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Fast 2D Simulation of Superconductors: a Multiscale Approach

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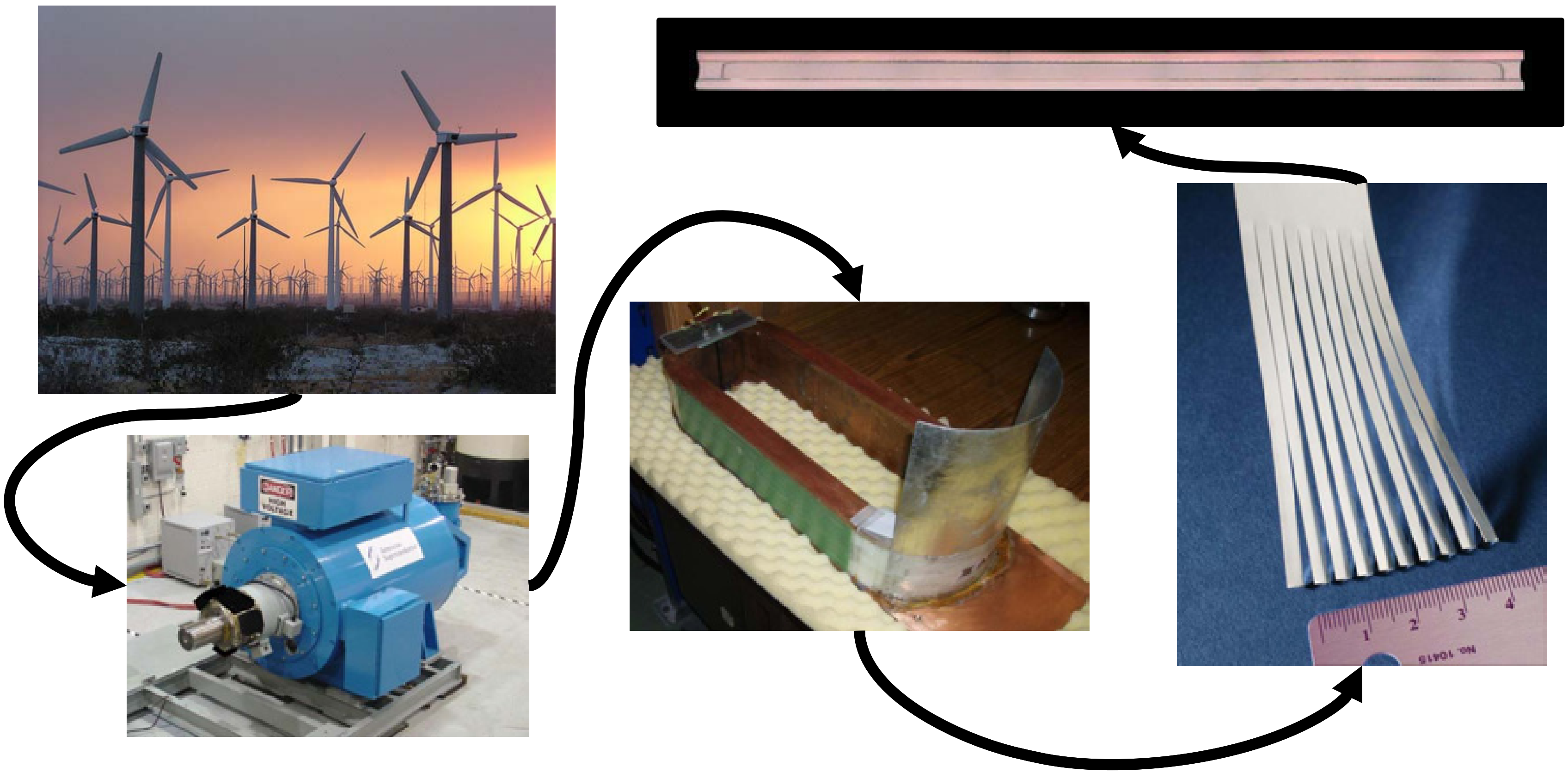
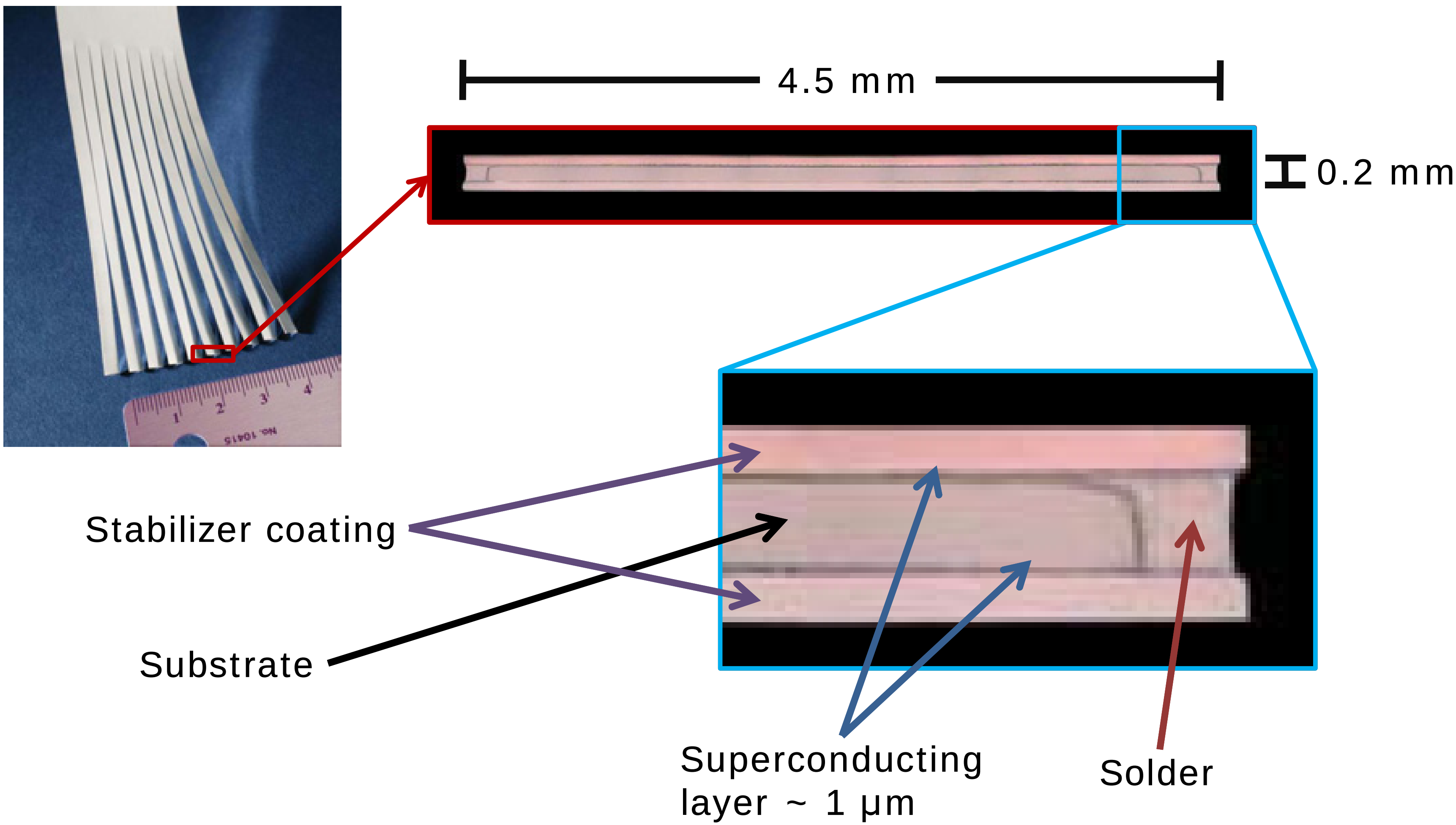
Motivation

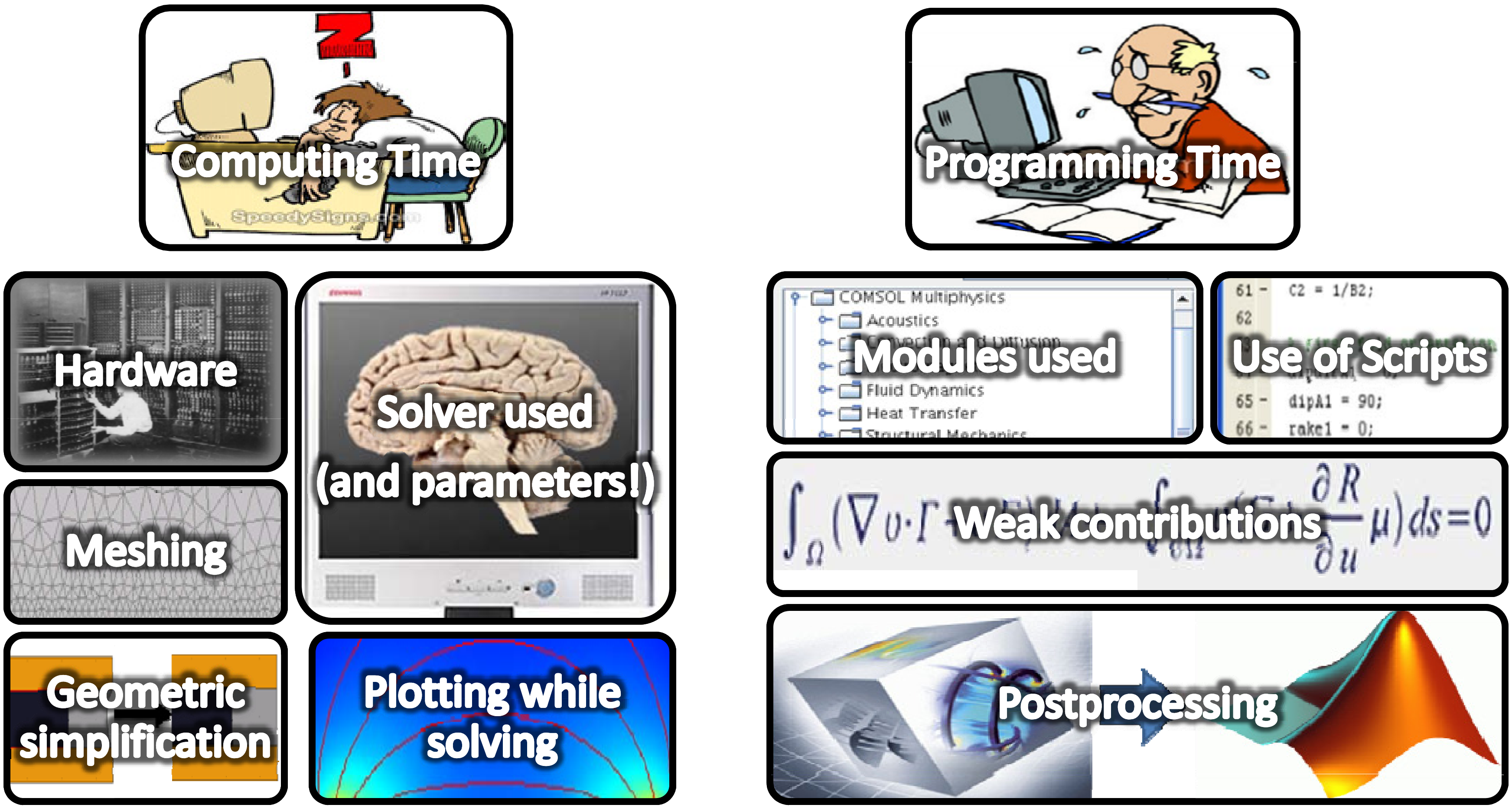
PhD Project: Computation of Superconducting Wind Turbine Generators.



Wire architecture



Time to solution: calculation of AC losses using COMSOL



Governing equation

$$\nabla \times (\rho \nabla \times \boldsymbol{H}) = -\mu \frac{\partial}{\partial t} \boldsymbol{H}$$

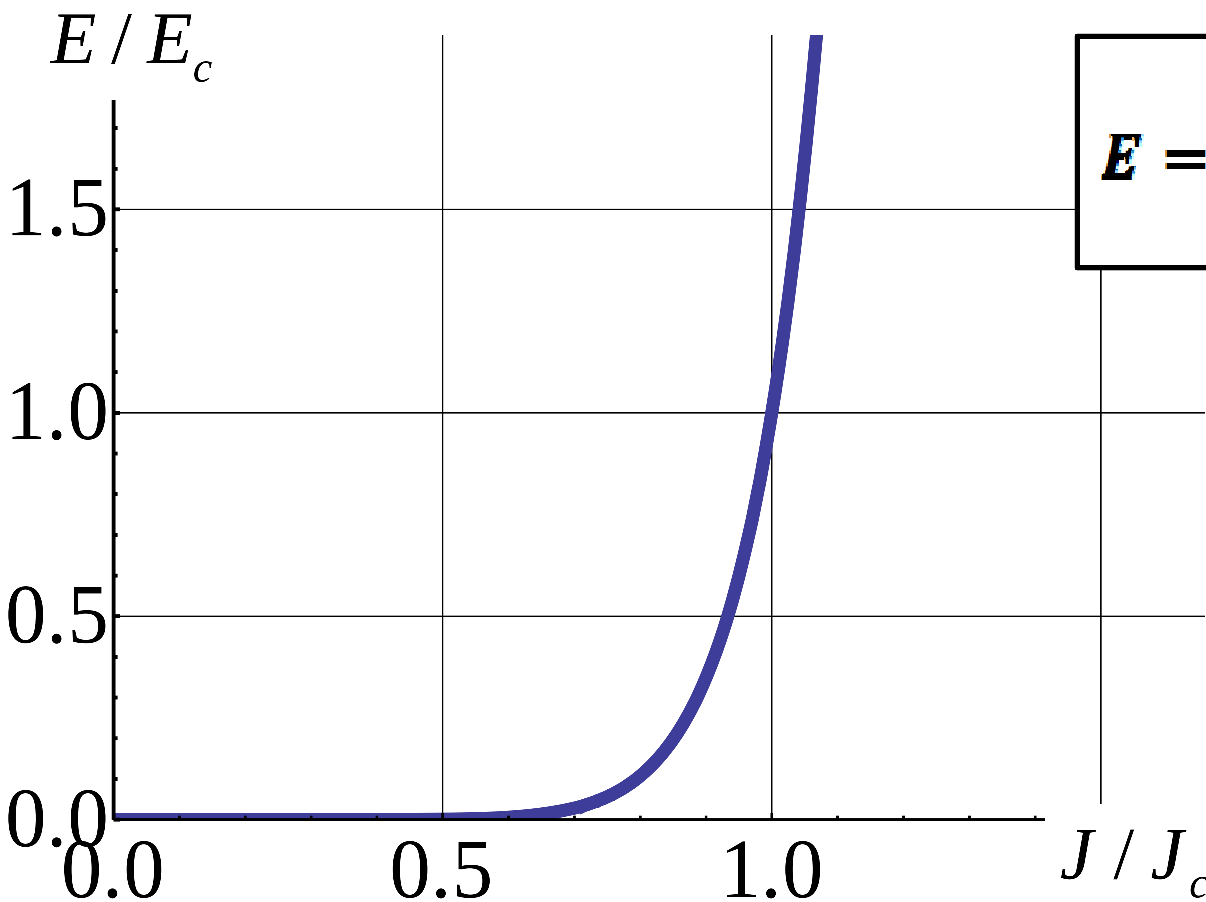
$$\rho = \frac{E_c}{J_c} \left(\frac{|\boldsymbol{J}|}{J_c} \right)^{n-1}$$

$$\boldsymbol{J} = \nabla \times \boldsymbol{H}$$

$$Q = \oint dt \int \boldsymbol{E} \cdot \boldsymbol{J} \, dS$$

$$\boldsymbol{E} = \rho \, \boldsymbol{J}$$

E-J relationship in Superconductors



$$\boldsymbol{E} = \boldsymbol{J}\frac{E_c}{J_c}\left(\frac{|\nabla \times \boldsymbol{H}|}{J_c}\right)^{n-1}$$

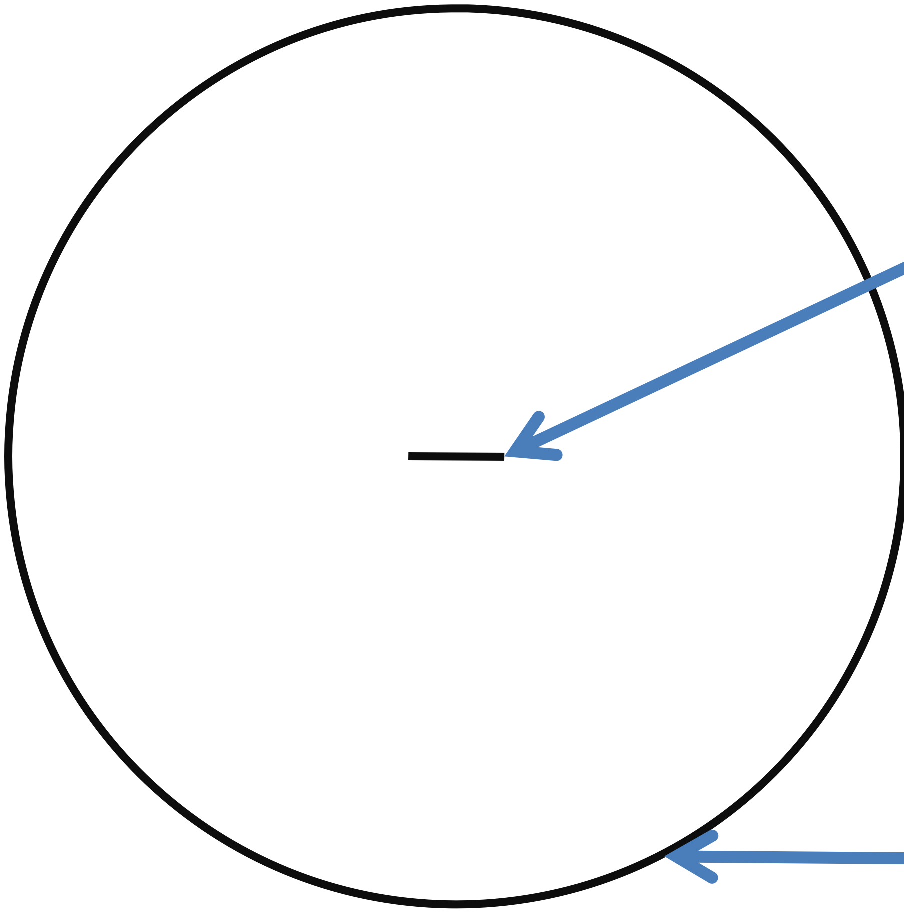
Use of COMSOL

$$e_a \frac{\partial^2 \mathbf{u}}{\partial t^2} + d_a \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \Gamma = F$$

$$\mathbf{u} = \boldsymbol{H} = \begin{bmatrix} H_x \\ H_y \end{bmatrix} \quad e_a = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad d_a = \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$$

$$\Gamma = \rho \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad F = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Conditions and Restrictions



$$I_k = \int_{C_k} J_z \, dS$$

$$\boldsymbol{H}|_{t=0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{H}|_{\partial\Omega} = \begin{bmatrix} H_{sx} + H_{ex} \\ H_{sy} + H_{ey} \end{bmatrix}$$

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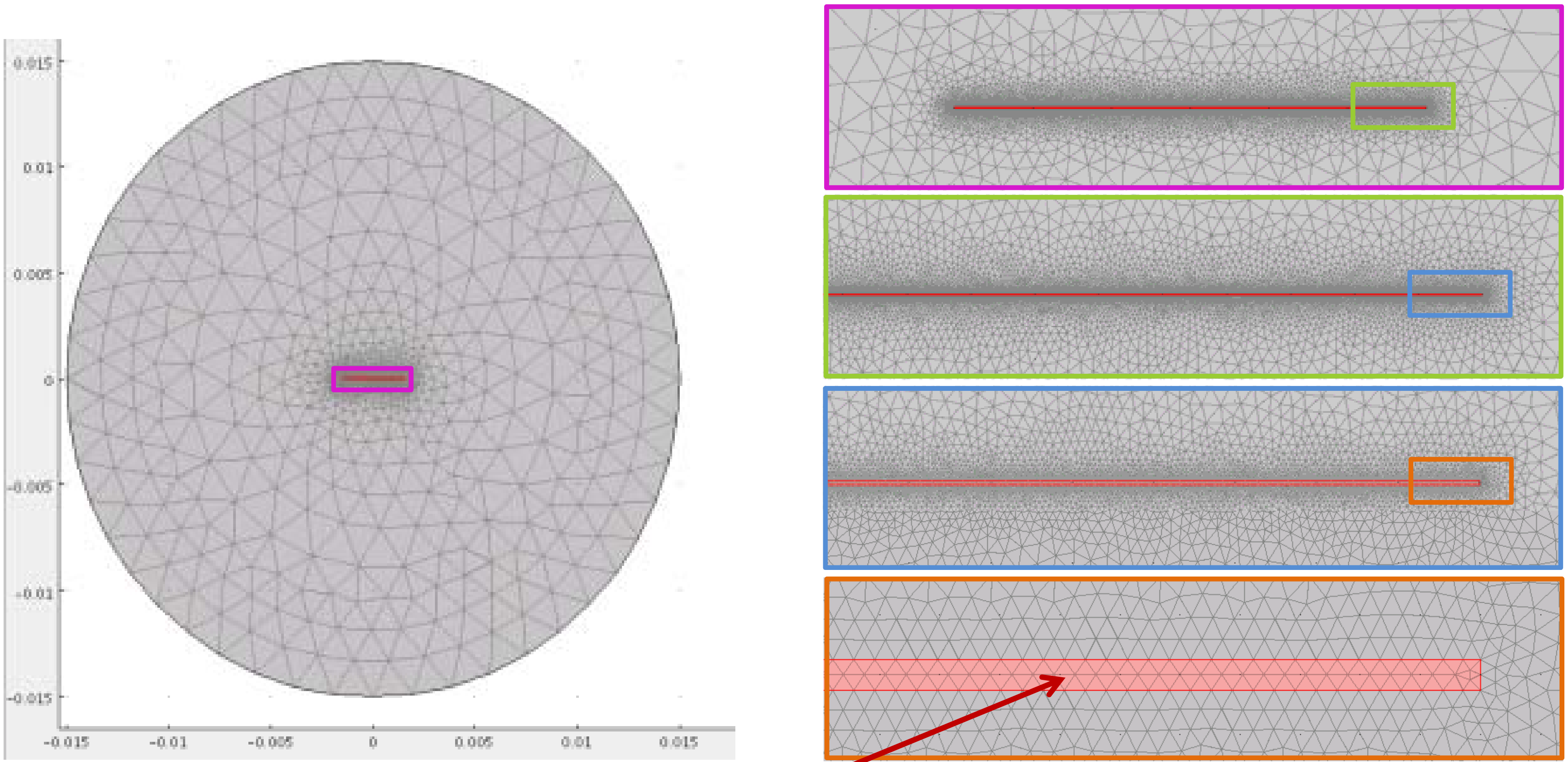
Using Free Meshes



17150 elements in the Superconducting region

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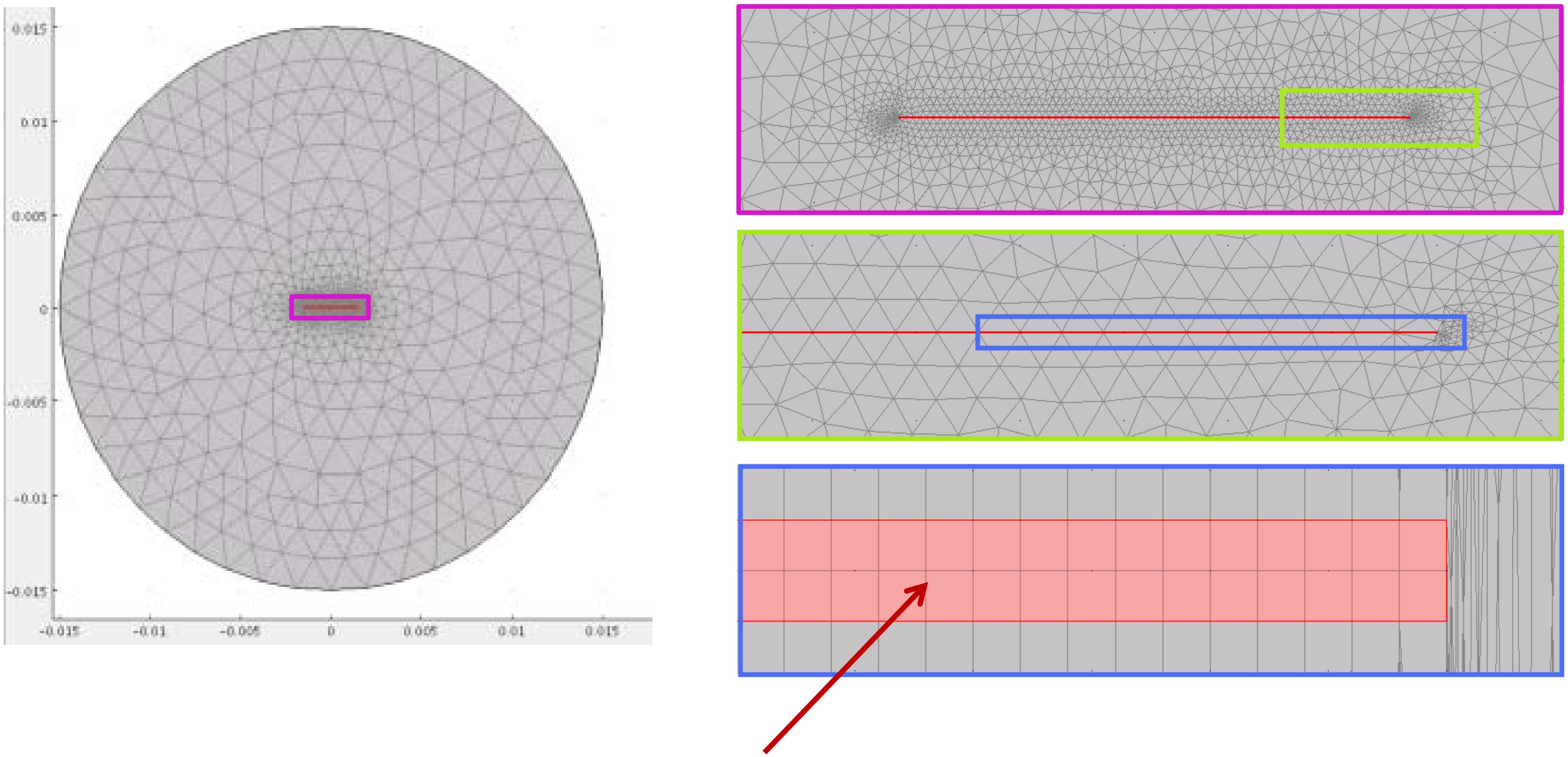
Using Mapped Meshes



Only 150 elements!

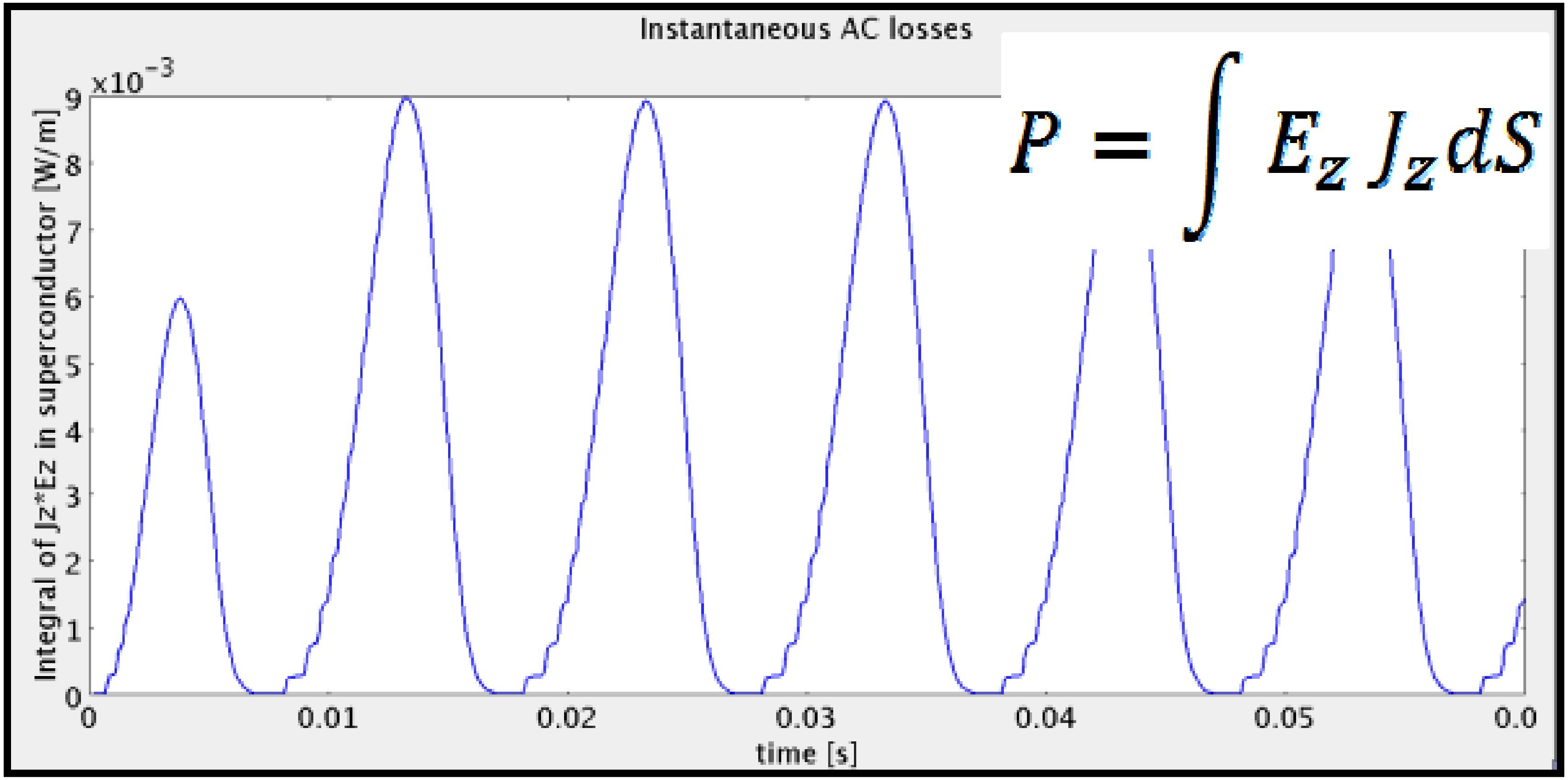
Calculation of AC losses

$$Q = \xi\left(t_0 + \frac{1}{f}\right)$$

$$\dot{\xi} = f\,P, \quad t \in \left(t_0, t_0 + \frac{1}{f}\right)$$

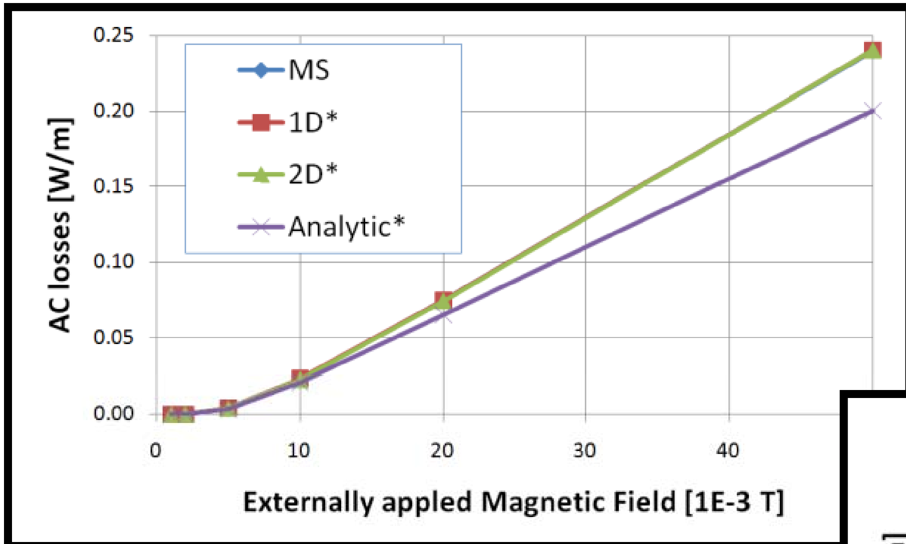
$$\xi(t_0) = 0 \qquad P = \int E_z J_z dS$$

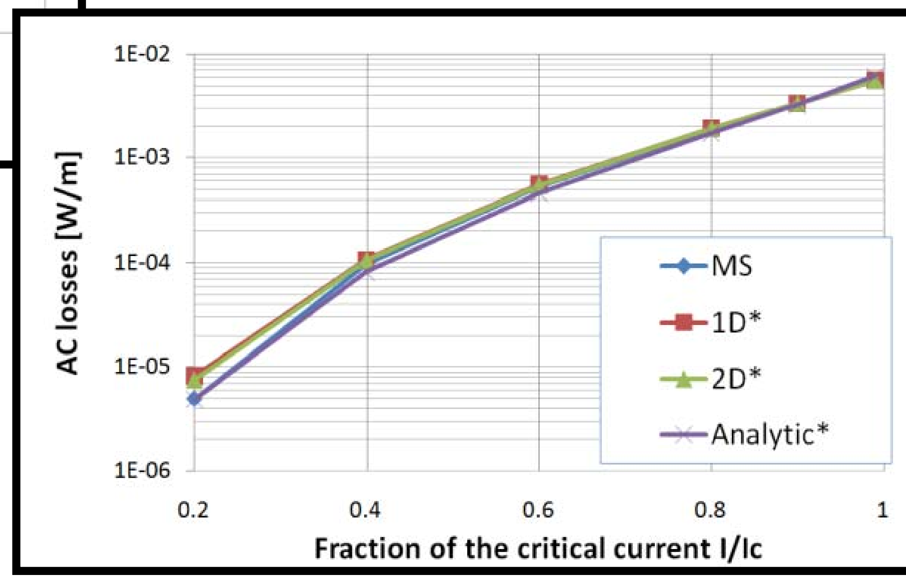
Results



$$P = \int E_z J_z dS$$

Validation of Results





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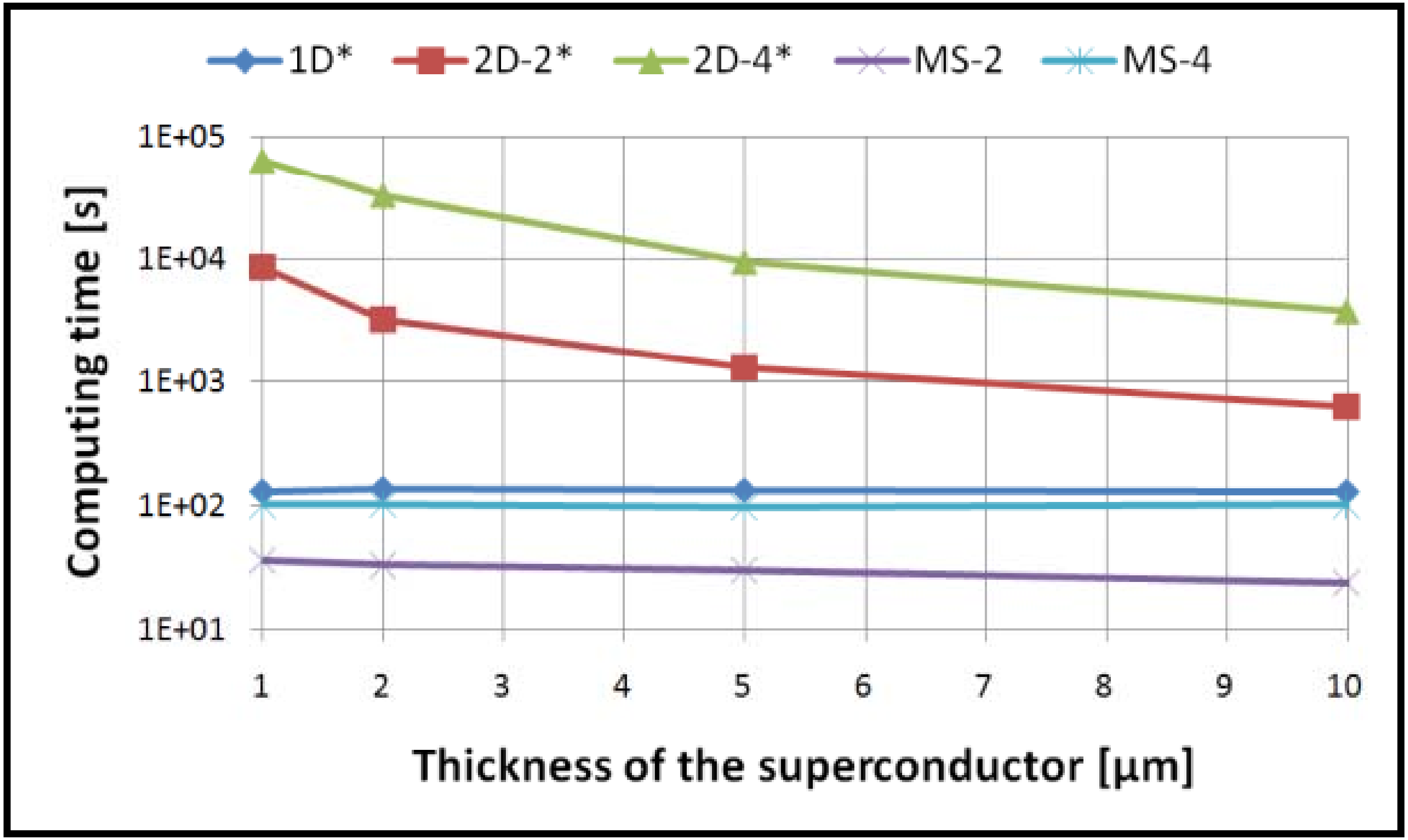
Computing Time



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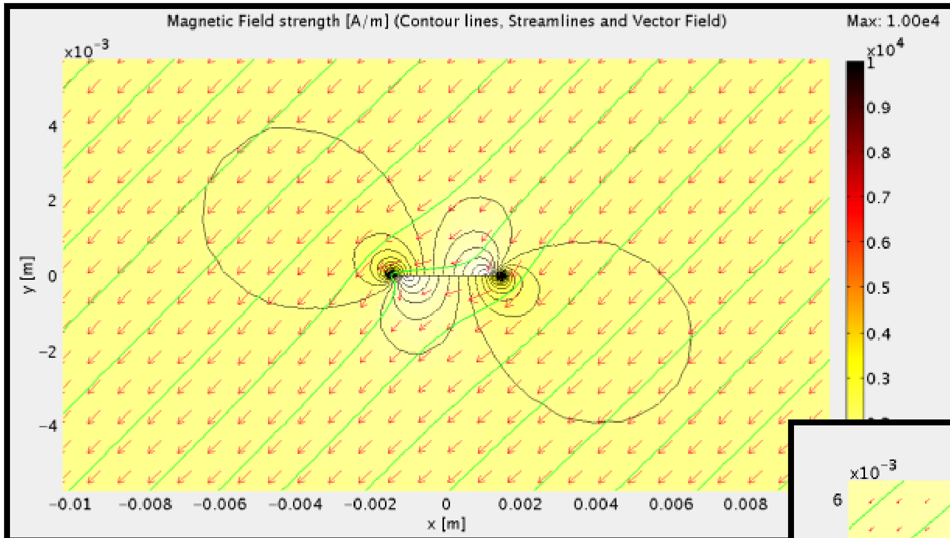
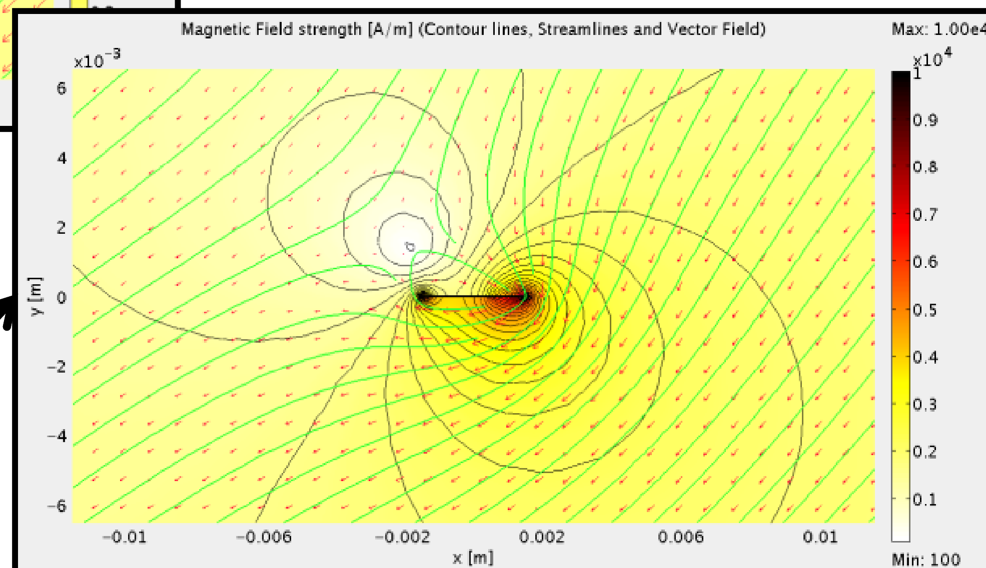
Externally applied field and current



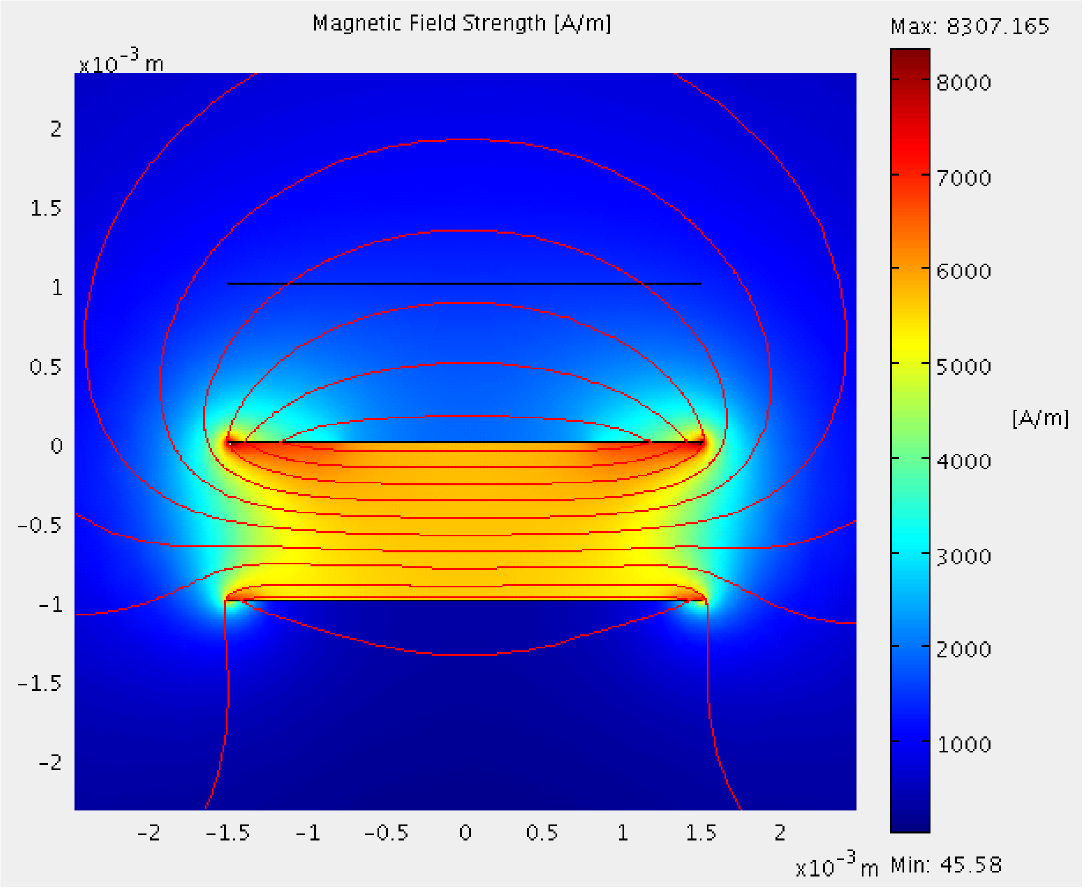
Magnetic Field Intensity (peak value shown) for a thin superconductor when a sinusoidal Magnetic Field of 2 mT (~1.6e3 A/m) is applied .

No external current applied.



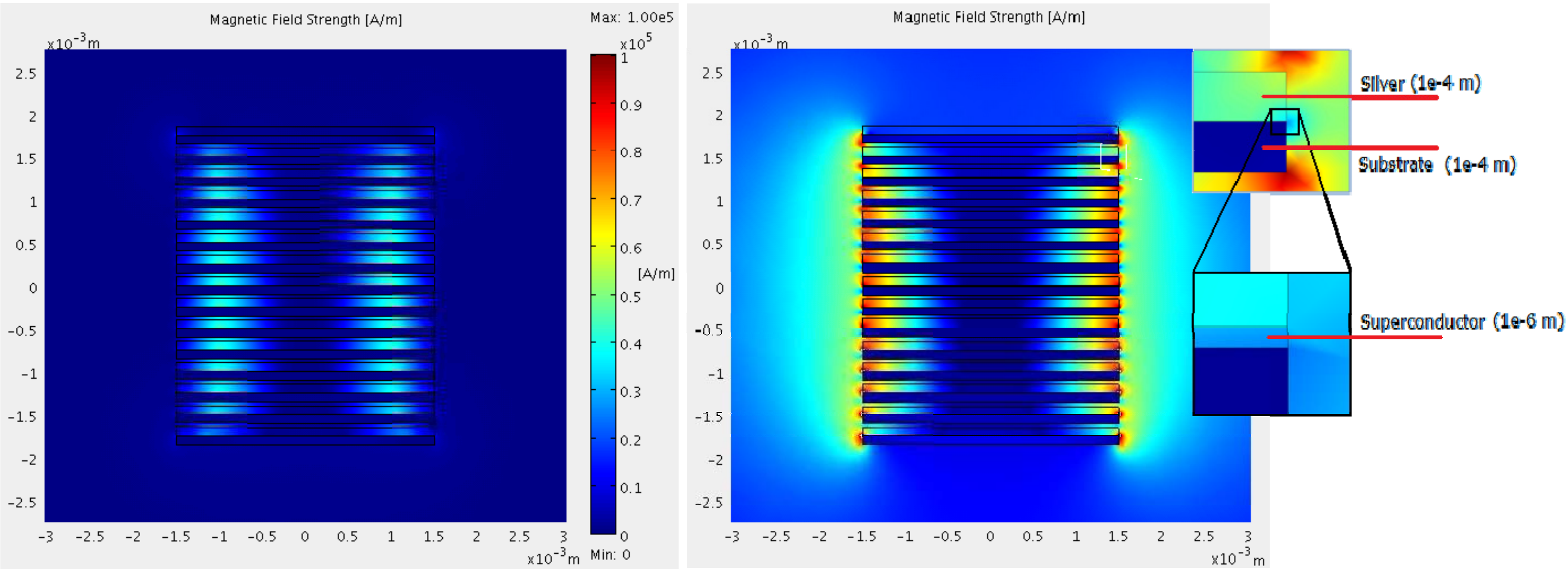
External current applied (80% critical current).

Interaction among several thin conductors



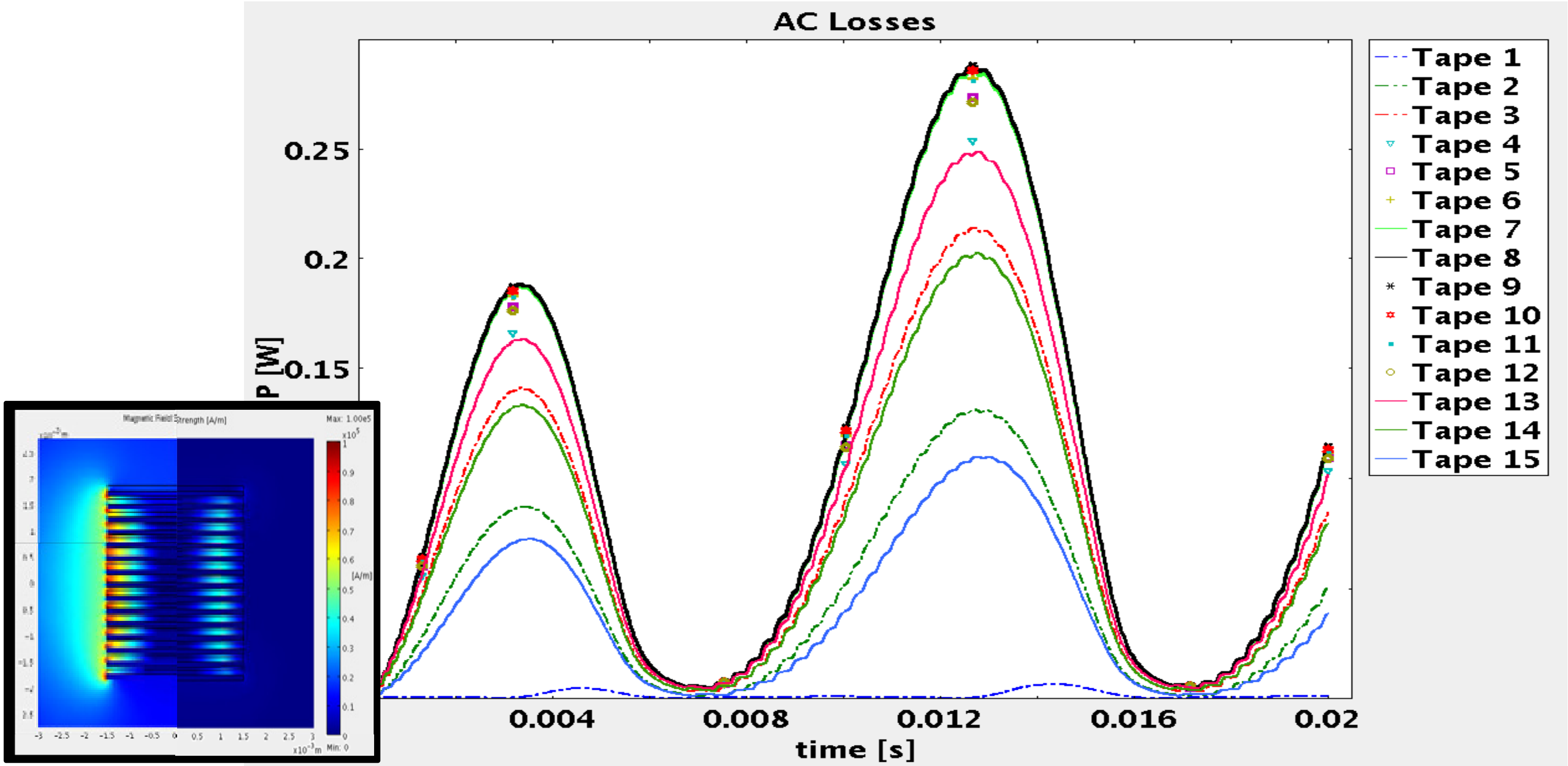
Top: Silver, no external current. Center: superconducting, imposed AC transport current (0.9 *Ic*, 50 Hz). Bottom: superconducting, no external transport current. Notice how the field is expelled from the bottom conductor.

Stack of tapes



AC electric currents (0.9 *Ic*, 50 Hz, in phase) are applied to a stack of 15 coated conductors. Magnetic field strength at 0 phase (left) and at peak value (right). A substrate with a relative permeability of 50 was considered. The insert shows the thickness of the different layers.

AC losses in a stack of tapes



Instantaneous AC losses in the previous stack of tapes (both phases are shown in the insert). Consider tapes to be enumerated from the top. Observe that the higher losses are experienced by the central conductors. Also, notice that the top conductors (tapes 1, 2 and 3) experience less loss than their bottom counterparts (tapes 15, 14 and 13).

Conclusions

Use mapped meshes and specifically, use of large aspect ratio elements, provides a considerable increase in the computing speed for calculation of AC losses in superconductors. Numerical simulations were performed showing a decrease of 2 to 3 orders of magnitude in the computing time when compared with other 2D simulation were no mapped meshes are used.

The time spent modeling every single application while using large aspect ratio elements does not depend heavily in the number of conductors and offers “similar” computing time than the 1D formulation. Therefore, the work presented here offers a faster time to solution strategy for calculating AC losses.

Finally, the ease to set a problem using the proposed formulation makes it possible to think of further applications such as superconducting coils, and induction machinery, among others in the near future.

Thank You!

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