Electromagnetic Modeling of a Millimeter-Wavelength Resonant Cavity

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Abstract: The measurement of dielectric constant at frequencies of 20 GHz or greater is important for specifying the optical properties of materials at millimeter wavelength. One method of measurement is to use a resonant cavity containing a sample of the material, and to relate the resonant frequency and quality factor of the resonance to the real and imaginary components of the complex dielectric constant through electromagnetic simulation. Because the actual measurement makes use of an external system such as a network analyzer and coupling elements, the electromagnetic simulation must include the effects of these external components. In the engineering literature, the external loading on the cavity is modeled by an equivalent circuit representation of the measurement system. In this paper, the input impedance is calculated directly from the fields in the cavity using Poynting's theorem. The physics-based approach provides a useful method to treat the effect of the measurement system on the idealized cavity.

Keywords: Dielectric measurement, millimeter wave imaging, electromagnetic permittivity, terahertz, Poynting's theorem.

1. Introduction

The dielectric constant of a nonmagnetic material is the key physical parameter for how it interacts with electromagnetic fields. Knowledge of the dielectric constant can quantify reflectance in imaging systems that use long wavelength, non-visible radiation, such as the Advanced Imaging Technology (AIT) systems used in airports. Because dielectric constant is frequency dependent, it needs to be measured at the frequency related to its application. The measurement of dielectric constant at frequencies of 20 GHz or greater is important for determining the optical properties of materials viewed in millimeter waves.

Dielectric constant is a complex quantity: the real part is associated with the wave phase velocity, and the imaginary part with absorption. Various experimental techniques are used to measure dielectric constant, including dielectric probes, free-space illumination, and resonant cavities.[1] Resonant cavities are particularly useful in detecting the small imaginary component in weakly absorbing materials, which is necessary to characterize semi-transparent objects. In one type of measurement using a resonant cavity, the *S*-parameter S_{11} – i.e., the reflection coefficient – is measured as a function of frequency using a network analyzer. In a weakly coupled system, the reflection is near unity except in the vicinity of an electromagnetic resonance in the cavity, where the reflection decreases sharply. The resonant frequency, and the frequency width of the resonance are measured, and these can be related through a simulation to the real and imaginary components of the complex dielectric constant of the materials filling the cavity.

For the computational reduction, it is important to take into account the effects of the measurement system to assure that the result extracted from the simulation corresponds with the physical measurement;[2] in our case, the measurement system involves a coupling iris and connecting waveguide. In this paper we show a method based on Poynting's theorem for harmonic fields that is able to characterize the input impedance at the iris aperture. The numerical calculation is complementary to an instrumental calibration that relates the experimental measurement of the reflection coefficient.

In Section 2, an experimental resonator for measuring configured samples of material is described. The numerical model and experimental results are given in Section 3. Application of Poynting's theorem to calculate reflection coefficients is presented in Section 4. Section 5 is the conclusion.

2. Experiment

For measurement of the complex dielectric constant in the frequency range of 15 – 22 GHz, a resonant cavity was designed using a section of WR51 waveguide.[3] The waveguide has a large dimension of a = 12.954 mm, small dimension b = 6.477 mm, and extends a depth c = 11.09 mm with flat metallic plates on both ends. A centered iris in the *ab*–plane couples the cavity with the input waveguide through the end wall of the waveguide. For measurement, an HDPE plastic fixture fills the cavity, designed to encapsulate a cylindrical sample of material in the center region of the cavity. The cavity geometry is illustrated in Figure 1. The experimental design was deliberately minimalistic to facilitate the simulation. The assembled experimental system is shown in Figure 2.

The reflectivity spectrum detected with HDPE as the sample is shown in Figure 3, measured across 10 – 40 GHz. The spectrum is well behaved between 11.6 and 26 GHz, as delimited by the cutoff frequencies for TE10 and TE11 waveguide modes. Two resonances are observed at 18.6 GHz and 24.1 GHz corresponding to cavity modes TE102 and TE301, respectively. The resonance spectrum as calculated from electromagnetic theory has seventeen resonant frequencies in the waveguide band, but only these two couple effectively through the iris.

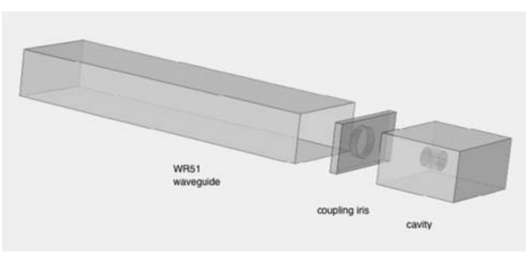

Figure 1. Schematic of WR51 cavity coupled to measurement waveguide. The placement of the sample inside the cavity is also illustrated.

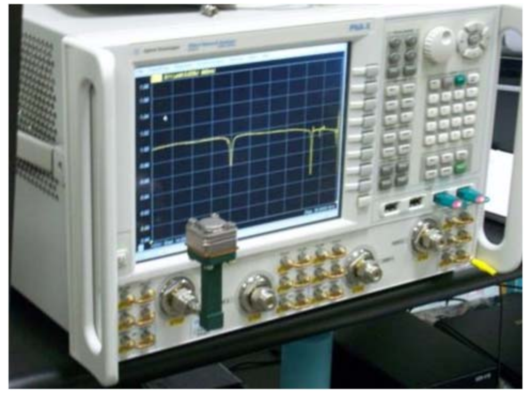

Figure 2. The WR51 resonator measurement system.

3. Simulation

The RF module in COMSOL was used to simulate the cavity system in a frequency domain study. The simulation uses an internal waveguide port midway in the WR51 input waveguide to detect S_{11}; the port is backed by a PML layer. The model includes the input waveguide, coupling iris, and cavity. For the HDPE fixture, a lossy dielectric of ε = 2.30 + 0.0062i is used. The metal walls are modeled with an impedance boundary condition with a finite conductivity. The waveguide boundary behind the PML has a scattering boundary condition. With this initialization, the wave propagating down the waveguide towards the measurement system does not reenter through the port: this properly corresponds to a matched termination of the waveguide. Figure 4 shows the electric field solution for the TE102 cavity mode.

Although COMSOL computes S_{11} at the internal port, this is not a good correspondence with the experiment. In the experiment, the calibration is done at the end of the waveguide where the coupling iris is mounted. We tried two methods to simulate the experimental measurement. In the first, we applied an instrument-like calibration for the port, using a simulated short, offset short, and a PML termination.[4] In the second method, we applied Poynting's theorem for harmonic fields to compute the input impedance at the iris. The Poynting's theorem method is discussed below.

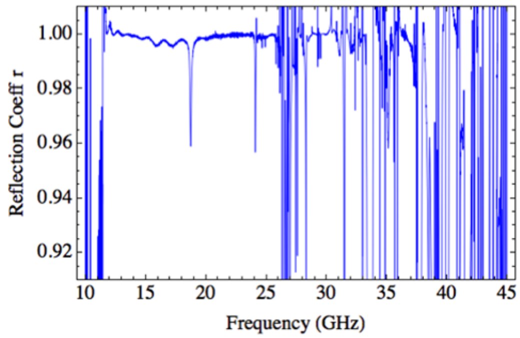

Figure 3. Reflection spectrum of cavity.

In order to detect the dielectric constant of the sample, the real part of the dielectric constant of the sample is varied in the simulation to match the resonant frequency, and the imaginary part of the dielectric constant is varied to match the frequency width. The correspondence between simulation and the experimental data is shown in Figure 5 (the offset of the experimental data can be disregarded). The dielectric of the sample in the experimental measurement is found to be $\varepsilon = 2.57 + 0.28i$. The simulated reflection spectra are comparable in the two calculations, one derived from the input impedance computed from the volume fields, and the second corresponding to the simulated S_{11} detected in a virtual port in a lossless input-waveguide.

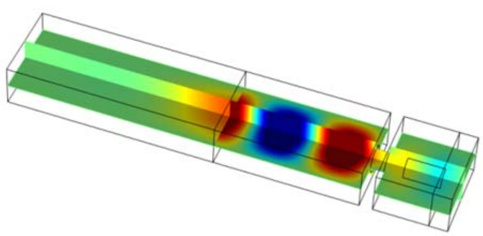

Figure 4. COMSOL solution for the electric field E_y at 19.51 GHz.

4. Poynting's Theorem Method

The input impedance of a two-terminal linear network can be found based on field concepts. Following Jackson, [5] at the input terminals of the linear, passive electromagnetic system the input impedance $Z = R - iX$ is given by

$$R = \frac{1}{|I_i|^2}\left\{Re\int_V \mathbf{J}^* \cdot \mathbf{E}\, d^3x + \oint_{S-S_i} \mathbf{S}\cdot\mathbf{n}\, da \right. \\ \left. + 4\,\omega\, Im \int_V (w_m - w_e)\, d^3x\right\}$$

and

$$X = \frac{1}{|I_i|^2}\left\{4\omega\, Re\int_V (w_m - w_e) d^3x \right. \\ \left. - Im\int_V \mathbf{J}^* \cdot \mathbf{E}\, d^3x\right\}$$

Here, the integrals are performed over a two-terminal system surrounded by a surface S enclosing the volume V. To match the experiment, the volume is chosen to encompass the cavity and iris; the surface integration excludes the iris aperture, which is the terminal surface, S_i.

The integrands are all expressions computed by COMSOL. In our example, the conductivity is zero in the interior volume, so the free current $\boldsymbol{J}$ is zero. The surface integral is derived from the resistive loss (Qrh) in the bounding surfaces. The time-averaged harmonic electric energy density, w_e, and magnetic energy density, w_m, are computed by COMSOL as (Weav) and (Wmav), respectively. The dielectric loss is incorporated through the imaginary component of the dielectric constant, and is exhibited in the imaginary part of the integrated electric energy density. The harmonic input current, I_i, is computed by integrating the vector displacement current over the iris aperture, S_i.

The input impedance Z relates to the reflection in a transmission line with characteristic impedance Z_0 as follows:

$$S_{11} = \frac{Z - Z_0}{Z + Z_0}$$

Figure 5 shows that the reflection coefficient computed from the cavity fields compares well with the "calibrated" port S_{11} measured at the internal waveguide port.

The significance of the physics-based calculation of the reflection coefficient is that it allows the reflection to be calculated at places that are not conveniently modeled by a simulation "port" or connected to an instrumental calibration. For example, placing the port at the iris (which corresponds to the experimental calibration plane) requires a numeric port, which restricts the solution to modes transmitted through the iris "waveguide." We are also studying the possibility of de-embedding the iris and fixture from the sample

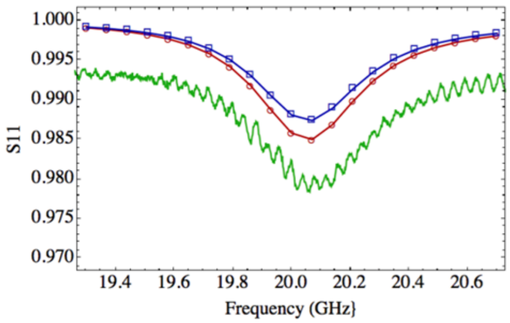

Figure 5. S11 reflection coefficient: (circle) calibrated internal port; (square) Poynting's theorem calculation; and (solid) experiment.

in order to improve measurement precision. This requires representing the embedding structures as a two-port network, and correcting the measured S-parameters. The correspondence to the simulation can then be accomplished with the appropriate choice of volume for the field integrations.

8. Conclusions

Frequency domain computer modeling is used to accomplish dielectric measurement in a resonant cavity: the real part of the dielectric constant of the measured material is varied in the simulation to match the resonant frequency, and the imaginary part of the dielectric constant is varied to match the frequency width (or resonant quality factor). In the experiment, a network analyzer is used to measure the reflection coefficient through a waveguide with an aperture (port) looking into the cavity. By applying Poynting's theorem to the simulation, the reflection coefficient is calculated from the field solutions in the simulation space beyond the port. The method should apply to an arbitrary configuration of the input port in order to match the instrumental calibration.

9. References

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