Equation-Based Modeling: Simulation of a Flow with Concentrated Vorticity in an Unbounded Domain

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Abstract

The curl of the velocity field in a fluid flow is called the vorticity and is best interpreted as twice the effective local angular rotation rate vector of the fluid at a point. The vorticity field is concentrated if it is nonzero only within a bounded region of space even if the region filled with fluid is unbounded. The trailing vortex sheet left behind a lifting wing that has been started from rest and has been in motion for a finite time is an example of a flow with concentrated vorticity. Turnkey fluid dynamics simulation packages, including the CFD Module in COMSOL Multiphysics® software are well suited to simulation of flows in bounded regions but require careful treatment when the region is unbounded. In the simulation of such flows there is an advantage to solving problems simple enough to admit an analytical solution to serve as a basis for comparison. The present model simulates Hill's Spherical Vortex (Hill, 1894), a flow in which the vorticity is nonzero within a sphere but zero outside it.

The region filled with fluid has no external boundary. The model employs a change of independent variable that maps points exterior to the sphere to points interior to another sphere of the same radius. I will call the latter region the proxy space. COMSOL software carries out a simultaneous solution for the flow in a subset of physical space (namely the interior of the sphere, where the vorticity is nonzero) and in the proxy space. Both computational domains are thus bounded by construction. The change of position variable that takes points in the exterior of the original sphere to the points in the proxy space is an example of so-called Kelvin Inversion (Kelvin, 1884). The simulation employs the General Form PDE physics interface for both the physical and proxy regions. The challenges in model development include: (a), the determination of boundary conditions sufficient to ensure that the solution for the velocity field is unique; (b), the derivation of the appropriate partial differential equation in the proxy region; and (c), the appropriate choice of dependent variable in the proxy region. Results of the simulation are presented (e.g. Figure 1) and compared with the analytical solutions and the results agree.

Reference

1. M.J.M. Hill On a Spherical Vortex. Philosophical Transactions of the Royal Society of London (A), Volume 185, pp 213--245, 1894.

2. Lord Kelvin Reprint of Papers on Electrostatics and Magnetism by Sir William Thomson, Second Edition, MacMillan & Co. Article IV, Electric Images pp 144--154, 1884 [Reprinted 2011 as part of the Cambridge Library Collection]

Figures used in the abstract

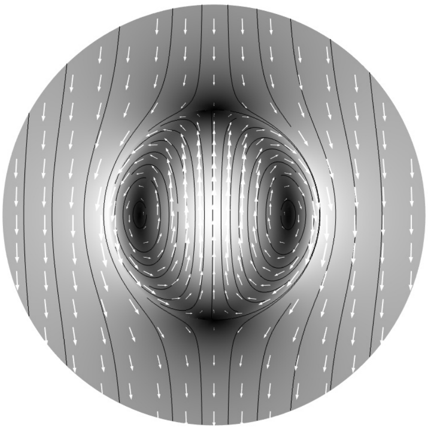

Figure 1: COMSOL simulation of Hill's spherical vortex. The vertical centerline is an axis of symmetry. The gray levels indicate fluid speed, the white arrows represent velocity vectors, and the black lines are streamlines.