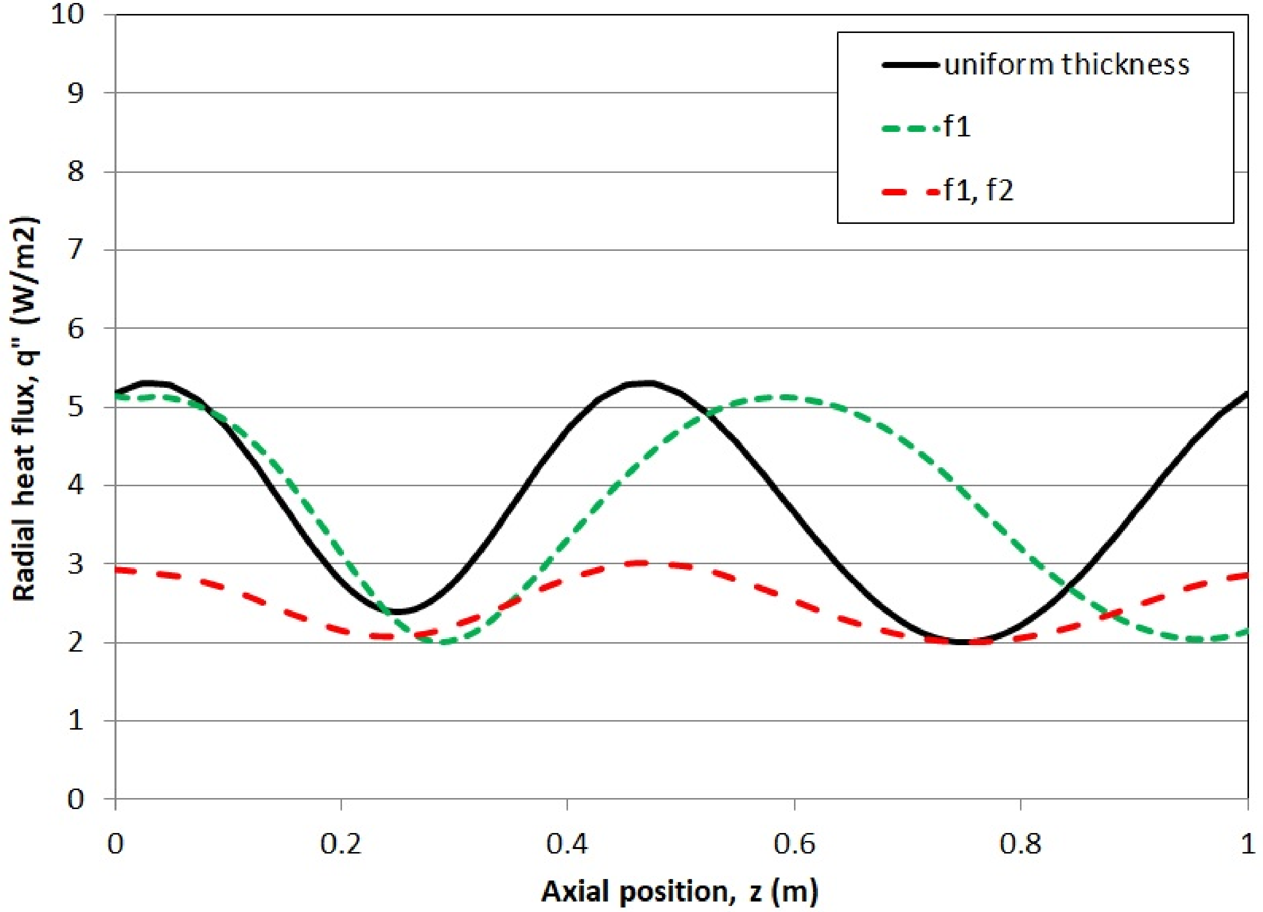
or ...

$$R + 3t + t\left[\sin\left(\frac{2z\pi}{L} + f1\right) - \cos\left(\frac{2\pi z}{L} + f_2\right)\right] f_1$$

Numerical Model II

COMSOL Optimization Module

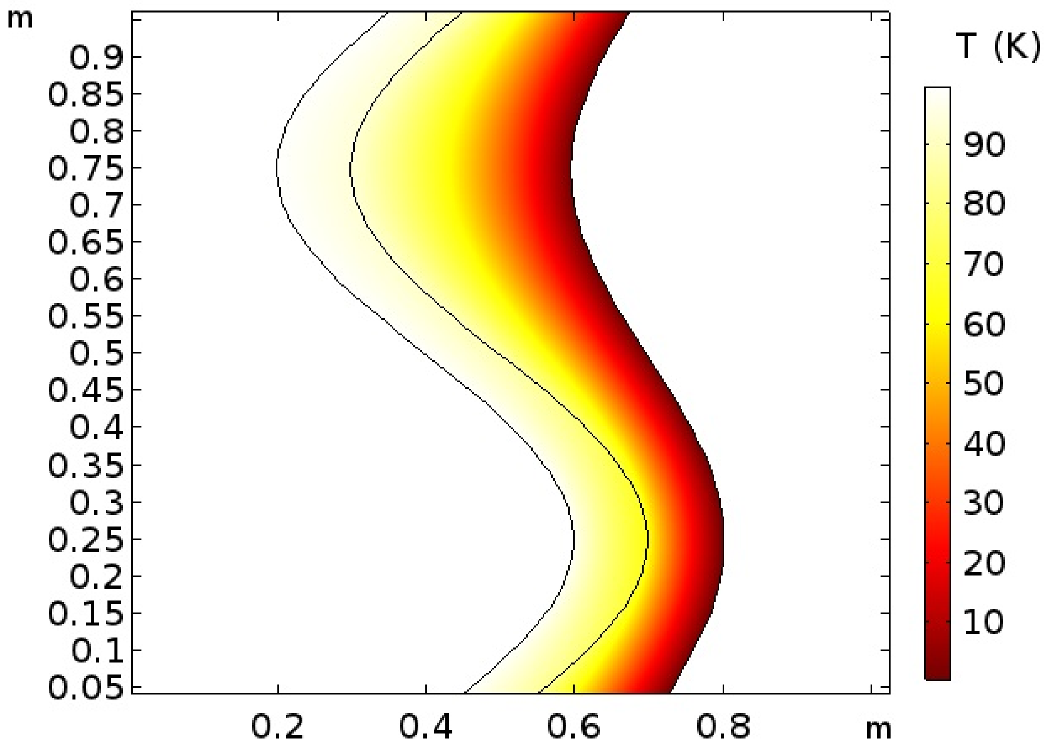
b) Function to describe insulation outer surface

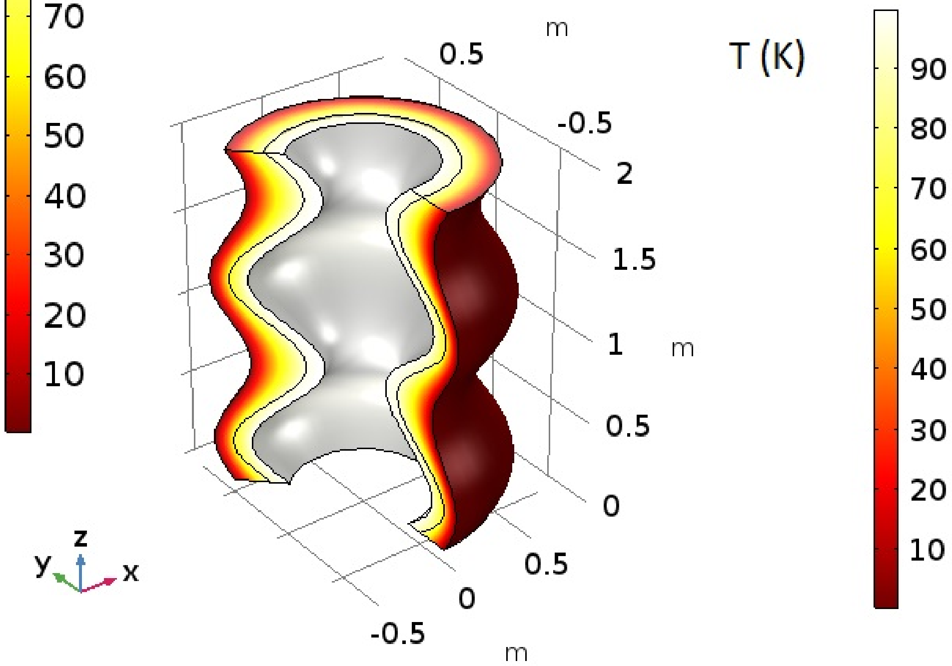


Numerical Model II

COMSOL Optimization Module

Temperature contours





Final Remarks

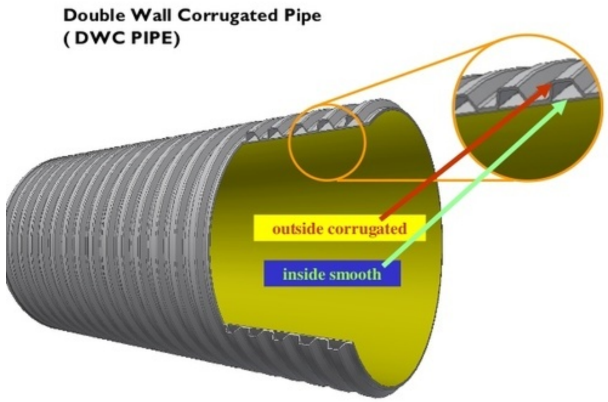
And ongoing work …

Numerical modification …

- Non-dimensionalize everything!
- Splines between (fewer) points
- Graded mesh (axially)
- Try Monte Carlo (and other methods)

Next …

- Add flow, internal and external
- Natural convection
- Daily/seasonal external temperature variation
- Layers of phase-change material



Thanks for listening!