Simulating Spiking Neurons Using a Simple Mathematical Model

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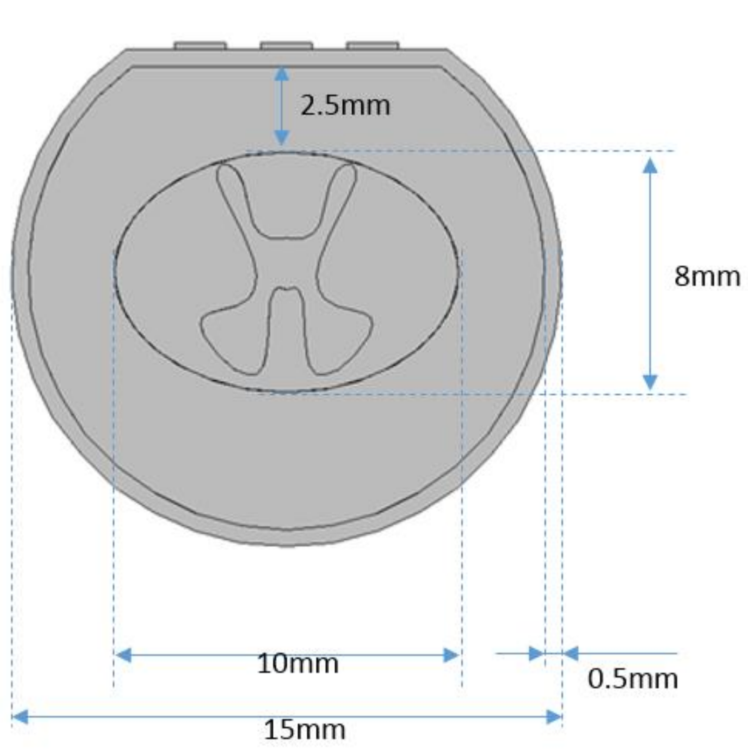
Introduction: Evolving studies related to external electrical stimulation of the spinal cord has been shown to enable voluntary motor function in paralyzed patients with spinal cord injury [1]. However, the relationship between stimulus and response is not completely understood. Simulations may be useful in predicting the response from particular stimulation parameters. Simulations may also aid in better understanding restorative spinal cord stimulation.

Computational Methods: Based on the average dimensions of the L1-L5 region [2] of the human spinal cord, a two dimensional longitudinal model was created. Two surface electrodes were modeled on the domain which represents the epidural fat.



(a)

(b)

Figure 1. (a) Cross section of the spinal cord with average dimensions of L1-L5.(b) 2D model used for simulations representing the longitudinal section of the spinal cord with two embedded electrodes.

Two model electrodes were used to apply an electrical stimulus to the model and the Maxwell's equations were solved to calculate the resulting potential distribution in the model.

$$\nabla \cdot \mathrm{J} = -\nabla(\sigma \nabla V) - \nabla\left(\epsilon_o \epsilon_r \nabla \frac{\partial V}{\partial t}\right) = 0 \qquad (1)$$

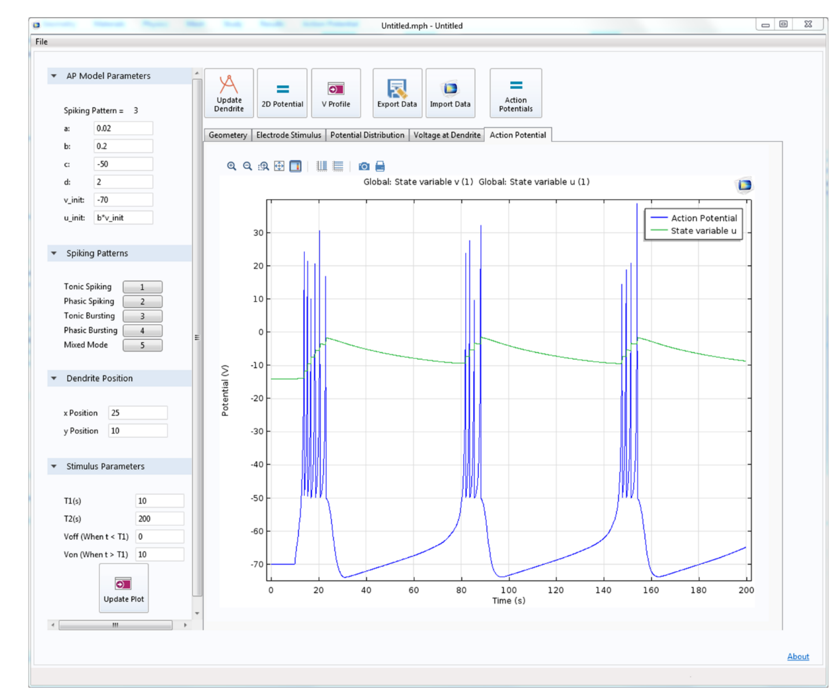
Calculated potential distribution is used as an input to the Izhikevich model (eq. 2-4)[3] to calculate the neuronal response in an axon embedded in the 2D spinal cord model.

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I \qquad (2)$$

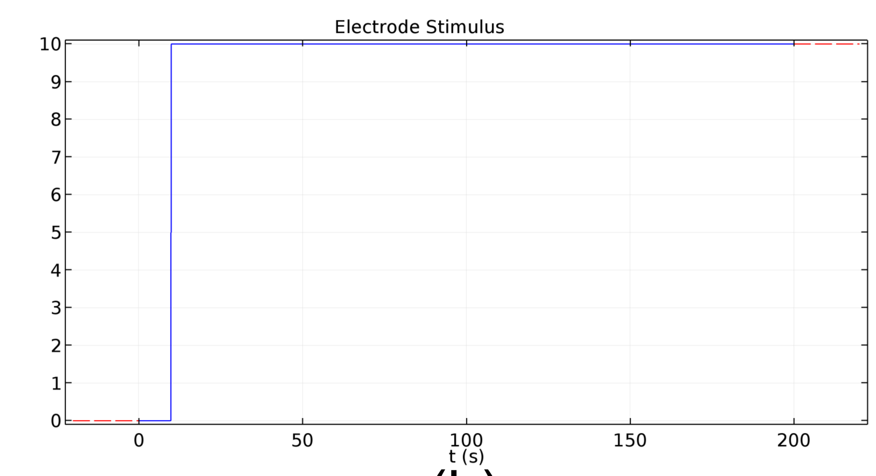
$$\frac{du}{dt} = a(bv - u) \qquad (3)$$

$$if\ v \geq 30mV, then \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases} \qquad (4)$$

Results: A COMSOL app was built using application builder to simplify the simulation process.



(a)



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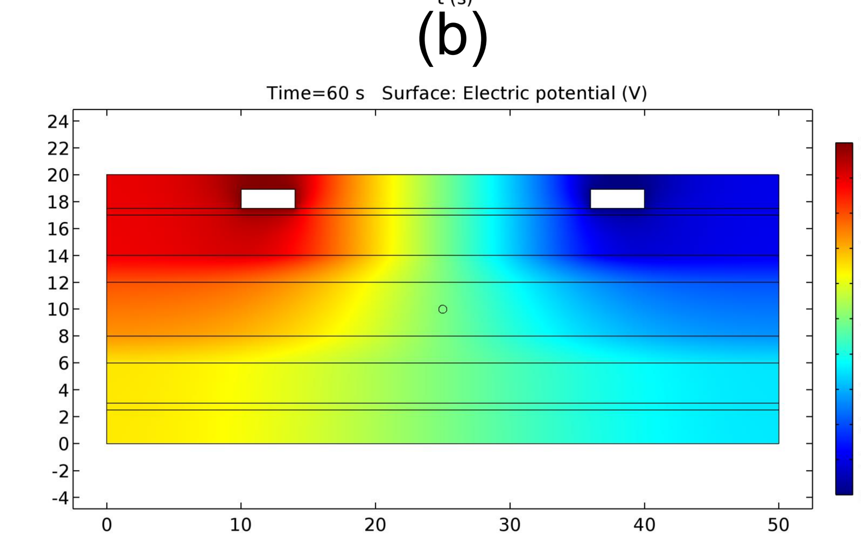

(c)

Figure 2. (a) Interface of the COMSOL app. (b) Stimulus used for the simulations. (c) 2D Potential distribution

A point potential was extracted from the potential distribution (figure 2.b) and used as an input to the Izhikevich model.

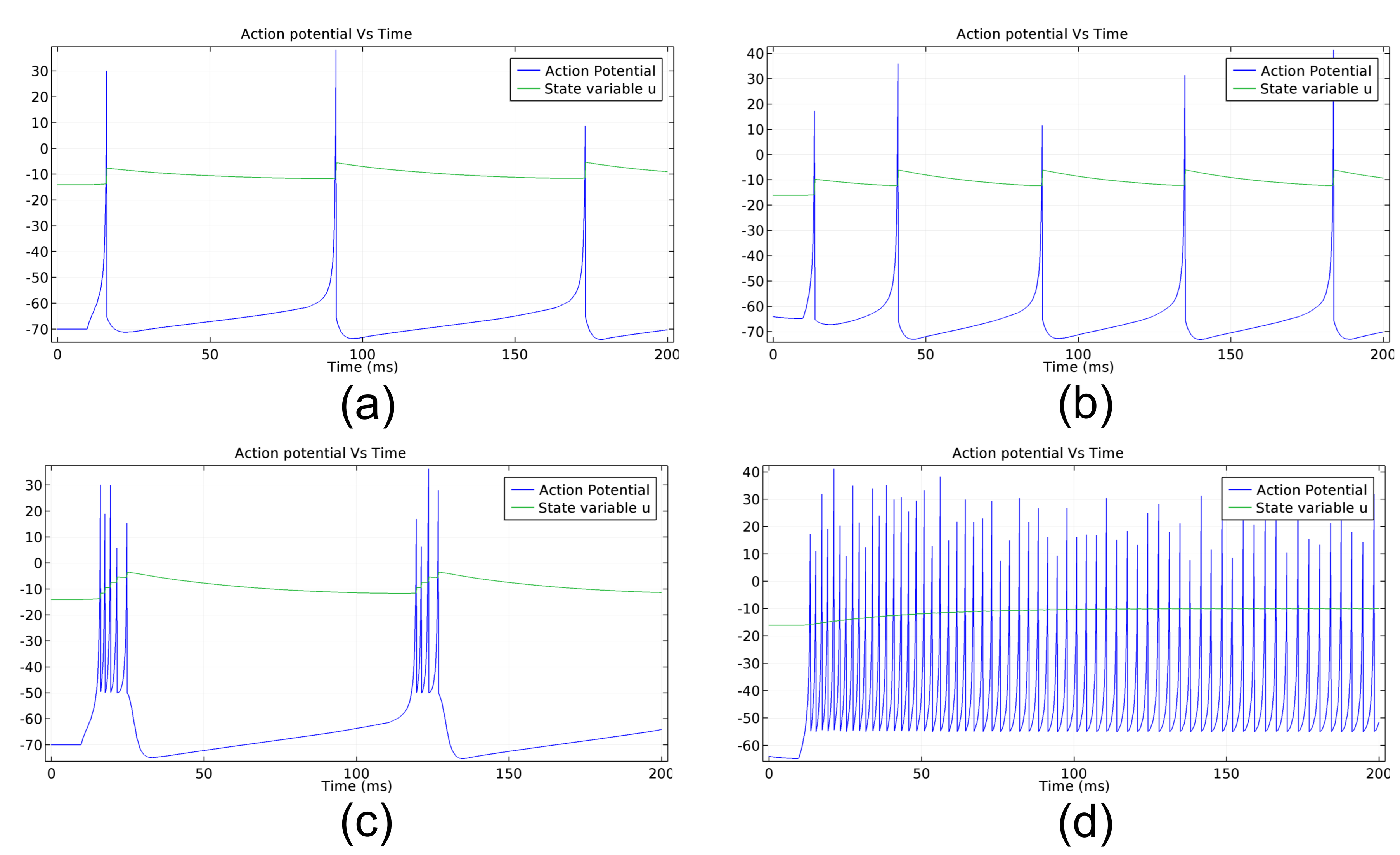

Figure 3. Four of the resulting action potential patterns (in mV) obtained from the simulations. (a) Tonic spiking. (b) Phasic spiking. (c) Tonic bursting. (d) Phasic bursting

Conclusions: Simulation results demonstrate the possibility of combining a geometry with a neural model to perform variety of simulations to better understand restorative spinal cord stimulation.

References:

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