Analysis of Electromagnetic Propagation for Evaluating the Dimensions of a Large Lossy Medium

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Abstract: In this paper the propagation of a plane wave in a large lossy medium is presented. The investigated geometry consists in a wedge-shaped lossy dielectric embedded in a lossy material with different electromagnetic properties. The aim of the study is to determine the feasibility of a radar technique for measuring the length of the dielectric wedge. In order to address this problem and to evaluate the radar echo signal, both the transient and harmonic analysis are performed.

Keywords: lossy dielectric substrate, large medium, plane wave propagation, transient and harmonic analysis.

1. Introduction

In this paper the study of electromagnetic propagation in a geometrically large lossy media, analyzed by using COMSOL Multiphysics, is presented.
In several application area, such as the geophysics prospection here considered, when a multilayer medium is analyzed, the dimensions of the investigated geometry are very important to be determined.
An useful approach to evaluate the length of large structures is illustrated in this paper; it consists in illuminating the bounded object by using an electromagnetic plane wave and observing the time that the signal takes to be received. The delay, needed to complete a round trip, provides information about the distance that the plane wave covers in the lossy medium.
The geometry shape is a constraint to take into account when the dimensions of the problem are large compared to the wavelength.
In this work COMSOL Multiphysics has been used to analyze time propagation and in-frequency distribution of the electric field associated with a plane wave that illuminates the investigated geometry.

When a plane wave propagates in a medium with losses, part of the energy is dissipated depending on the material properties. Moreover, for a bounded medium, the reflection by the boundaries has to be taken into account. Consequently, an electromagnetic signal can be detected not only if a reflection appears, but also if the signal is not strongly attenuated.

The paper is organized as following: in the next section the main concepts involved in the propagation theory are presented, successfully the investigated geometry and the COMSOL numerical model are discussed, and finally some preliminary numerical results are shown.

2. Physics of propagation in a lossy medium

In order to evaluate if a transmitted electromagnetic signal, once received, is useful to determine the dimensions of a lossy dielectric wedge, let us consider a homogeneous medium characterized by an electrical conductivity σ and dielectrical permittivity ε, moreover the magnetic permeability μ is assumed to be coincident with the free space one μ_0.
In the time-harmonic analysis the z-component of the electric field associated with the incident plane wave is expressed as following:

$$E_i = E_0 e^{-jkx}\hat{z} \tag{1}$$

where E_0 is the amplitude of electric field and x is the position vector in rectangular coordinates. k represents the wave number whose real k_r and imaginary k_i parts are defined, respectively, as the phase and attenuation constants [1]. These quantities are expressed as:

$$\begin{aligned} k_r &= \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]^{1/2}; \\ k_i &= \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]^{1/2} \end{aligned} \tag{2}$$

It is important to note that the latter introduces an attenuation depending on the electrical properties of the medium, in terms of electric conductivity and permittivity. The reflection of the electromagnetic field depends on the difference of the electromagnetic properties of the two media as:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \tag{3}$$

where the intrinsic impedance η_i is calculated as:

$$\sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \tag{4}$$

In the transient analysis, the electric field source has been synthesized as the first derivative of Blackman-Harris pulse function, shown in Figure 1 with center frequency 100 MHz [2], expressed as following:

$$E_z = -0.7174\sin(5.511 * e8t) +$$
$$0.4264 * \sin(11.023e8t) - 0.0451 * \sin(16.534e8t)$$
for $t<1.14e\text{-}8$ s. (5)

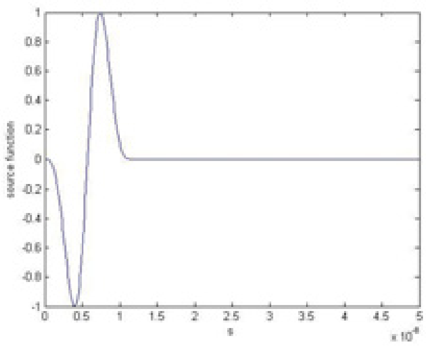


Figure 1: the first derivative of Blackmann-Harris windows function used to synthesize the source

The reflected and attenuated electromagnetic signal is received with a delay T, therefore the length L of the wedge is calculated as:

$$L = \frac{c}{\sqrt{\varepsilon_r \mu_r}} T \tag{6}$$

where L is the length of a round trip covered by the wave propagating inside the wedge.

3. Geometry

The investigated geometry is a large composite structure consisting in a lossy dielectric box which contains a wedge-shaped lossy dielectric medium. For the sake of clarity, the geometry is shown in Figure 2, where the two materials are represented by different colors.

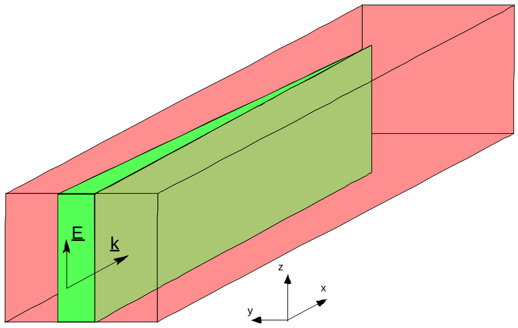


Figure 2: the investigated geometry consists in a lossy dielectric box and a wedge-shaped lossy dielectric

Let us suppose to model the source as a plane wave that illuminates the left side (*yz* plane) of the structure. The electromagnetic field *E* propagates along +*x* direction (*k* represents the wavenumber), undergoing an attenuation depending on the medium losses. The electric field is reflected by the wall discontinuity (wedge-box) towards the source (receiver) undergoing a further attenuation. By measuring the delay of the reflected and attenuated electromagnetic field, it is possible to obtain the covered path that represents the length of the structure.

4. Numerical model

COMSOL electromagnetic module has been used to analyze the geometry shown in Figure 2. In particular, the 3D problem has been reduced to the equivalent two-dimensional one by assuming that no guiding effects on z-axis appear. The simplified model is shown in Figure 3; where the box is assumed to be 21 m length by 2 m wide, while the wedge is 20 m length by 0.5 m wide corresponding to the shorter side.
The rectangular box is a dielectric material characterized by a complex dielectric constant $\varepsilon=5\text{-}j0.359$. The cuneiform wedge has a complex dielectric permittivity $\varepsilon=10\text{-}j0.071$.

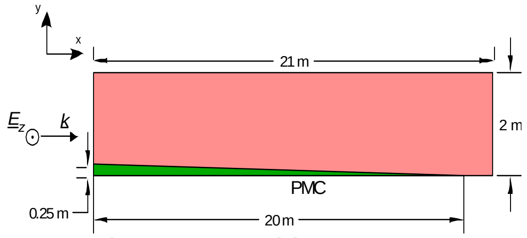


Figure 3: the COMSOL model

Since the dimensions of the investigated geometry are large compared with the wavelength, evaluated at the operative frequency of 100 MHz, in order to reduce the computational domain to half of the original one, the PMC (PMC - Perfect Magnetic Conductor) has been used as symmetry conditions.

The boundary conditions, imposed on each side of the structure, are shown in Figure 4.

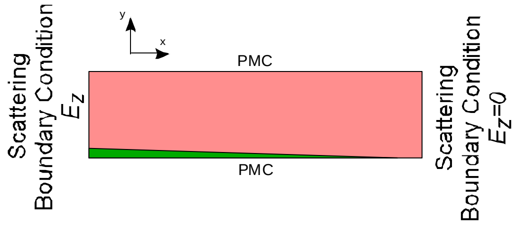


Figure 4: boundary conditions imposed on each side of the structure

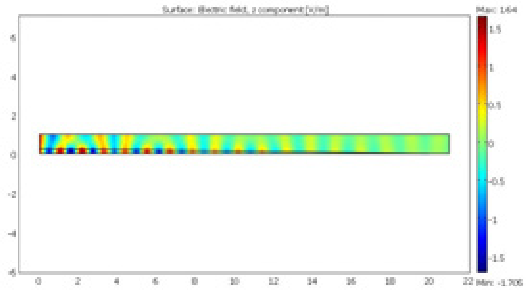
As mentioned above, the PMC condition on the down side allows us to reduce the investigated structure to half equivalent structure decreasing the computational effort. Moreover the PMC condition on the top side assures the continuity of the tangential electric field avoiding spurious reflections from walls. The structure is illuminated by using an incident wave propagating from left side toward the right side. To this aim the scattering boundary conditions with incident wave have been used. In particular, in the time harmonic analysis the z-component of the electric field is set to 1, while in the time dependent analysis the electric field assumes the expression indicated in (5). The scattering boundary condition with electric field set to zero has been used on the right side. PML have not been used as boundary conditions in order to do not further increase the large investigated structure.

5. Results

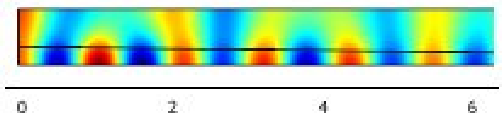
The harmonic propagation analysis has allowed us to evaluate the distribution of the electric field in the computational domain, pointing out which region is interested in the propagation. Transient analysis has permitted us to evaluate the propagation of the plane wave by determining the distribution of the electric field corresponding to several time steps.
The geometry has been modeled by using a mesh of 39447 elements corresponding to 80554 freedom degree. The previously mentioned boundary conditions, shown in Figure 4, have been imposed.

5.1 Time harmonic analysis

The time harmonic analysis allows us to determine if the electromagnetic signal is able to propagate inside the wedge or it tends to remain outside. As previously mentioned the analysis has been performed at a working frequency of 100 MHz.
The distribution of the z component of the electric field, evaluated in time harmonic domain, is shown in Figure 5 (a), while a zoom of the initial part of the wedge is shown in Figure **5** (b).



(a)



(b)
Figure 5: amplitude distribution of z-component of electric field (a) and zoom of the initial part of the wedge (b)

It is important to note that large part of the signal is confined into the cuneiform medium; this is

due to two phenomena. Firstly, the losses in the wedge are less than the one in the surrounding region, consequently the internal electromagnetic signal is not as attenuated as the external one. Secondly, the electromagnetic field impinges on the wall discontinuity with an incident angle greater than the critical one [3], therefore a quasi-total reflection effect appears.
The entity of the attenuation is shown in Figure **6**, where the amplitude of the electric field [dB] is determined as function of the distance from the geometry origin (lower left corner) inside the wedge-shaped dielectric (blue curve) and inside the environment (green curve).

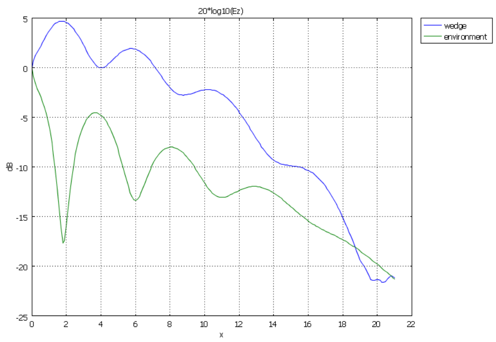


Figure 6: amplitude of z-component of the electric field evaluated at several distances from the origin inside (blue curve) and outside the wedge (green curve)

By observing Figure 6, it is important to note that the signal outside the wedge involves in a greater attenuation respect to the one in the wedge.

5.2 Time dependent analysis

The time-dependent analysis has permitted us to determine if the electromagnetic signal is reflected back towards the receiver and the dimensions of wedge-shaped medium are allowed to be determined. The simulation has been performed by fixing the observation time window at 500 ns, enough to receive the reflected electric field after a round trip of 40 m. The source has been modeled as a pulse function according the eq. (5).
In Figure 7 a zoom of the electric field distribution on the geometry is shown for several sampling intervals: 63 ns (a), 140 ns (b), 390 ns (c).

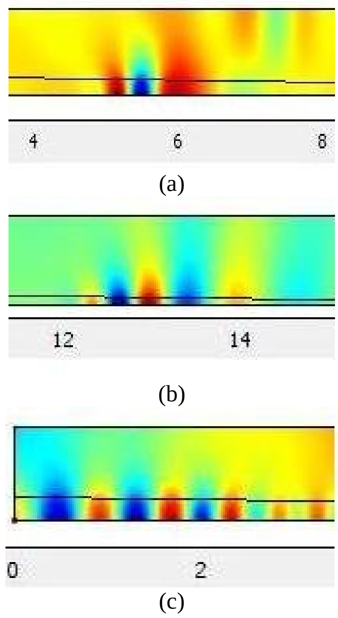


Figure 7: zoom of the amplitude of z-component of the electric field corresponding several time steps: 63 ns(a), 140 ns(b), 390 ns(c).

It is important to observe that, according the eq. (6), the wave propagation delays correspond to the distances covered in the cuneiform dielectric wedge characterized by a relative dielectric constant of 10. In particular, by referring to Figure 7, the electromagnetic wave has propagated for a distance of 6 m, 13 m and 37 m, respectively.
In Figure 8 the amplitude [dB] of the z-component of the electric field, calculated as time function in a point placed on the origin of the geometry, is shown.

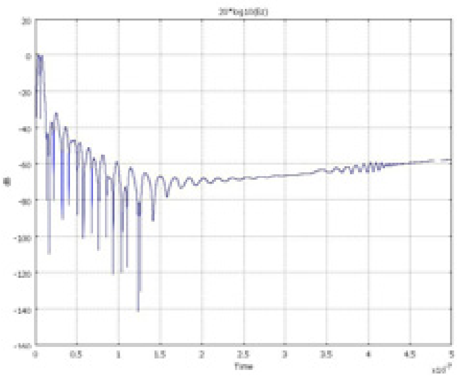


Figure 8: amplitude of z-component of the electric field evaluated as time function

In order to follow the variations of the electric field and to minimize the instability of the solution, the sampling step has been chosen 5 ps.
It is important to observe that the electromagnetic signal received with a delay of 420 ns represents the scattering from the end of the wedge corresponding to a covered distance of 40 m according to the equation (6). The calculation of this delay is in agreement with the length of the cuneiform dielectric medium.
Therefore, the study of a reflection and attenuation effect allowed us to determine the dimensions of a complex large structure.

6. Conclusions

Transient and harmonic propagation of the Electromagnetics module of COMSOL Multiphysics are very suitable for analyzing large structures.
The combination of the in-frequency and in-time domain analysis allowed us to well understand the propagation phenomena in a lossy bounded dielectric medium. The detection of the reflected and attenuated electromagnetic signal permitted us to determine the dimensions of the structure according to the electromagnetic characteristics of the geometry. In particular, the detection of an electromagnetic signal is mainly permitted depending on the losses entity for conductive media and the difference of complex dielectric constant. Moreover, the dimensions of the geometry, compared to the wavelength, are very important to determine the propagation of waves.

7 References

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