Numerical Study of Loudspeaker Diaphragm Geometries

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Introduction

Objective of this paper is to illustrate the modal performances of the loudspeaker diaphragm geometry subjected to geometry modifications. Geometry modification is carried out using mass distribution which is further explained in the following chapters with governing equation models. This paper will also discuss about the validation of numerical method at start and dynamic responses of those modified geometries in the end.

Modal values that are calculated by the numerical method using COMSOL will be validated using analytical expressions. Modal frequency of a structure is also called as natural frequency, when the structure excited with the natural frequency, the resonance happens. It is usually a frequency that helps in understanding the characteristics of a structure and must be controlled since resonance will lead to higher deflections which is often not an interest.

A 2D plate model will be used in 2D Axi-symmetrical coordinates and this 2D plate is a very general and plain representation of flat diaphragm. The end conditions will be explained later. The geometry is studied by keeping its mass constant so that the change of geometry achieved by distributing the mass in the radial direction

Validation of Numerical Method

The ideology behind the model development in COMSOL is to create a 2D plate model and evaluating the results of the model and this results will be compared with analytical results simulated in MATLAB.

A plate which is having finite length in x and z axis with defined thickness of 0.5mm is modelled in MATLAB following the reference [1]. This analytical results obtained using MATLAB representing the modal values for the unit width in the z direction. The mode shapes are extracted by solution function that that represent sinusoidal waves in each axis. To find modes of a simple rectangular plate the below equation is used based on the reference [1].

$$\omega_n = K_n^2 \sqrt{\left(\frac{B'}{M''}\right)} = \left[\left(\frac{n_1\pi}{L_1}\right)^2 + \left(\frac{n_2\pi}{L_2}\right)^2\right]\sqrt{\left(\frac{B'}{M''}\right)} \quad \dots (1)$$

Where the n_1 and n_2 are nodal lines that splits the rectangular plate into segments and that represents the

sinusoidal wave solution form with crests and troughs. K_n is wave number components on the x and z axis. B' is bending stiffness per unit width and M'' mass per unit area which are expressed in the below equations.

$$m'' = \rho h; B' = \frac{Eh^3}{12(1-\nu^2)} \quad \dots\dots\dots\dots (2)$$

Where E young's modulus of a plate material and 'v' is Poisson ratio. The bending stiffness of a 1D beam is usually defined as B'=EI where, I is mass moment of inertia. I=h^3/12 per unit width. The potential difference noticed between a 1D beam and a 2D plate is the term in the denominator $(1-v^2)$, this value is usually lesser that 1 and increases the bending stiffness for a 2D plate. This lead to the explanation of Poisson ratio which defines the cross contraction of the 2D plate geometry that adds resistance to the plate [1].

The natural frequencies obtained from the COMSOL can be compared with the values obtained using equation 1. This equation will be used with input values and it is assumed that the length in the Z direction is unity for extracting the results in only x axis. The model is used with simply supported end condition and the formulas used above are also representing the modes for a simply supported plate.

COMSOL Model used for Validation

The model used for validation process is created to only validate the numerical method used in COMSOL and the model for the study has different boundary conditions from the model being used for validation. The validation is helps understanding the accuracy of the numerical tool also to what extent the result could be trusted based on the mesh size. It was found from the study that comparing the errors between the analytical mode shapes and the simulation mode shapes were unnecessary since the amplitude in the analytical method is simply arbitrary [5]!

Validation of Modal Results

The below table compares the results of the Eigen values obtained from the simulation and analytical methods. The first mode shown in the table 1 is in good agreement and there is slight deviation observed from the mode 2 in the simulation results yet it is still considered as good correlation. These natural frequencies were obtained without any external excitation and it was also reflected in the mode shapes obtained, which will be discussed in the following sections.

Modes	n_1, n_2 Values	Analytical[Hz]	Simulation[Hz]
1	1,1	32.4	32.6
2	2,2	129.6	130.48
3	3,3	291.7	293.54
4	4,4	518.6	521.74
5	5,5	810.2	815.02

Table 1. Comparison of simulation and analytical results

Validation Mode Shapes

Mode shapes can also be validated using mode shape function derived using the plate equations and the mode shapes obtained by simulation will be compared with analytical solution using the MATLAB. The plate equation that models the plate in our cases is following,

$$D(\frac{\partial^4 W}{\partial x^4} + \frac{\partial^4 W}{\partial z^4}) + Kw = P \quad \ldots\ldots\ldots\ldots (3)$$

This fourth order partial differential equations can be reduced to represent the thin plate model that represents the validation model. Using thin plate assumption from the reference [2] the equation will be reduced as below,

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial z^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots (4)$$

Since Eigen frequency analysis will not be requiring any external load since the free vibration is leads to the natural frequencies and the 2D plate now subjected to pure bending! When the plate becomes very thin and simply supported the longitudinal deflection is negligible. Using method of separation of variable for the above equation 4 that solution function will look like below,

$$W(x, z) = X(x)Z(z) \quad \ldots\ldots\ldots\ldots (5)$$

The boundary conditions to solve the problem are at x=0, L_1, W(x, z) =0 W"=0. At z=0, L_2, W(x, z) =0 W"=0. And the final solution obtained after having used the boundary condition is,

$$W(x, z) = \sin K_x X \sin K_z Z \quad \ldots\ldots\ldots (6)$$

The wave number components K_x and K_z are obtained as below and the final solution function will be,

$$W(x, z) = \sin \frac{n_1 \pi}{L_1} X \sin \frac{n_2 \pi}{L_2} Z \quad \ldots\ldots\ldots (7)$$

$$K_x = K_{n1} = \frac{n_1 \pi}{L_1}; K_z = K_{n2} = \frac{n_2 \pi}{L_2} n_1, n_2 = 1, 2, 3, \ldots \quad \ldots\ldots (8)$$

The modal displacements from the simulations were obtained 100 cut points with equal distance and same number of solution points were analytically solved using MATLAB and hence this mode shapes can be compared. Since the objective is just to validate the mode shape it is enough study one of the mode shape as it is stated earlier the amplitudes were arbitrary and mode shapes obtained from analytical and simulation method can be observed in the below figure 1.
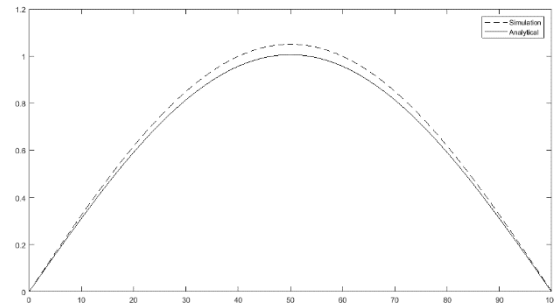

Figure 1. First mode shape Analytical Vs Simulations

Diaphragm Geometry Modification

Before going into the simulation setup, it is important to explain how the geometry modification is performed. The basic idea behind the geometry modification is that to keep the mass of the plate to be constant since the natural frequency of a plate depends on the mass and the stiffness. When the mass is kept constant, changes happening over modal performances can be observed based on stiffness change of structure. The geometry modifications were done by distributing the mass along the radial direction of the diaphragm and this leads a plate that has concentrated mass at some locations and reduced mass or thinner 2D structure at some locations along the radial direction.

Performing such mass distribution along the radial direction need some method that helps is keeping the total mass of the structure to be constant also provides modified geometry. This is achieved by segmenting the geometry into four or more parts and now the volume of the each segments can be controlled independently also dependently to keep the continuity of the structure.

By this way there are six modified structures were created that are having different stiffness values calculated based the boundary conditions that resembles the Loudspeaker's end conditions. These structures were presented below.

The most complicated part of keeping the volume to constant is that in axisymmetric coordinates the cross

section considered for the study is naturally having mass that is distributed more in the outer periphery while in the inner periphery it is less. This can be countered by developing the mathematical models that helps in controlling the mass of each segments also the
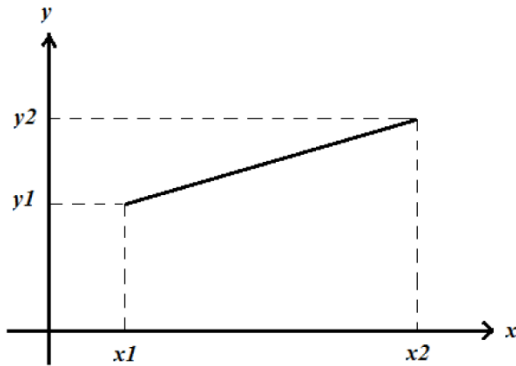
thickness at the both ends of the segments. A line functions is developed as shown in the below figure.

Figure 2. Segmented Geometry –Line function

For the ease of representations boundary curve is developed in XY coordinate and lately incorporated with integrals with polar coordinates. This coordinate system used must be considered as local coordinate, this idea would be helping to control the height of the lines defined above so that we have the heights (thickness at x_1 and x_2) that resulting a particular fixed volume also the volume that resulting in particular heights when one of the heights given. This whole calculations will be controlled using MATLAB. The function representing the line in the above figure 2 will only depend on r not theta since using COMSOL it was decided to simulate the 2D axisymmetric cross section per theta value. Which means the volume calculations were performed for per theta values.

Mathematical equation of a line that has slope 'a' and that intersects y axis at b is represented as 'ax+b' and this is hereafter called as function H(x). This function H(x) can be written at position x_1 and x_2 as follows.

$$H(x_1) = ax_1+b = y_1 \ldots\ldots (9)$$
$$H(x_2) = ax_2+b = y_2 \ldots\ldots (10)$$

Solving above two equations will be resulting the values of 'a' and 'b'. Substituting these values in one of the above equation 9 or 10 will be resulting a

$$a = \frac{(y_1 - y_2)}{(x_1 - x_2)} \quad b = y_1 - x_1\frac{(y_1 - y_2)}{(x_1 - x_2)}$$

function that has variable to control the ends of the segments. This function can further be used as an integrand to find the volume of the segment and the

deduced equations for the height control variables will be used to calculate the height of the segments. The integral equation that will used to calculate the volume of the boundary curve in the figure 2 is,

$$V = \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} f(r,\theta) r dr d\theta \quad \ldots\ldots\ldots (11)$$

In the above double integral the function f(r, ɵ) is formed as below and it does not depend on ɵ.

$$V = \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} ((\frac{(y_1 - y_2)}{(x_1 - x_2)})r + y_1 - x_1\frac{(y_1 - y_2)}{(x_1 - x_2)}) r dr d\theta \quad .. (12)$$

When we want to find one of the height variables y_1 or y_2, the equation has to be written explicitly to the required variable. The deduced equations of height control variables can be written as below.

$$y_2 = y_1 - \left[\frac{\frac{180V}{\pi} - y_1(\frac{r_2^2-r_1^2}{2})}{\frac{r_2^3-r_1^3}{3} + x_1(\frac{r_2^2-r_2^2}{2})}(x_1 - x_2)\right] \quad (13)$$

Similarly y_1 is written as below,

$$y_1 = \frac{\frac{180V}{\pi}(x_1 - x_2) + y_2\left[\frac{r_2^3-r_1^3}{3} + x_1(\frac{r_1^2-r_2^2}{2})\right]}{\frac{r_2^3-r_1^3}{3} + x_1(\frac{r_2^2-r_2^2}{2}) + (x_1 - x_2)(\frac{r_2^2-r_1^2}{2})} \quad .. (14)$$

In the above equations r_1, r_2 and x_1, x_2 values simply defining the position of the segmented elemental length of the circular plate. Where r_1, r_2 comes from the limits of the integral that changes to every segments along the radial direction and difference between x_1, x_2 values are going to be same for all the cases which is 25mm.

The segmentation is illustrated in the below figure that helps in understanding how the mass distribution will be carried out though out the segments. The below geometry gives an idea of how mass distribution could be gradually applied in figures!
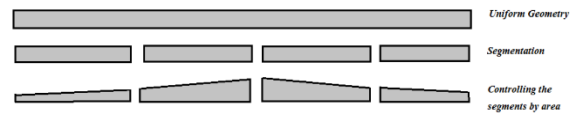

Figure 3. Geometry Modification

COMSOL Model Set-up

Roller Guide
Subjected to External Excitation
Spring Mounted k=5 N/mm

Figure 4. Uniform Geometry with 0.5mm thickness

The above figure represents the case 1 also the end conditions that are going to be followed for all the cases. Roller Guide is applied at the left end in the 2D axisymmetric model since the diaphragm has a coil that moves between the magnet and the rest, this roller allows the motion in only vertical directions and resembles the voice coil motion. And the excitation happens in the same end while the other end will be having spring mounted end that resembles loudspeaker's suspension. The stiffness of the spring is 5 N/mm. The list of different cases that are subjected to mass distribution were listed in the below table 2.

Table 2. Geometry Structure and Stiffness

The above table has list of all the cases that are subjected to Eigen frequency analysis and also to the dynamic analysis. Stiffness values of each structure is calculated using the same boundary conditions specified. 1N force will be applied on the excitation end and maximum displacement will be calculated and hence the stiffness values are calculated using load/deflection formula. Note that case 4 has the highest stiffness value for the specified boundary conditions while the case 6 has least stiffness value. Since concentration mass is at center for the case 4 and

concentration of mass is at the right edge for the case 6. The more the mass moves away from the loading end the lesser stiff it is. And the case 3 and 2 are having mass closer to the loading but facing rolling resistance that restrict the motions resulting in the lesser displacements, hence lesser stiff than case 4.

This paper has little introduction on the influence of the acoustic interaction with the 2D plate and hence there is another model used for the study which is presented below.
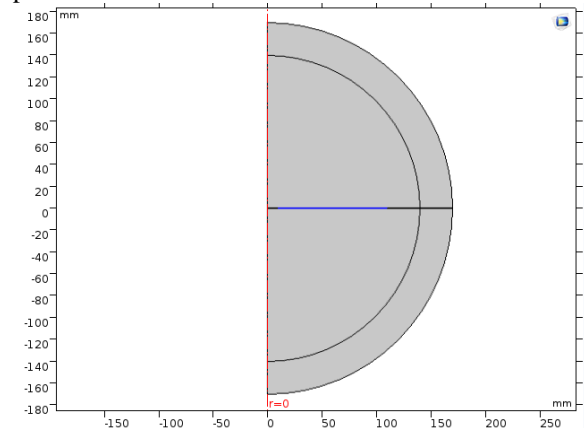

Figure 5. Uniform Geometry with Acoustic Interaction

The same boundary conditions were used in the above figure 5 while the 2D plate is just surrounded by Air now with that the plate is placed in an infinite baffle! The study for this paper is so limited with this model. This model will be helping in comparing the dynamic responses with and without acoustics domain presence.

Simulation Results and Discussion

Results of the Eigen frequency analysis of all the cases will be compared and discussed below. The below figure shows modal performances of all the cases where case 6 stays lowest of all and 4 stays highest of all the cases except the second mode.

Cases(flat)	First Mode [Hz]	Second Mode[Hz]	Third Mode[Hz]	Fourth Mode[Hz]	Fifth Mode [Hz]	Sixth Mode [Hz]
Case 1	14.123	79.486	170.669	292.925	491.429	763.232
Case 2	13.754	86.679	207.931	383.703	623.410	941.675
Case 3	12.225	99.256	202.721	327.553	531.347	822.491
Case 4	17.945	89.407	248.723	364.446	612.664	878.376
Case 5	14.176	79.897	168.514	310.820	515.300	775.983
Case 6	14.111	72.514	151.886	252.332	441.292	706.329
Case 7	14.236	85.951	176.713	338.089	581.227	843.744

Table 3. Modal performances of all cases

From the above table it is possible to notice that the highest first mode is happening for case 4 with 17.945Hz. And the lowest first mode happens for the case 6 with 14.111Hz the mass of the every single case is kept constant while the stiffness of the each cases varies as shown in the table 2 and that results in a deviation of the first fundamental mode from 14Hz to 18Hz.
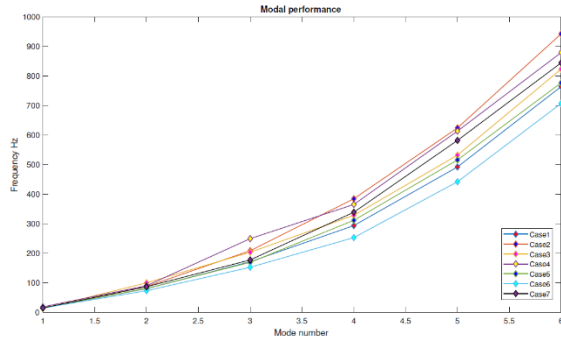
Figure 6. Modal comparison of all cases

From the above figure 6, there was an important modal line crossings were happening between the case 3 and 4. And in order to look at other modal crossings the cases were split into groups for the convenience. The modal performances first group is 2, 3, 4 and 7 is below.
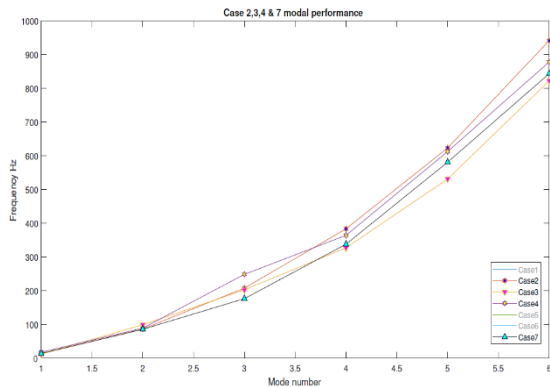

Figure 7. Modal comparison of cases 2,3,4,7

Case 4 second modal value 89.407Hz is lower than case 3's 99.256Hz. The reason could be elaborated using mode shape diagram of those two cases for the particular mode. Since it is a second mode it is expected to have bending of plate in the middle or close the middle. Case 4 has concentrated mass at the center while the case 3 has concentrated mass shared between first two segments also placed in the middle of the first two segments, which means that case 3 has mass that is closer to excitation end. Below is the mode shape diagram of the case 3 and case 4 for the second mode.

From figure 8 notice that case 3 has lesser displacement at excited since the mass is closer to the roller constraint and case 4 starts with higher amplitude and falls at middle due mass concentration at middle. At the right end case 3 gets more amplitude than case 4. For case 4 bending at middle provides more resistance to the displacement while case 3 provides lesser resistance to the displacement and this resulting in a reduced second mode for case 4.
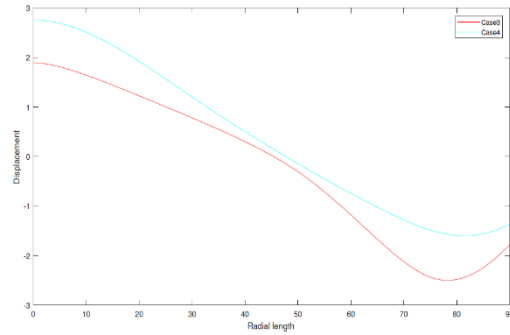

Figure 8. Second mode shape of case 3 and case 4

On the other hand case 3 modal values are falling after mode 3 compared to all the cases. This leads to an important conclusion that '**Concentrated mass closer to the excitation end reduces the higher order amplitude much significantly than lower order modal amplitudes'**. This idea actually builds a though of controlling the modes of particular geometry. This modal study covers only first 6 modes and higher order modes could be even more interesting. Suppressing the lower modes for woofer and middle modes for middle range drivers and higher modes at high amplitude for tweeters could harvest the plan of long term study. This ideology must be comprehensively studied with dynamic responses also with the Acoustical responses. The below figure compares the dynamic responses of the case 1 with and without acoustical domain influence.
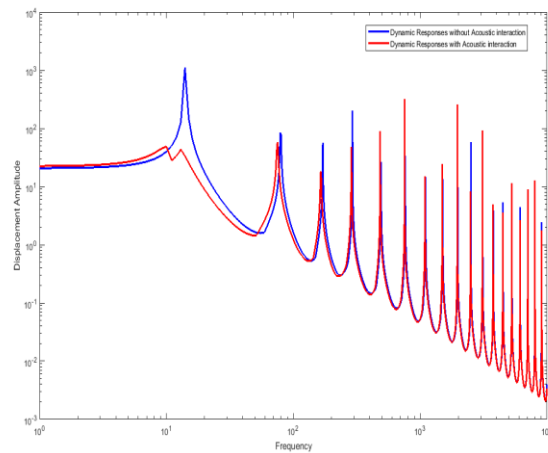

Figure 9. Dynamic Responses

From the above figure 9 it is possible to observe the dynamic response of the case 1 uniform geometry. The blue line is without acoustical interaction while the red curve is representing the influence of the acoustical interaction. Observe that slight fall of modes in the red curve compared to the blue curve which is result of added acoustic mass that provides more resistance to the displacement of the diaphragm. And the first mode is totally suppressed by introduction of acoustic

domain in the model. This study can be further established with other cases and acoustic pressure responses for every cases must be related with modal performances so that the idea of controlling the modes is even more viable in the future.

Conclusions

1. The validation of numerical method giving an idea of how accurate the numerical tool is while comparing with an analytical method.

2. Geometry modification using Mathematical models enables to model the geometry of the loudspeaker diaphragm. At present the this geometry operations were carried out using MATLAB while this could be Optimized so that even efficient geometry with required input parameters can be modeled.

3. The acoustic pressure response at 1m distance for all the different cases will be further related with its characteristic modal responses and results will be deduced. And also the dynamic responses of the each cases could substantiate the idea of controlling the modes!

4. The idea of controlling the modes of the particular geometry or designing the geometry based on the required **operating frequency range** is more viable. Since this is the first step and only first six modes were studied and yet to be related with acoustical performance of the geometry cases.

5. Another important study could be the same geometry subjected to inclinations which will be resulting completely different results since the when it becomes a cone, the stiffness values of the plate drastically changes.

Figure 10. Structure subjected to Inclination with same boundary conditions

The figure 10 is actual representation of a diaphragm in conical shape and results of such cases are very interesting to study in the future!

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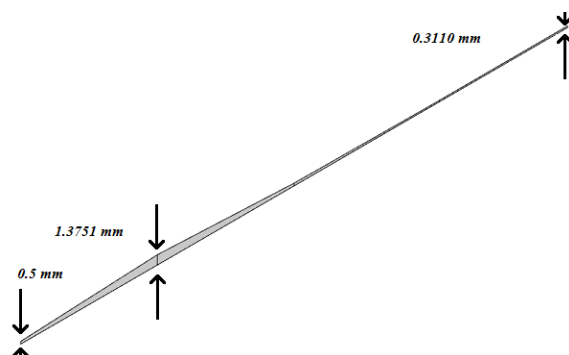5. Numerical study of loudspeaker diaphragm geometries, Arularasan Mageswaran, DTU, Denmark.