COMSOL Modeling and Tensile Loading of Aluminum Material Test Samples

Joseph L. Palladino, Ph.D.
Department of Engineering, Trinity College, Hartford, CT 06106, USA
joseph.palladino@trincoll.edu

Abstract— **COMSOL Multiphysics was used to model "dog bone" aluminum material test samples for tensile loading. A 3D linear solid model was studied to quantify axial and transverse strains under axial tensile loading conditions for both 6061 and 7075 aluminum alloys. General purpose Constantan alloy strain gages were installed in both axial and transverse directions at the midpoint of the sample test section. Axial and transverse strain was measured for applied loads ranging from 0–2000 lb for 6061 aluminum and 0–4000 lb for 7075 aluminum, applied as a tensile load with an Instron material test machine. Model strains were compared to measured strains and were found to agree within $\pm 2\%$. The elastic modulus was calculated for each test sample by linear regression of the axial stress and strain, and the Poisson ratio by linear regression of the axial and transverse strains, which were within 2% and 3%, respectively, of the model parameter values.**

Keywords: COMSOL Multiphysics, solid mechanics, material testing, strain gage, elastic modulus, Poisson ratio, Saint-Venant's principle.

I. Introduction

MATERIAL test samples are commonly subjected to tensile testing, with an extensometer used to accurately measure changes in length of the material to determine strain. Lacking an extensometer, stain gages were used to measure axial and transverse strain, with the location and magnitudes of strain guided by a COMSOL finite element model. Test samples were loaded with an Instron tensile test machine (model 5500R), and model strains were validated by comparison with measured strains in both the axial and transverse directions for different aluminum alloys.

II. Methods

1) Equations: Modeling the material test sample requires three equations: an equilibrium balance, a constitutive relation relating stress and strain, and a kinematic relation relating displacement to strain. Newton's second law serves as the equilibrium equation, which in tensor form is:

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{F_v} = \boldsymbol{\rho}\, \ddot{\boldsymbol{u}} \tag{1}$$

where $\boldsymbol{\sigma}$ is stress, $\boldsymbol{F_v}$ is body force per volume, $\boldsymbol{\rho}$ is density, and $\ddot{\boldsymbol{u}}$ is acceleration. For static analysis, the right-hand side of this equation goes to zero.

The constitutive equation relating the stress tensor $\boldsymbol{\sigma}$ to strain $\boldsymbol{\epsilon}$ is the generalized Hooke's law

$$\boldsymbol{\sigma} = \boldsymbol{C} : \boldsymbol{\epsilon} \tag{2}$$

where $\boldsymbol{C}$ is the fourth-order elasticity tensor and : denotes the double dot tensor product. In COMSOL, this relation is expanded to

$$\boldsymbol{\sigma} - \boldsymbol{\sigma_0} = \boldsymbol{C} : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon_0} - \boldsymbol{\epsilon_{\mathrm{inel}}}) \tag{3}$$

For this application, initial stress $\boldsymbol{\sigma_0}$, initial strain $\boldsymbol{\epsilon_0}$, and inelastic strain $\boldsymbol{\epsilon_{\mathrm{inel}}}$ are all zero. For isotropic material, the elasticity tensor reduces to the 6×6 elasticity matrix:

$$\begin{bmatrix} 2\mu+\lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2\mu+\lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2\mu+\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \tag{4}$$

where λ and μ are the Lamé constants, E is the elastic modulus, and ν is Poisson's ratio, with material properties listed in Table I.

The final required equation is the kinematic relation between displacements $\boldsymbol{u}$ and strains $\boldsymbol{\epsilon}$. In tensor form

$$\boldsymbol{\epsilon} = \frac{1}{2}\left[\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T\right] \tag{5}$$

where ^{T} denotes the tensor transpose. For rectangular Cartesian coordinates the strain tensor may be written in indicial notation [1]

$$\epsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} - \frac{\partial u_\alpha}{\partial x_i}\frac{\partial u_\alpha}{\partial x_j}\right] \qquad (6)$$

where α = 1,2,3,.... For small deformations the higher order terms are negligible and ϵ_{ij} reduces to Cauchy's infinitesimal strain tensor:

$$\epsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right] \qquad (7)$$

TABLE I
MATERIAL PROPERTIES FOR 6061 AND 7075 ALUMINUM ALLOYS USED IN THE COMSOL MODEL [2].

Parameter	Symbol	6061	7075
Elastic Modulus	E	10,000 ksi	10,400 ksi
Poisson Ratio	ν	0.33	0.33

2) COMSOL Multiphysics Model: A three dimensional Solid Mechanics model was built, with the test sample profile geometry drawn using a CAD program (Ashlar-Vellum Graphite) as shown in Fig. 1. The profile was imported using the COMSOL CAD Import Module, extruded into a 1/8 in. thick 3D bar and modeled as homogeneous, linearly elastic 6061 or 7075 aluminum.

An extra fine physics-controlled mesh was generated (Fig. 2) and a stationary analysis was performed, using default solver settings.

3) Model Verification: Aluminum test samples milled from 6061 and 7075 alloys were used. Strain gages used were general purpose CEA series polyimide encapsulated Constantan alloy (Vishay Micro-Measurements CEA-13-240UZ-120) with 120 ohm resistance and a 2.2 gage factor. The gages were installed using standard surface preparation: degreasing, abrading, layout, conditioning, and neutralizing steps, following the methods in [3]. They were bonded to the aluminum test samples with M-Bond 200 methyl-2-cyanoacrylate adhesive.

The strain gages were wired with 27 AWG polyurethane insulated solid copper wire and gage lead wires were kept of uniform length to prevent unwanted lead resistance differences. The gages were wired as quarter bridges and connected to a bridge amplifier (Vishay P3 Strain Indicator and Recorder) [4]. The bridge amplifier provides the excitation voltage, completion resistances and, after the bridge is balanced and the gage factor is input, produces an output voltage directly in units of micro-strain $(10^{-6}\epsilon)$.

The instrumented aluminum test samples were loaded in axial tension using an Instron 5500R material test machine, and loaded between 0–2000 lb, and 0–4000 lb for the 6061 and 7075 alloys, respectively, below their yield stresses.

III. RESULTS

Figure 3 shows first principal strain arising from an applied load of 600 lb. As expected, the stress in the test region is uniform. Figure 4 shows a contour plot of first and second principal strains. It is clear from this plot that despite complex local stresses near the grips (ends), there is a uniform central test region, 2 in. long, where strain gages may be placed for accurate strain measurements.

Figure 5 shows measured axial and transverse strains for a 6061 test sample loaded with 600 lb in tension. Over the entire range of 0–2000 lb applied load, measured strain was highly linear.

A linear regression of the measured axial stress and strain gives the test sample's elastic modulus, as shown in Fig. 6. Measured elastic modulus E was 9.785×10^6 ksi for 6061 aluminum, which is within 2% of the literature value of 10.0×10^6 ksi. Similar agreement between experiment and theory was found for the 7075 alloy samples. A linear regression of the measured transverse strain and axial strain gives the test sample's Poisson ratio, as shown in Fig. 7. Measured Poisson ratio ν was 0.3217 for 6061 aluminum, which is within 3% of the literature value of 0.33. Similar agreement between experiment and theory was found for the 7075 alloy samples.

COMSOL FEA models are useful for allowing students to "look inside" of structures. For example, introductory textbooks in mechanics of materials use a constant average normal stress across the width of an axially loaded bar. Intuition suggests that for an applied point load, there should be significantly different local stresses depending on location. Saint-Venant's principle predicts that the difference between the effects of two different

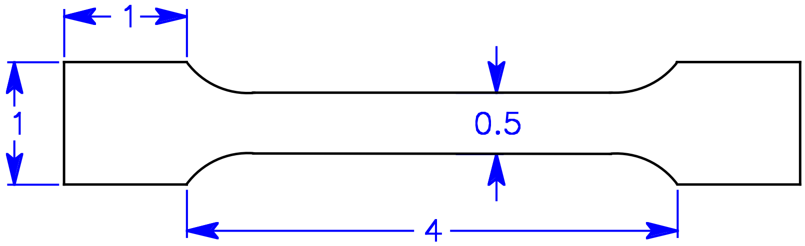

Fig. 1. Test sample profile geometry drawn in Graphite, then imported into COMSOL with CAD Import Module. Units are in inches.

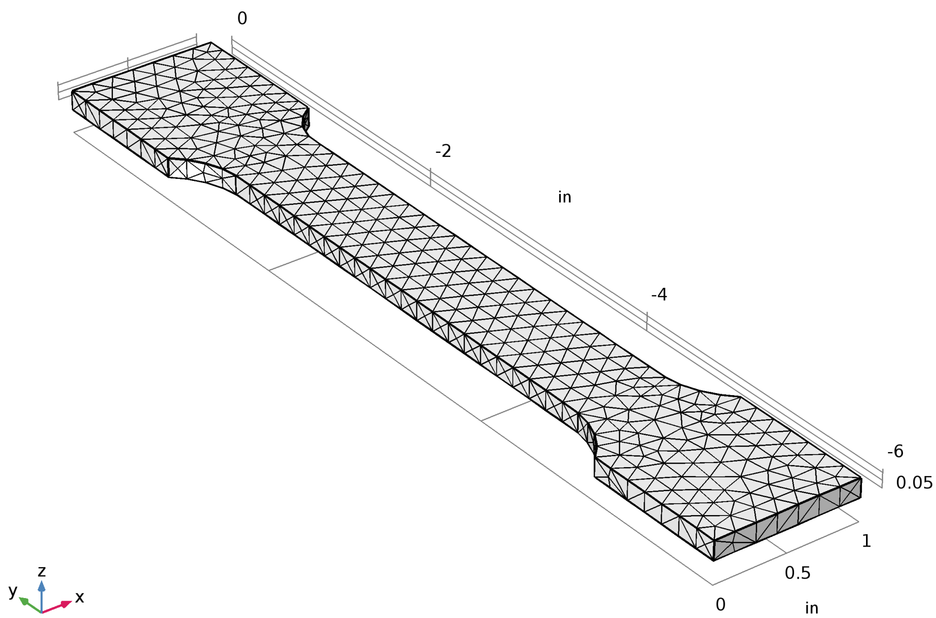

Fig. 2. COMSOL extra fine mesh of material test sample, yielding 2,192 tetrahedral elements with 13,368 degrees of freedom.

but statically equivalent loads becomes small at sufficiently large distance from the load [5]. Figure 8 shows axial stress across a uniform bar of aluminum with an applied axial point load. The blue curve was calculated from the FEA model at a distance $b/4$ away from the applied load, where b is the bar's width, and shows substantial stress variation. The green curve, measured at $b/2$, is much more uniform, while the red curve, measured at a distance b away from the applied load, is nearly uniform. The average normal stress, equal to the applied force divided by the bar's cross sectional area, is equal to the areas under these curves. For this particular example, the bar width was 1 m, thickness was 0.1 m, and applied force was 10,000 N, giving average normal stress of 1×10^5 Pa.

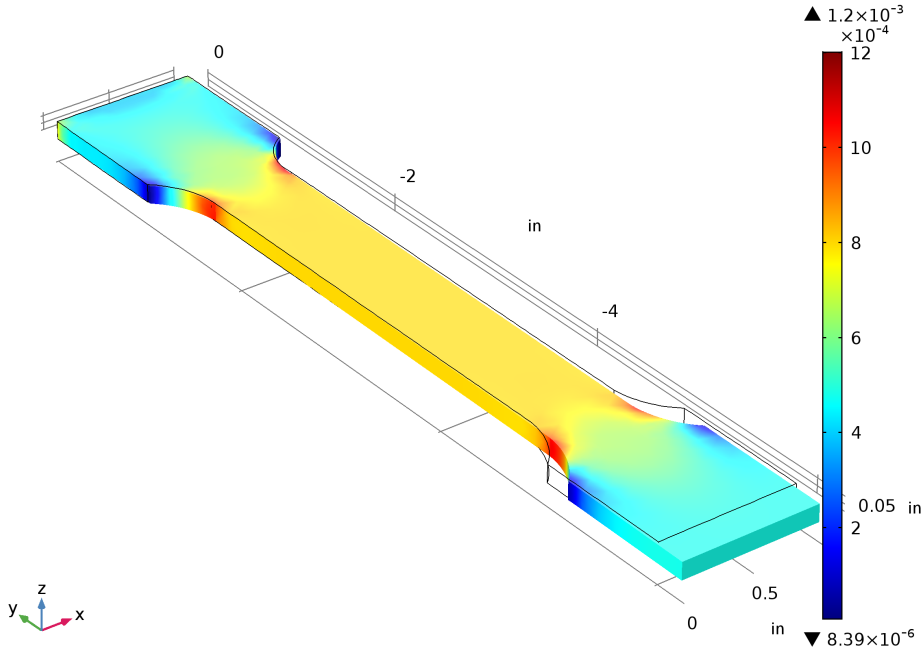

Fig. 3. First principal strain for a 6061 aluminum test sample with a 600 lb tensile load. The test section width for this particular sample was 0.6 in. giving a test section stress of 8000 psi and first principal strain of 0.0008.

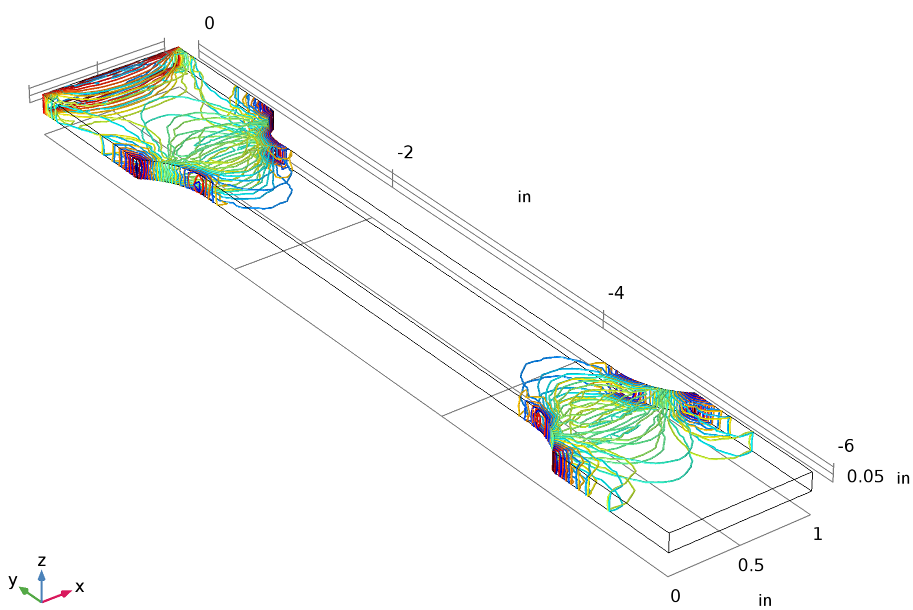

Fig. 4. Modeled first and second principal strain contours for a 6061 sample with a 600 lb load, showing complex local strains near the test sample grip ends and a uniform central test region 2 in. long.

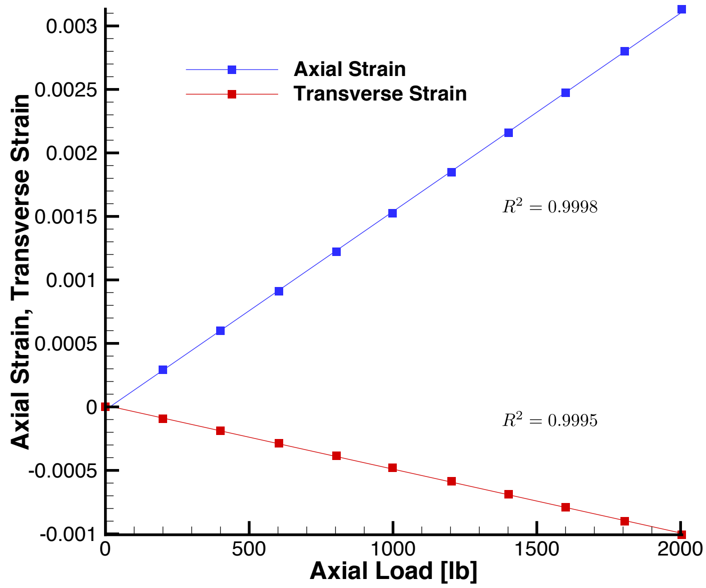

Fig. 5. Measured axial and transverse strains of a 6061 test sample for the load range 0-2000 lb, showing a high degree of linearity.

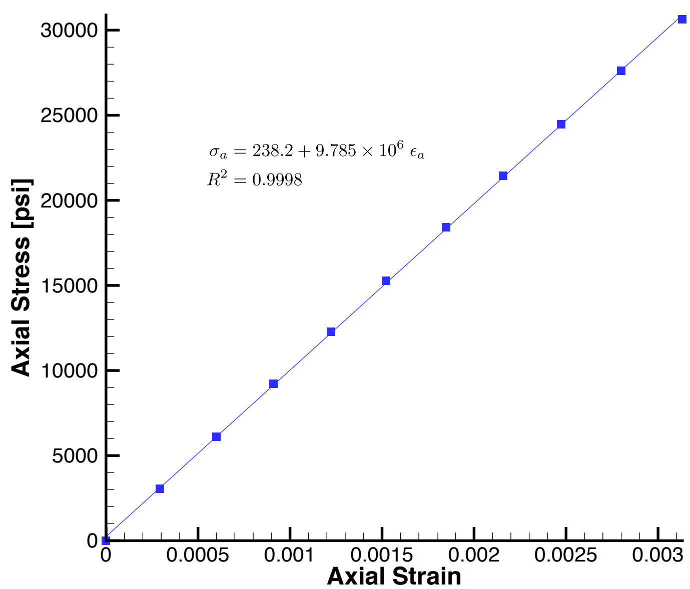

Fig. 6. Elastic modulus, E, calculated from a linear regression of measured axial stress and strain for a 6061 test sample. Measured E was 9.785×10^6 ksi, which compares favorably to the literature value of 10.0×10^6 ksi.

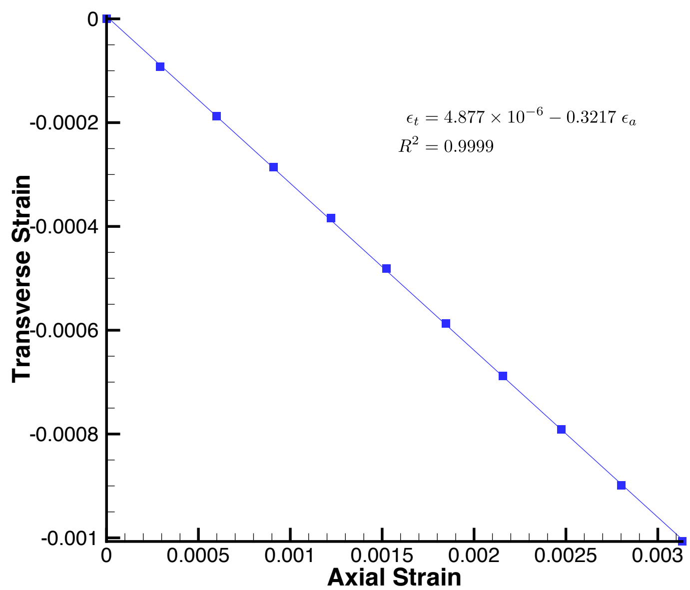

Fig. 7. The Poisson ratio, ν, calculated from a linear regression of measured transverse strain and axial strain for a 6061 test sample. Measured ν was 0.3217, which compares favorably to the literature value of 0.33.

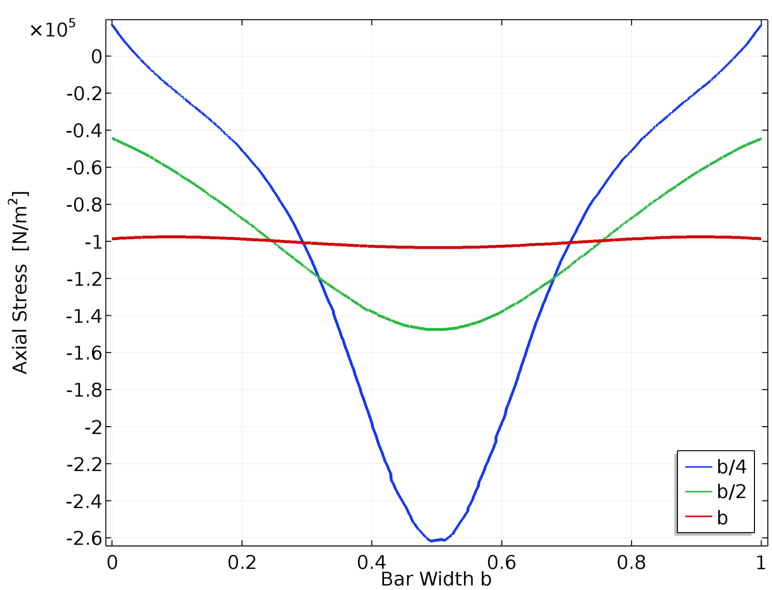

Fig. 8. Axial stress calculated from a COMSOL model of a uniform bar of aluminum of width b subjected to an axial point load. Results show that stress is not uniform near the applied load, but becomes so at a distance b from the load (red curve), demonstrating Saint-Venant's principle. The negative sign in the plot denotes compression.

IV. Discussion and Conclusions

Despite its simplicity, this COMSOL model was useful in predicting and visualizing experimental stresses and strains, particularly the stress concentrations between the grip regions at the test sample ends. Model results predicted a uniform stress region of 2 in., which guided placement of the strain gages. The experimentally determined elastic modulus was found to be within 2% of literature values, and the Poisson ratio within 3%, showing that this method can accurately measure both material properties for aluminum test samples.

The model permits visualization of phenomena such as Saint-Venant's principle, a topic that now has meaning for students. Close agreement between theory, model, and experiment validates the model, giving students confidence in this approach before moving on to more complex multiphysics models.

References

[1] Fung, Y.C., *A First Course in Continuum Mechanics (2ed)*, Prentice-Hall, Englewood Cliffs, NJ, 1977.

[2] Davis, J.R. ed., *Metals Handbook Desk Edition (2ed)*, ASM International, Materials Park, OH, 1998.

[3] *Student Manual for Strain Gage Technology*, Bulletin 309E, Vishay Measurements Group, Raleigh, NC, 1992.

[4] *Model P3 Strain Indicator and Recorder*, instruction manual, Vishay Micro-Measurements Group, Raleigh, NC, 2005.

[5] Saint-Venant, A.J.C.B., *Memoire sur la Torsion des Prismes*, Mem. Divers Savants, 14:233–560, 1855.