Tunable Flat-Plate Absorber Design for Active Sound Absorption

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1 Introduction

Low frequency noise (20-200 Hz) is an issue in many workplaces and indoor environments and can negatively impact individuals' work performance and well-being [1]. In rooms and enclosed spaces, the annoyance of low frequency noise can be exacerbated by strong room resonances leading to highly non-diffuse sound fields and prolonged decay times [2]. Effective means of damping these modes and dissipating the low frequency sound energy are then required to equalize the room response. As opposed to high frequency noise, dissipating low frequency sound energy by passive means is problematic. Porous absorbers are impractical for sound absorption at frequencies towards the lower end of the audible spectrum and require thicknesses on the scale of meters for effective sound absorption. Helmholz resonators and membrane absorbers offer low frequency sound dissipation at a relatively compact size but are limited by their narrow and fixed frequency range.

Active sound absorption is an attractive alternative to passive sound absorption at low frequencies. The concept of an electroacoustic absorber is one interesting example. It consists of a loudspeaker, used as a membrane absorber, the acoustic impedance of which can be modified to target optimal absorption in the low-frequency regime. Using feedback control such absorbers have been shown to be capable of providing broadband sound absorption in a tunable manner [3]. In this paper the design of a tunable flat-plate absorber is introduced. The mechanical system is an elastically suspended plate that is developed to exhibit rigid behavior in the low frequency range. An electrodynamic inertial exciter is attached to this plate and with feedback control, implemented in the same manner as in [3], this system is turned into a tunable electroacoustic absorber. For simplicity, the absorber's performance is evaluated by assuming incident plane waves, i.e. the system is modeled in a one-dimensional waveguide. We present the design and modeling of this electro-mechano-acoustic system, including the feedback control law which allows to turn it into a tunable electroacoustic absorber. The simulated performance of the system is compared to analytical predictions of a lumped model and the performance of a prototype is discussed.

2 Lumped modeling of the system

2.1 Passive system

A schematic of the lumped system configuration is shown in Figure 1. The system subject to the analysis is composed of a suspended plate, an air back cavity and an electrodynamic inertial exciter. We consider the placement of the system in a waveguide excited by a loudspeaker at one end.

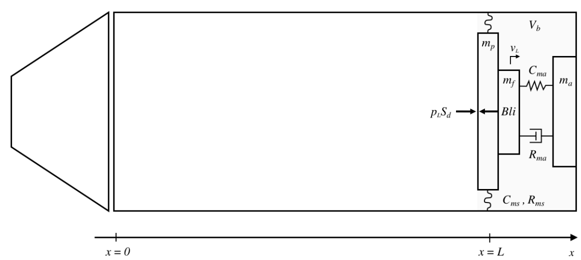

Figure 1: Schematic of lumped system configuration.

The elastic plate is modeled as a rigid piston with mass m_p and effective area S_d. The piston suspension includes a mechanical compliance C_{ms} and a mechanical resistance R_{ms}. The mechanical compliance due to the closed-box air volume V_b is modeled as $C_{mb} = V_b/(\rho c^2 S_d^2)$ where ρ is the density of air and c is the speed of sound in air. The inertial exciter consists of a frame mass m_f rigidly attached to the piston, a proof mass m_a rigidly attached to the rear end of the cabinet and a voice coil with a force factor Bl. The total moving mass of the system is then $m_{tot} = m_p + m_f$ and the total mechanical resistance and total compliance of the system are $R_{tot} = R_{ms} + R_{ma}$ and $C_{tot} = (1/C_{ms} + 1/C_{ma} + 1/C_{mb})^{-1}$ respectively. The total mechanical impedance of the moving mass, relating the force on the piston and its velocity, can be derived as,

$$Z_m = R_{tot} + j\left(\omega m_{tot} - \frac{1}{\omega C_{tot}}\right) \tag{1}$$

2.2 Active system

In the case of active absorption a force is exerted on the piston so that its response is modified and its specific acoustic impedance is closer to a target value. The equation of motion of the moving mass can be derived from Newton's second law of motion. In the frequency domain it writes,

$$Z_m \hat{v}_L = \hat{p}_L S_d - Bl\hat{\imath} \tag{2}$$

which after some manipulation yields,

$$\frac{\hat{\imath}}{\hat{p}_L} = \frac{1}{Bl}\left(S_d - \frac{Z_m}{Z_s}\right) \tag{3}$$

where $Z_s(\omega) = \hat{p}_L(\omega)/\hat{v}_L(\omega)$ is the specific acoustic impedance of the controlled system. We now define the transfer function $\Theta(\omega) = \hat{\imath}(\omega)/\hat{p}_L(\omega)$. Measuring the total pressure near the piston surface we can use this transfer function to determine the electrical current required to a achieve some target specific acoustic impedance $Z_{st}(\omega)$,

$$\Theta = \frac{\hat{\imath}}{\hat{p}_L} = \frac{1}{Bl}\left(S_d - \frac{Z_m}{Z_{st}}\right) \tag{4}$$

For an electroacoustic absorber placed in a waveguide, perfect absorption is achieved when the piston's specific acoustic impedance equals the characteristic impedance of air, i.e. $Z_{st} = \rho c$. The characteristic impedance of air is a real number whereas, however, the specific acoustic impedance is generally not due to the presence of mass and compliance. In practice, it will be impossible to completely cancel the effects of the mass and the compliance. To account for this we define the target specific acoustic impedance in the following way,

$$Z_{st} = R_{st} + j\left(\mu_1 \frac{\omega m_{tot}}{S_d} - \mu_2 \frac{1}{S_d \omega C_{tot}}\right) \tag{5}$$

where μ_1 and μ_2 are positive real coefficients that decrease (or increase) the effective mass $\mu_1 m_{tot}$ and the effective stiffness μ_2/C_{tot}. By using different μ_1 and μ_2 factors it is possible to change the ratio of the effective mass and the effective stiffness and thus shift the resonance frequency of the active system from the resonance of the passive system [3]. In practice, the choice of μ_1, μ_2 and R_{st}, will determine the stability of the controlled system.

2.3 Sound absorption in a waveguide

At low frequencies we can assume plane waves within the duct. The pressure within the duct then follows the one-dimensional wave equation. Transforming to the frequency domain yields the Helmholtz equation,

$$\left(\frac{d^2}{dx^2} + k^2\right)\hat{p}(x,\omega) = 0 \tag{6}$$

We consider a loudspeaker located at $x = 0$, represented by a velocity boundary condition. The loudspeaker is a sound source providing low frequency sound energy to be dissipated by the absorber. The absorber itself is located at $x = L$ and is represented by an impedance boundary condition. Given (6) and these boundary conditions, the expression for the complex pressure within the duct can be derived as,

$$\hat{p}(x) = \rho c \hat{v}_0 \frac{e^{+jk(L-x)} + |R|e^{-jk(L-x)}}{e^{+jkL} - |R|e^{-jkL}} \tag{7}$$

where the reflection coefficient R is the complex ratio between the reflected and incident pressures [4]. It's magnitude is given by,

$$|R| = \frac{Z_s - \rho c}{Z_s + \rho c} \tag{8}$$

The absorber's absorption coefficient, the ratio of the acoustic power that is absorbed by the absorber and the acoustic power that is incident to it, can be derived as,

$$\alpha = 1 - |R|^2 \tag{9}$$

We define effective sound absorption as being the sound absorption resulting in sound energy in front of the absorber that is less than twice the corresponding sound energy for a perfect absorber. The minimum absorption coefficient for effective sound absorption can then be shown to be,

$$\alpha = 1 - (\sqrt{2} - 1)^2 \approx 0.83 \tag{10}$$

2.4 System design

The lumped system parameters are chosen so that the absorber resonates at around $f_0 = 100$ Hz and is capable of achieving effective sound absorption in the frequency range of 50–200 Hz by tunable means. The system is designed to be as compact as possible, which constrains the closed-box volume. This will in turn increase the total compliance requiring a plate of relatively large mass for obtaining resonance around $f_0 = 100$ Hz. The

Table 1: Lumped system parameters. Properties of the inertial exciter correspond to the HIAX25C10-8/HS by HiWave.

Parameter	Symbol	Value	Unit
Closed-box volume	V_b	9.8	L
Effective piston area	S_d	491	cm^2
Piston mass	m_p	114	g
Proof mass	m_a	84	g
Frame mass	m_f	1	g
Compliance of air	C_{mb}	$2.77{\cdot}10^{-5}$	m/N
Compliance of surround	C_{ms}	$1.18{\cdot}10^{-4}$	m/N
Compliance of exciter	C_{ma}	$6.30{\cdot}10^{-4}$	m/N
Resistance of surround	R_{ms}	2.00	N·s/m
Resistance of exciter	R_{ma}	0.06	N·s/m
Force factor	Bl	4.28	N/A
Density of air	ρ	1.21	kg/m^3
Speed of sound in air	c	343	m/s
Resonance frequency	f_0	100.8	Hz

lumped system parameters are listed in Table 1. An important assumption in the lumped modeling is that the plate behaves like a rigid object. If bending modes of the circular plate are induced the simplistic approach of modeling the system like a rigid object is no longer adequate and more sophisticated methods of modeling and control are required. In order to avoid this predicament the circular plate is designed to exhibit rigid behavior at low frequencies. For a circular plate of homogeneous and isotropic material with design constraints of fixed area and mass, the following scaling can be derived for the resonance frequencies of the plate,

$$f_r \sim \sqrt{\frac{E}{\rho^3(1-\nu^2)}} \tag{11}$$

indicating that the plate density is the critical material property. We have identified balsa wood as an interesting candidate due to its low density and furthermore, low cost. Due to the highly anisotropic material behavior of wood, further investigation of the suitability of balsa for such an application is carried out numerically.

3 Numerical modeling

3.1 Model setup

Multiphysics simulations are carried out coupling two acoustical domains with a structural domain. The COMSOL model is illustrated in Figure 2. This model is created as a replica of an experimental setup for testing a prototype of the system. A velocity boundary condition is applied to one end of the waveguide and serves the role of a loudspeaker providing sound energy to be dissipated. The waveguide is modeled as a perfectly rigid structure using sound hard boundaries, as is the cabinet of the resonator. The plate and the surround suspension are implemented as shells.

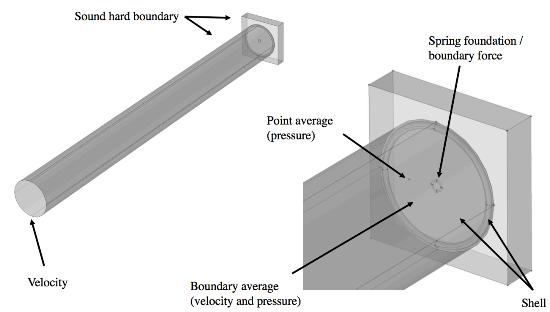

Figure 2: COMSOL model. Two acoustic domains are coupled with a structural domain.

The exciter's compliance and resistance are modeled with a spring foundation boundary condition. In the case of control, a force $Bl\hat{\imath}(\omega)$ is applied on the exciter boundary. Pressure is monitored at a point at a distance of 1 cm from the plate, mimicking the conditions that would typically be realized in physical measurements. From the monitored values of the pressure the control current is determined through the transfer function Θ in (4). Table 2 lists the expressions for these boundary conditions. The mechanical impedance of the passive system, Z_m, is extracted by curve fitting the response of the passive system and the target specific acoustic impedance, Z_{st}, is calculated through (5). Both pressure and velocity are monitored with an average of properties over the plate area allowing for an evaluation of the specific acoustic impedance of the system, from which the absorption coefficient is calculated via (8) and (9). As the shell is adjacent to two acoustic domains, the boundary over which properties are evaluated is not uniquely defined. To circumvent this problem the side operator is utilized. This operator takes as input the number of the domain on which side one wants to determine properties.

Table 2: Expressions for the stiffness, damping and control force, applied on the exciter's boundary.

Stiffness	$1/C_{ma}$
Damping	R_{ma}
Force	$\hat{p}\left(S_d - \frac{Z_m}{Z_{st}}\right)$

A cross-plied balsa plate consisting of two 8 mm plies is considered. The balsa is approximated as being an orthotropic material having different properties along and across the grain. The material properties are listed in Table 3. A cross-plied plate has been chosen over a single-ply one to counteract the anisotropy of the wood which would otherwise result in lowered resonance frequencies of the plate. The plate is modeled as a two layer shell in COMSOL. For one shell the material properties are defined with respect to the global reference axes. For the other shell, the properties are defined with respect to reference axes that have been rotated 90° about one of the global axes, perpendicular to the plate. The circular plate is suspended from the cabinet with a foam surround suspension in a similar manner as in the COMSOL loudspeaker driver tutorial [5].

Table 3: Orthotropic elastic constants corresponding to balsa wood of $\rho = 150$ kg/m^3.

	[GPa]		[–]
E_L	3.1	ν_{LT}	0.375
E_T	0.1	ν_T	0.030
G_{LT}	0.230		
G_T	0.012		

3.2 Results and discussion

An eigenfrequency study has been carried out to investigate the modal behavior of the system. In Figure 3 one can see the piston mode of the system corresponding to resonance frequency f_0 of the resonator. In Figure 4 one of the rocking modes present in the system is shown. These modes, which occur at low frequencies, are problematic to the system's performance and stability and indicate that preventive measures against rocking motion are required. Note, however, that although these rocking modes appear in the modal analysis of the system their effect is underestimated in the frequency response as they are not excited due to perfectly symmetric geometry and loading conditions. Figure 5 illustrates a coupled acoustic and structural mode. The system exhibits strong coupling between the physical domains with bending modes of the plate occurring at frequencies far below what is predicted by the uncoupled physics alone.

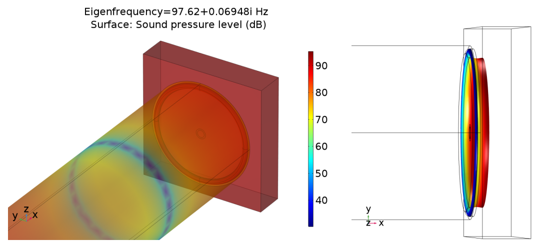

Figure 3: Piston mode.

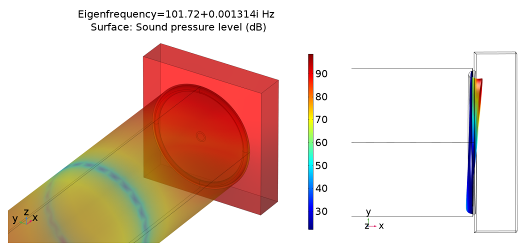

Figure 4: Rocking mode.

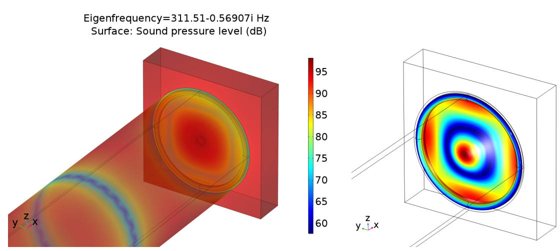

Figure 5: Coupled mode.

A frequency domain study has been carried out for the control configurations listed in Table 4. In Figure 6, simulated absorption coefficients are shown and compared to predicted values by the lumped model. The more broadband-like behavior of the numerical model can be attributed to the effective piston area being defined more realistically in the COMSOL model where it includes a portion of the projected area of the surround. At higher frequencies the numerical model deviates significantly from the lumped model, this behavior being associated with bending of the plate. As mentioned before, rocking modes are not excited due to perfect symmetry. As perfect symmetry is never the case in reality, it would be an improvement to include small non-symmetries in the geometry and loading conditions to see their effect on the frequency response of the system.

Table 4: Control configurations.

	R_{st}	μ_1	μ_2
A	ρc	0.15	0.15
B	ρc	0.55	0.55
C	ρc	1.00	0.30

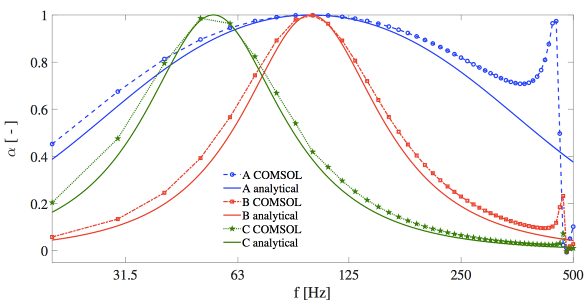

Figure 6: Calculated absorption coefficient for configurations A, B and C.

4 Assessment of a prototype

A prototype of the absorber has been built and tested to validate the concept. The prototype was built using a carbon fiber sandwich which renders it somewhat incomparable with the numerical model. Experimentally obtained absorption coefficients for different control configurations are shown in Figure 7. Overall, the experimentally obtained absorption coefficients are in good agreement predictions by the lumped model, with some noticeable deviations. The large deviation from the lumped model occurring at low frequencies is considered to be attributed to a rocking mode. Deviations at higher frequencies could be associated with other rocking modes and/or bending modes or measurement noise. The measurements show that the system is tunable and indicate that it can reach effective absorption in the frequency range of 37-224 Hz.

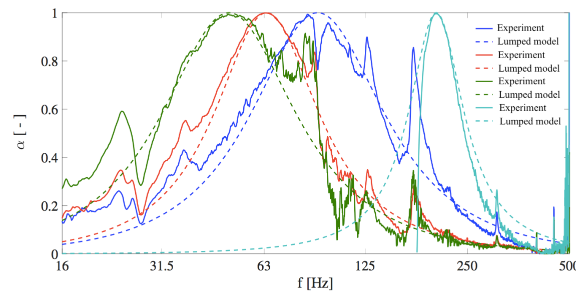

Figure 7: Experimentally obtained absorption coefficients of a prototype, for different control configurations. The corresponding predictions by the lumped model are shown.

5 Conclusion

The design and modeling of a new type of electroacoustic absorber using an inertial exciter as a driving unit has been covered. We have presented the design and modeling of this electro-mechano-acoustic system, including the feedback control law which allows to turn it into a tunable electroacoustic absorber. The COMSOL model of the system suggests deviations above 200 Hz from an idealized lumped model behavior due to bending of the plate. The numerical model fails to take into account the effect of rocking modes on the frequency response of the system due to perfect symmetry in geometry and boundary conditions. An experimental assessment of a prototype suggests that rocking modes affect the system performance. The prototype is still capable of delivering effective absorption in the frequency range of 37-224 Hz by tunable means.

References

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