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# Numerical Simulation of Granular Solids Rheology: Set-up and Experimental Validation

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In the challenging task to describe granular materials' behaviour various simulation techniques have been tested (DEM, Material Mechanics, Equivalent fluid).

Some of the numerical models describing the flow of granular materials like a fluid (such as Bingham's [1] or Cross') relates viscosity to the shear rate by means of various laws.





The model proposed here it's uses the same “equivalent fluid” approach, but takes into account in a better way particle mobility using a scalar called “granular temperature” ( $\theta$ ).

The model has been previously validated [2] on simple geometries and will be now tested on more complex ones, based on a silo existing at Danieli R&D.





Model assumptions:

- Interstitial fluid's effects are neglected
- Incompressible fluid
- No memory effects
- Symmetric stress tensor

Under these assumption, it is possible to write the conservation equations as follows



# Governing Equations - 2



Continuity equation:

$$\nabla \cdot \bar{\mathbf{v}} = 0$$

Navier-Stokes equation:

$$\rho \frac{\partial \bar{\mathbf{v}}}{\partial t} + \rho \bar{\mathbf{v}} \cdot \nabla \bar{\mathbf{v}} = -\nabla p - \nabla \cdot \Pi + \rho \mathbf{g}$$

Granular temperature transport equation:

$$\rho \frac{\partial \mathcal{G}}{\partial t} + \rho \bar{\mathbf{v}} \cdot \nabla \mathcal{G} = -\Pi : \nabla \bar{\mathbf{v}} - \nabla \cdot \hat{\mathbf{q}}^T - z^T$$

The last 2 equation are related by the stress tensor  $\Pi$ , which can be related to shear rate once a viscosity definition is given. In this case the viscosity  $\eta$  is obtained from an exponential function of the granular temperature  $\theta$ .



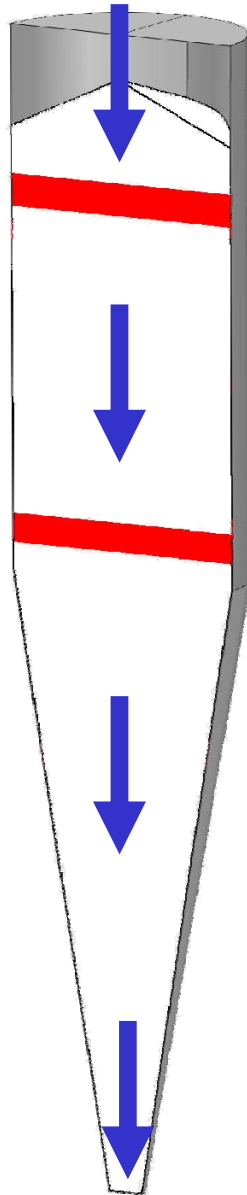


The silo was only an half, cut vertically on a symmetry plane made of Plexiglas to observe the evolution of some tracer bands.

Silo filled with 3mm diameter PE pellets of two different colors:

- White for the bulk
- Red for the tracer bands used to study descent profiles





- Various geometric configurations:
  - Hollow silo
  - With different kinds of flow aids:
    - ❖ One kind made of three tubes passing from side to side in the silo,
    - ❖ Another kind made of two truncated cones one below the other.
- Steady-state flow
- Wall stresses according to theory [3,4,5]



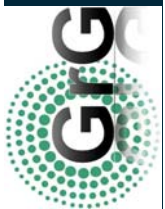


Granular materials can slip at walls.

Thus Coulomb condition ( $\tau = \mu_w \sigma$ ) should be applied to describe material boundary behaviour.

This condition has shown some problem during implementation, so the Navier condition ( $u_t = \lambda^* |\partial u_t / \partial n|$ ), along with weak formulation, has been used.

This kind of boundary condition relates wall tangential velocity to its derivative along wall normal direction by means of a parameter called “slip length”.



# Boundary Conditions - 2



The top of the experimental silo was open to air



Pressure inlet with atmospheric pressure as a boundary condition for the upper side

Controlled mass flow rate discharge for the experimental silo



Velocity outlet for the lower boundary





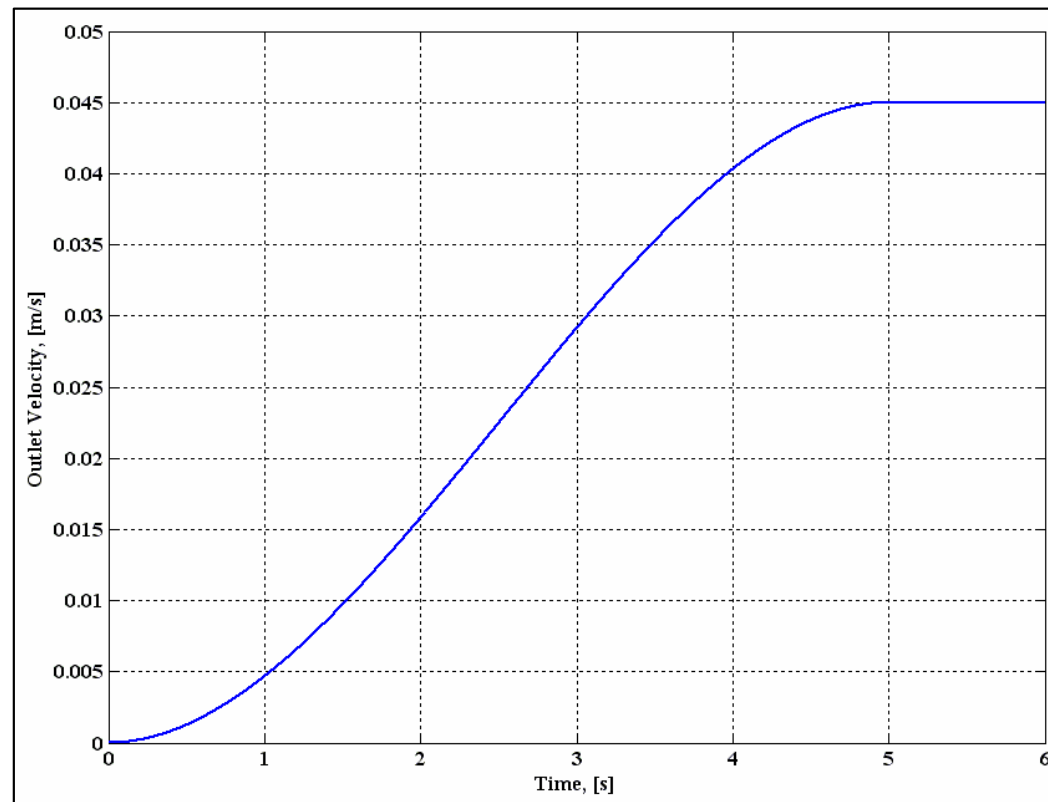
## COMSOL Multiphysics Application modes and other parameters:

- Incompressible Navier-Stokes (ns)
  - General Heat Transfer (htgh)
- Time dependent simulation (2h)
  - 2000-4000 triangular elements



## Precautions:

- 2-D axisymmetric simulation
- “Soft start” function for the velocity outlet boundary



**Figure.** Example of the outlet velocity values along time according to the “soft start” function

Model set-up : parameters determined from the experiments with the silo without any internal device

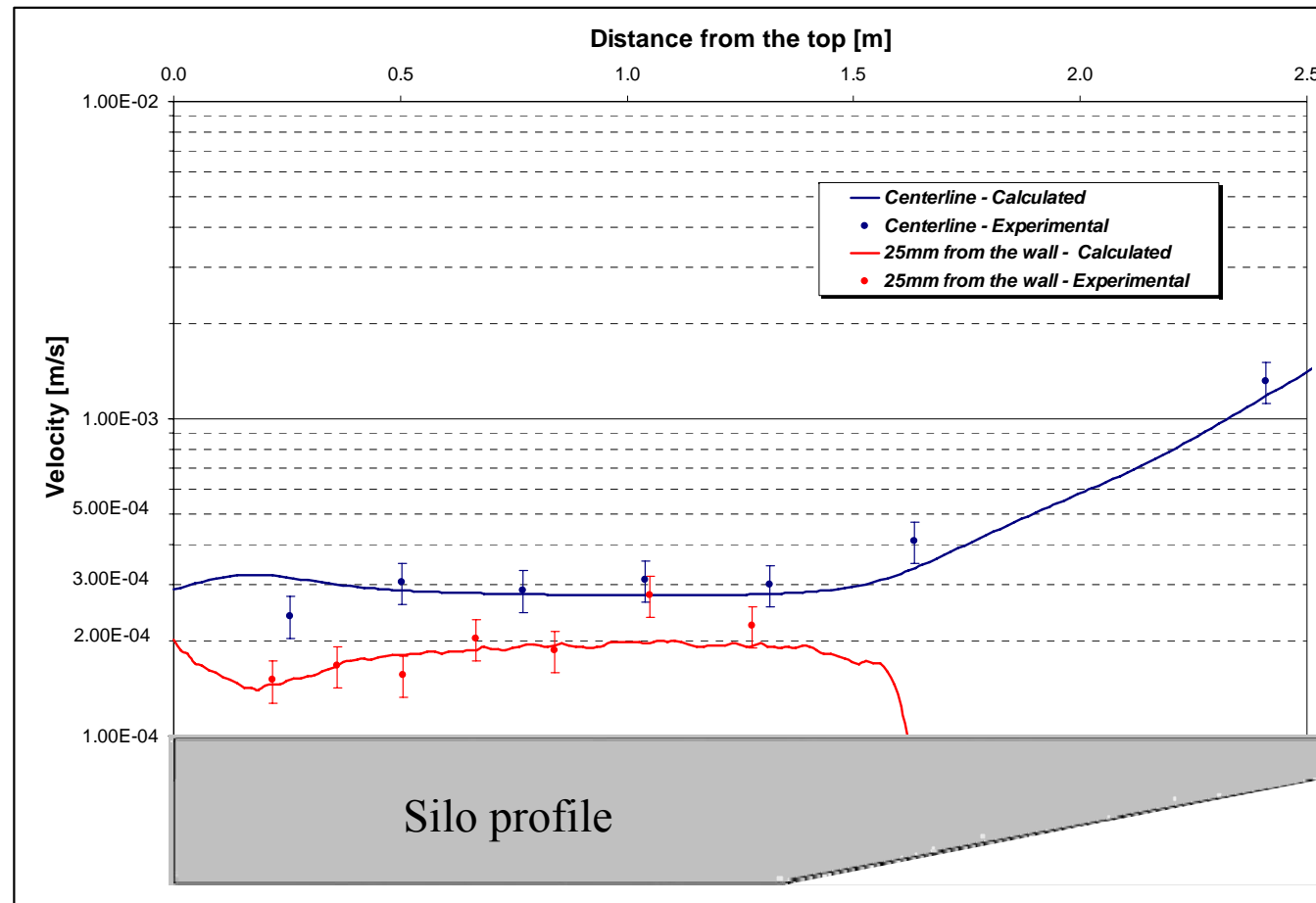
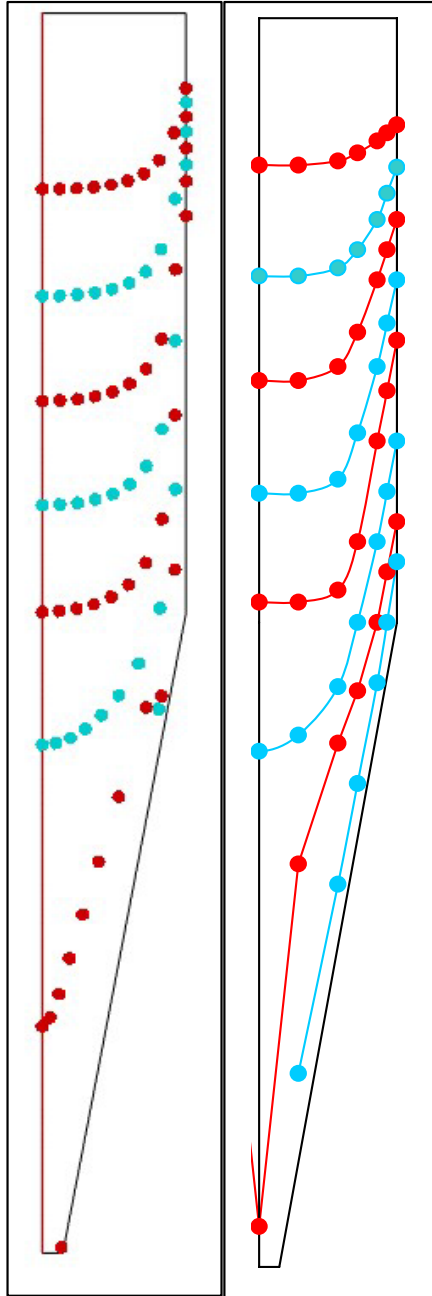


Figure. PE pellets velocity profiles along vertical direction for the base case. Error bars for experimental results  $\pm 15\%$ .



Hollow silo case – Descent profiles at the same times  
(every 15 minutes)



It is possible to see how the simulated case (on the left) and the experimental one (on the right) show good agreement with each other.

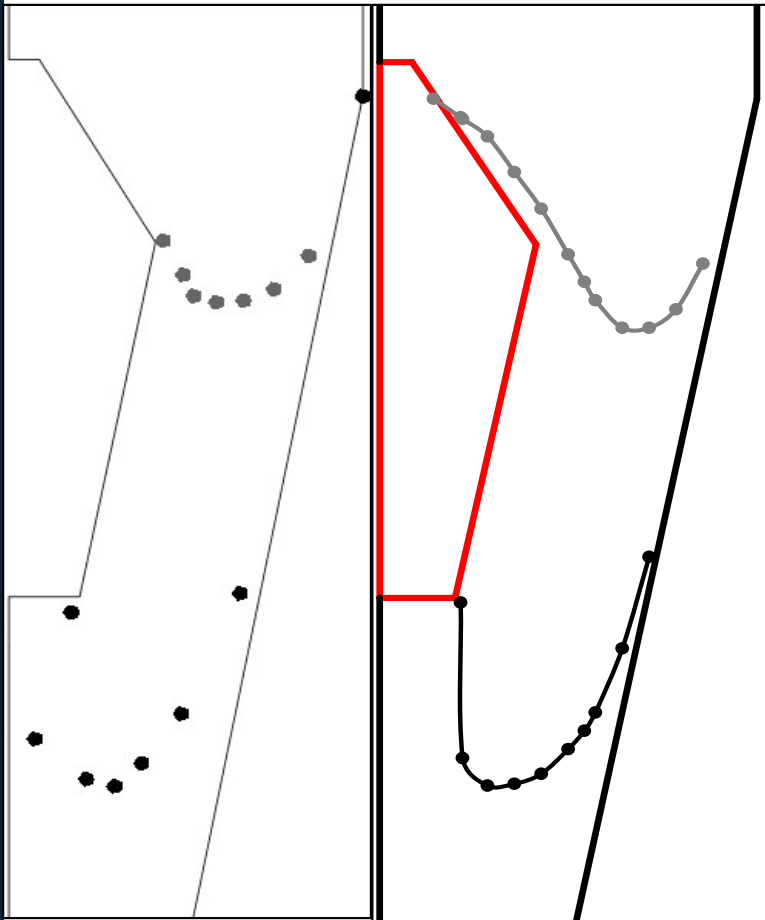
The different behaviour in the low part of the conic zone is due to bed density variations, not taken into account in these calculations.



## “Double cone” flow aid

Simulations

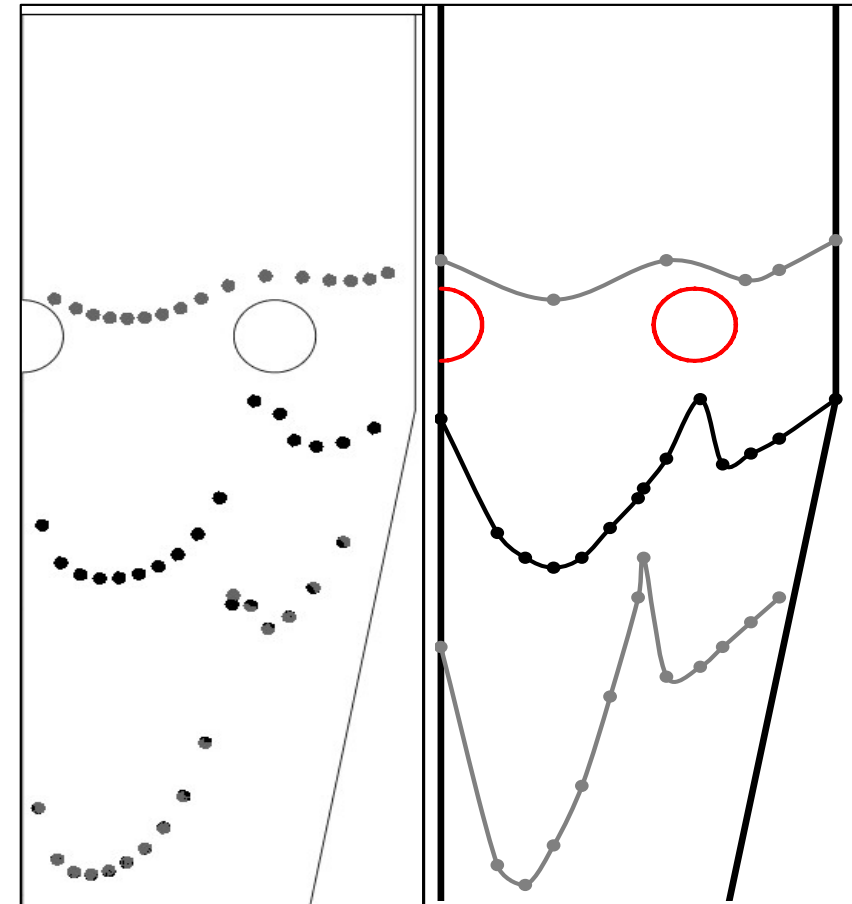
Experimental



## “Passing tubes” flow aid

Simulations

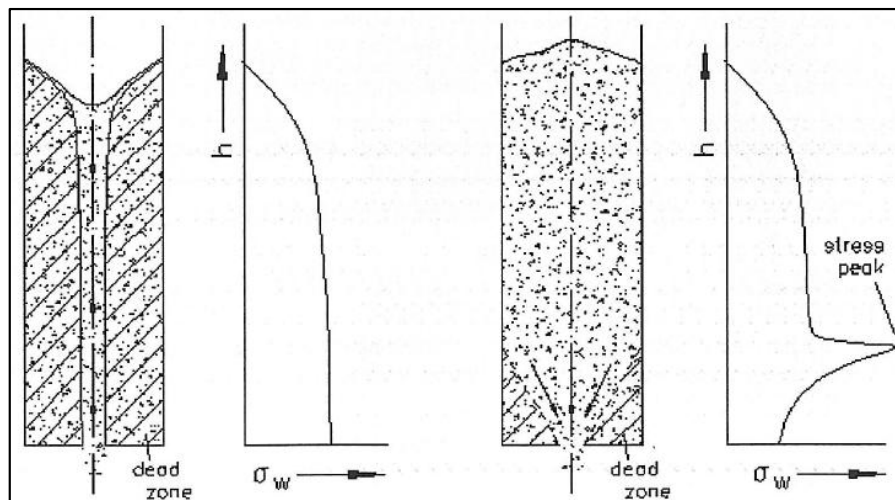
Experimental



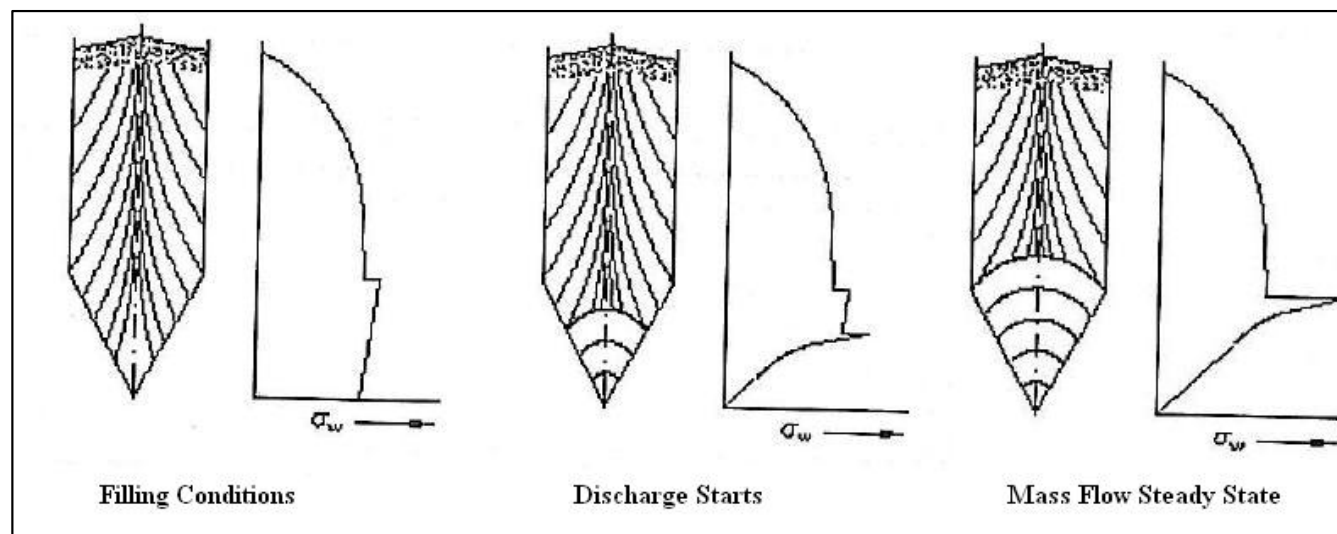
With the same parameters determined with the hollow silo, simulations and experiments maintain a good agreement, thus validating the model



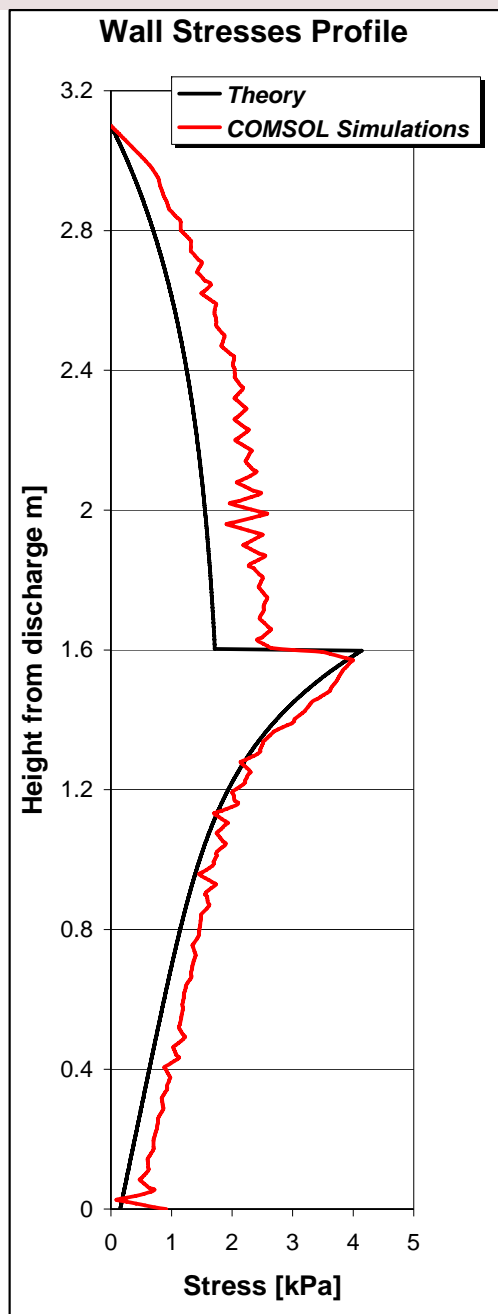
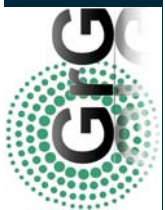
# Results - 4



Comparison between simulation-predicted wall stresses and those obtained following theory.







Results shown a good agreement with theory, especially in determining stress peak at cylinder-cone transition.

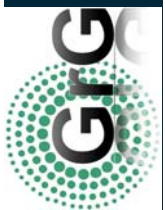


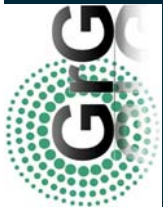
It has been shown that with COMSOL Multiphysics the model proposed gives good representation of granular solids rheology either for the descent profiles and for the wall stresses.

Possibility to handle complex geometries, such as silo with internal devices

Further developments could be:

- Not neglecting the interstitial fluid effect
- Taking into account bed density variations during discharge





***Thank you for your attention***



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3. Jenike, W., Gravity Flow of Bulk Solids, *Utah Eng. Exp. Stn.*, **Bull. 108**, 108 (1961).
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