Cracking in Quasi-Brittle Materials Using Isotropic Damage Mechanics

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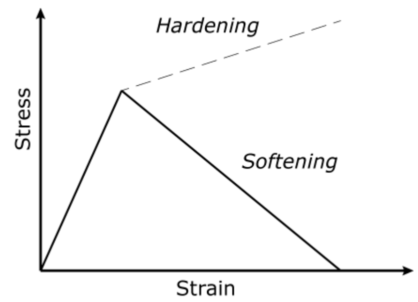
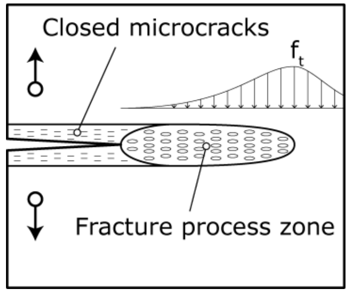
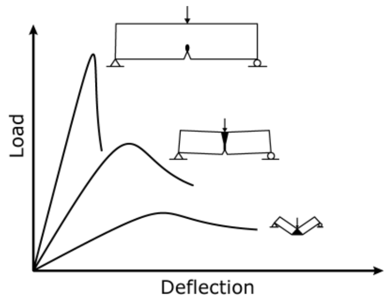
Contents

Introduction

- What is a quasi-brittle material?
  - Strain softening
  - Fracture process zone (FPZ)
  - Strong deterministic size effect
- All models presented are applicable to such materials but the presentation will focus on **concrete**
  - Other examples are rocks, ceramics, ice …
- Why analyze cracking of concrete?
  - Failure
  - Performance
  - Durability
  - …







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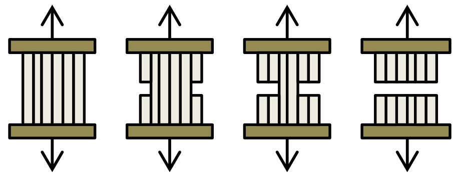
Isotropic Damage Mechanics

- Progressive loss of material integrity due to propagation of material defects
  - For example voids, cracks …
- Leads to a degradation of the macroscopic stiffness
  → Non-linear response

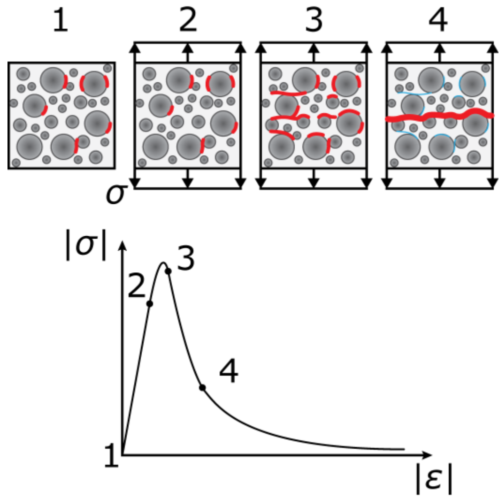
- The intact material carries a stress $\bar{\boldsymbol{\sigma}}$, often called the effective stress
- Over a unit volume of material the stress is then:

$$\boldsymbol{\sigma} = (1-\omega)\bar{\boldsymbol{\sigma}}$$

where $(1-\omega)$ describes the relative amount of intact material, i.e. $0 \leq \omega \leq 1$



concrete = cement matrix + aggregates



In a general formulation of damage mechanics, the scalar $(1-\omega)$ is replaced by a 4th order tensor

Isotropic Damage Mechanics

- Quasi-static formulation of the momentum balance and small strain kinematics:

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{F}_\mathrm{V} = \mathbf{0};\ \boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^\mathrm{T}\right);\ + \text{B.C.}$$

- Assuming that the intact material is linear elastic, the constitute equation is given as:

$$\boldsymbol{\sigma} = (1-\omega)\bar{\boldsymbol{\sigma}} = (1-\omega)\mathbf{C}_\mathrm{el} : \boldsymbol{\varepsilon}$$

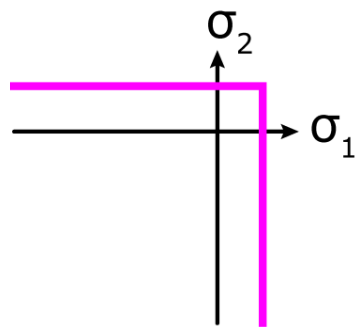
- The non-linear response of the material is thus given by the evolution of the damage parameter ω

Isotropic Damage Mechanics

- A formulation following the framework by Oliver et al. (1990)
- Loading function f with the internal variable κ:

$$f(\boldsymbol{\varepsilon}, \kappa) \equiv \tilde{\varepsilon}(\boldsymbol{\varepsilon}) + \kappa \leq 0$$

- The elastic domain is controlled by the equivalent strain $\tilde{\varepsilon}$:



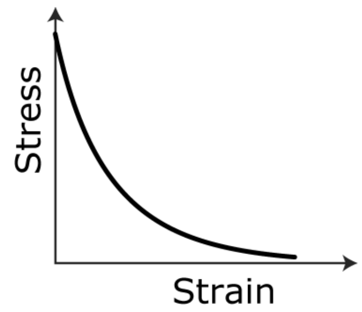
$$\tilde{\varepsilon}(\boldsymbol{\varepsilon}) = \frac{1}{E} \max_{I=1,2,3} \langle \mathbf{C}_{\text{el}} : \boldsymbol{\varepsilon} \rangle_I = \frac{1}{E} \max_{I=1,2,3} \langle \sigma \rangle_I$$

- Loading/unloading conditions on the Kuhn-Tucker form:

$$f \leq 0, \ \dot{\kappa} \geq 0, \ \dot{\kappa} f = 0$$

- Damage evolution law $\omega(\kappa)$ for exponential softening:

$$\omega(\kappa) = 1 - \frac{\varepsilon_0}{\kappa} \exp\left(-\frac{\kappa - \varepsilon_0}{\varepsilon_f}\right)$$



In the Paper an additional defintion of $\tilde{\varepsilon}$ is given and also another damage evolution law $\omega(\kappa)$

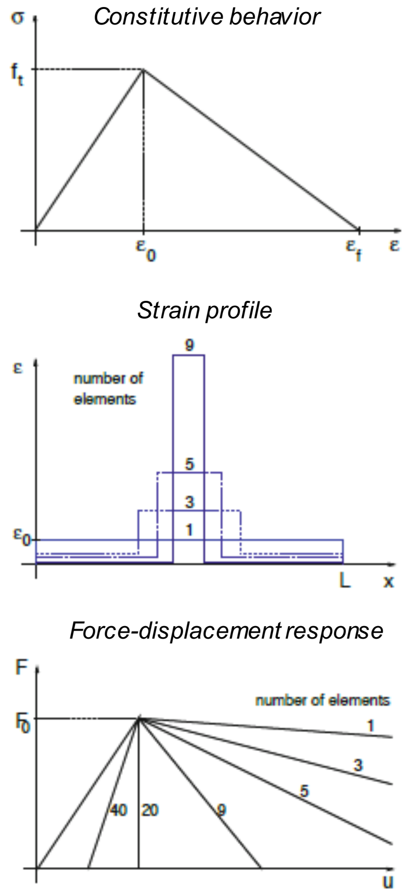
Lack of mesh objectivity

- The stress-strain formulation leads to a strong mesh dependency of results
  - No converging result upon mesh refinement

- Strains will localize in the narrowest possible region, i.e. a single element
- The amount of energy dissipated decrease with the element size
  - Eventually the response becomes unstable

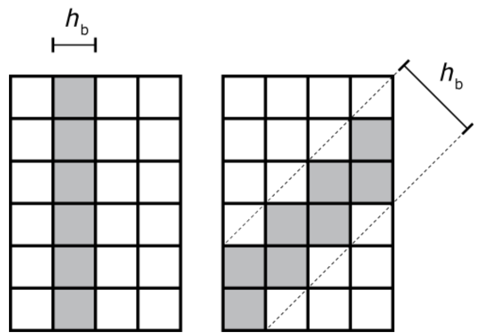
- More information is needed about the material and/or fracture process in needed!!!

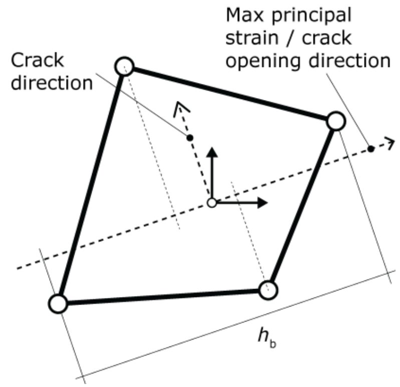
Example from Jirásek (2011)



Local formulation

- Crack-band method by Bažant and Oh (1983)
- At each material point (Gauss point):
  - → Supply information about the simulated FPZ, the crack-band width h_b
  - → Construct unique stress-strain law from a stress-crack opening law given by G_f
  - → $\varepsilon_f = G_\mathrm{f}/(f_\mathrm{t} h_\mathrm{b}) + \varepsilon_0/2$
- How to find an appropriate value of h_b?
  - Depends on for example interpolation order, element size and shape and the stress sate
- Here a projection method is used as proposed by Cervenka et al. (1990)





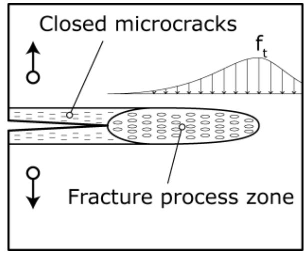
Non-local formulation

- Supply information about the material structure
  - Width of the experimentally observed FPZ
- Non-local continuum that averages some variable/s over its spatial neighborhood
- Following Peerlings et al. (1996), higher order gradients are introduced in the constitutive law
  - Non-local equivalent strain $\bar{\varepsilon}$ calculated as:

$$\bar{\varepsilon} - c\nabla^2\bar{\varepsilon} = \tilde{\varepsilon} \text{ with the B.C. } \nabla\bar{\varepsilon}\cdot\mathbf{n} = 0$$

which replaces its local counterpart in the loading function

  - Parameter c can be related to the width of the FPZ



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- Isotropic damage mechanics and localization
- Implementation in Comsol Multiphysics
- Examples
- Conclusions

Implementation in Comsol Multiphysics

- Implemented in Comsol v5.2 (v5.2a)
- Utilizes the Linear Elastic material model of the Solid Mechanics interface, but:
  - Introduces a new stress `dmg.Slxx` ($\boldsymbol{\sigma}$) which replaces the default stress `solid.slxx` ($\bar{\boldsymbol{\sigma}}$) in the weak expression

$$\bar{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}}{1-\omega}$$

- This new stress is defined using equation-based-modelling:
  - Domain ODE with the internal variable at the previously converged step κ_{old} as dependent variable
  - Discretized using Gauss point data shape functions
  - Previous solution node
  - The current state of damage calculated as $\kappa = \max(\tilde{\varepsilon}, \kappa_{old})$

Implementation in Comsol Multiphysics

- To calculate the crack-band width using the projection method the `atlocal` operator is used to obtain information about element coordinates and stress states

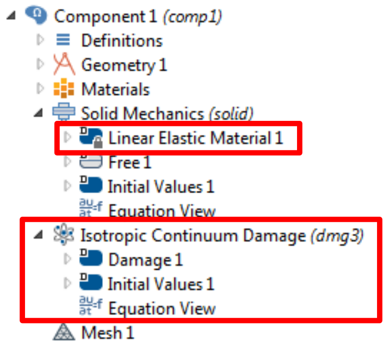
- The non-local model introduces an additional PDE to be solved with the non-local equivalent strain $\bar{\varepsilon}$ as dependent variable.
  - Discretized using Lagrange shape functions

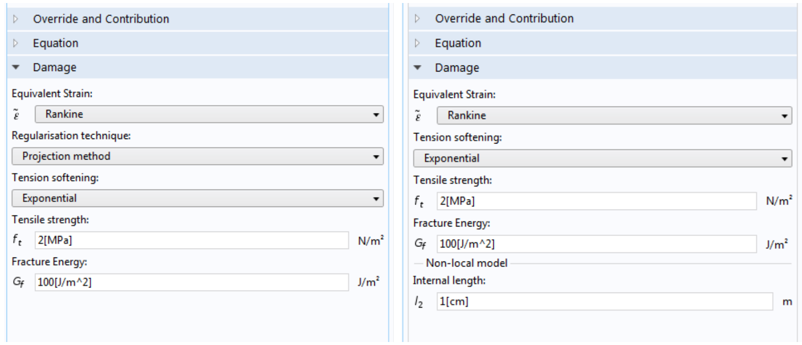
- Only major difference is in the variable definition of κ

Local	Non-local
$\kappa = \max(\tilde{\varepsilon}, \kappa_{old})$	$\kappa = \max(\bar{\varepsilon}, \kappa_{old})$

Custom physics interface

- Created using the Physics Builder
- Currently includes several definitions of:
  - The equivalent strain $\tilde{\varepsilon}$
  - The damage evolution law $\omega(\kappa)$
- Local version:
  - Different regularization techniques
- Non-local version:
  - One additional material parameter
- Both versions in the same interface



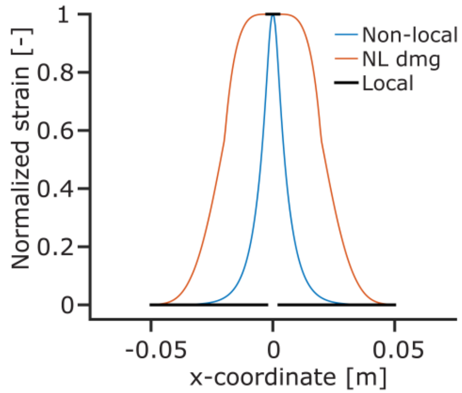


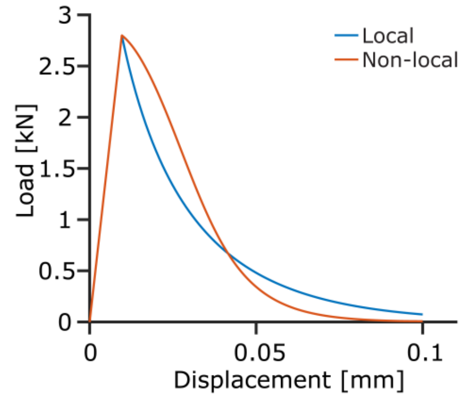
Local version Non-local version

Contents

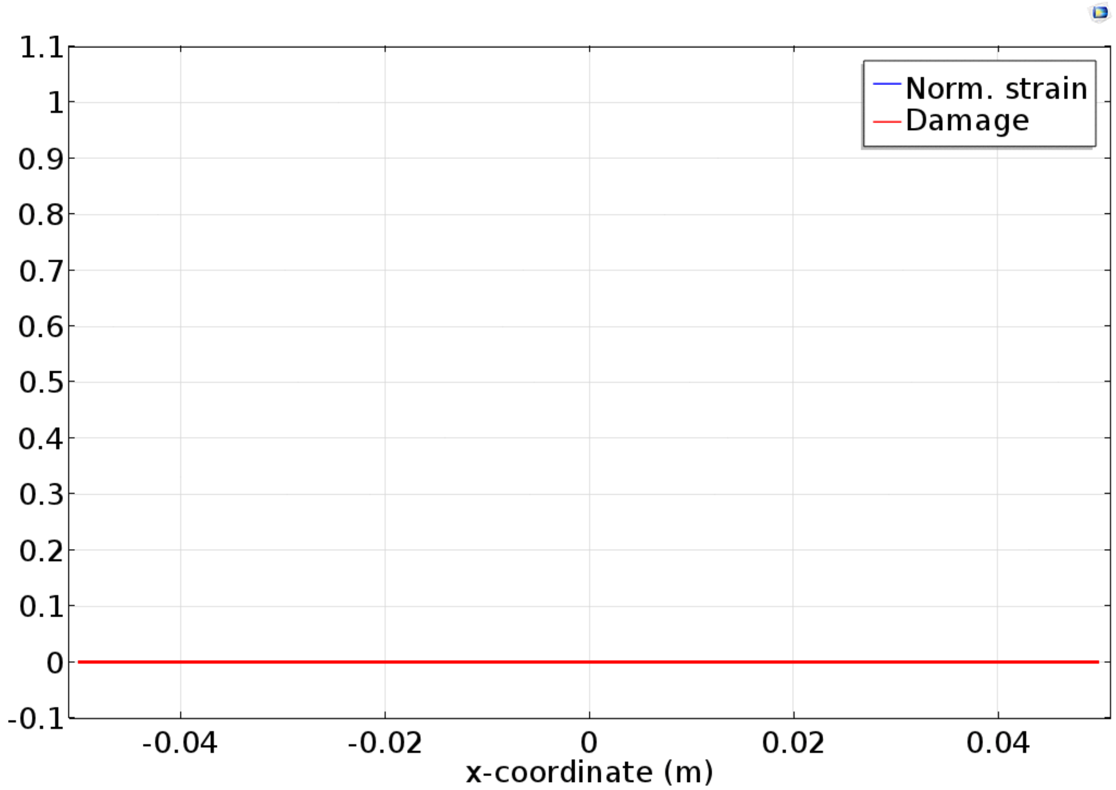
Plain Concrete – Uniaxial tension

- Extension of a bar by a prescribed displacement
- Highlight the differences of the two formulations
- Tensile strength of a single element (red) reduced by 5 % to force strains to localize
- Local model:
  - Strains localize in one element
  - Load-displacement curve has the same shape as the strain softening curve
- Non-local model:
  - Strain localization distributed over elements
  - Load-displacement curve influenced by the development of the localization zone



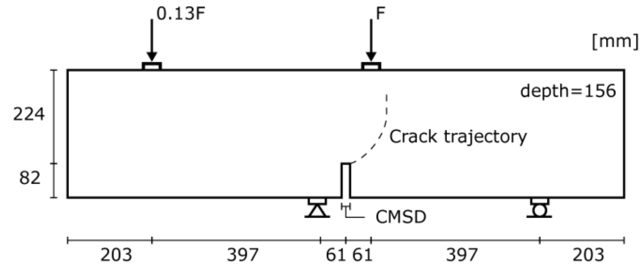


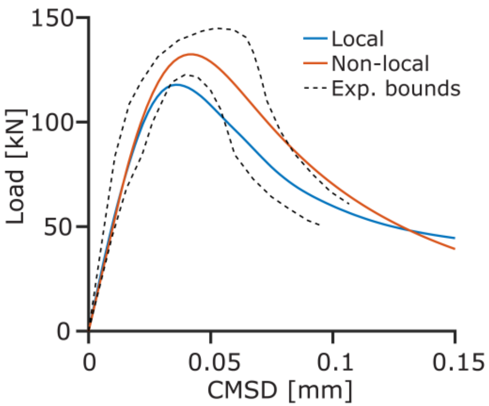
Plain Concrete – Uniaxial tension



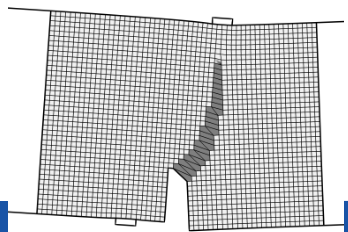
Plain Concrete – Mixed mode fracture

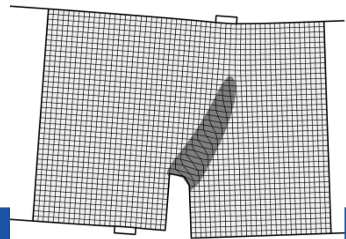
- Test series by Arrea and Ingraffea (1982)
- Notched beam under 4-point bending to simulate a curved crack trajectory
- Local and Non-local model with same mesh (7.5 mm) and material parameters
  - NL model uses quadratic interpolation
- NL gives better estimate of peak load but underestimates the softening
  - Due to difference in crack trajectory?





Local model

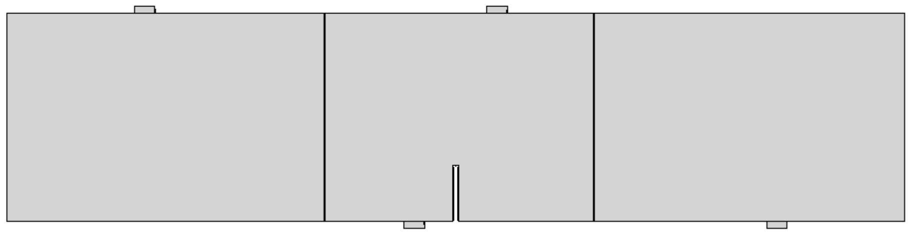




Non-local model

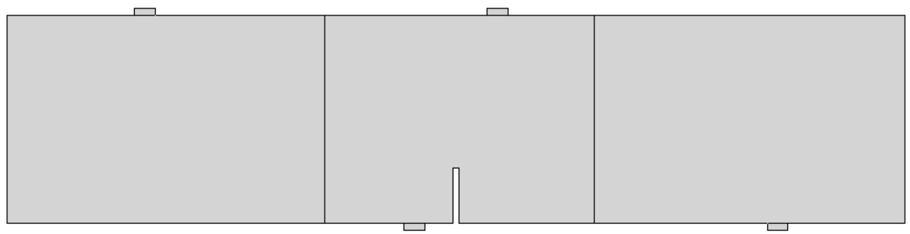
Plain Concrete – Mixed mode fracture

Local model



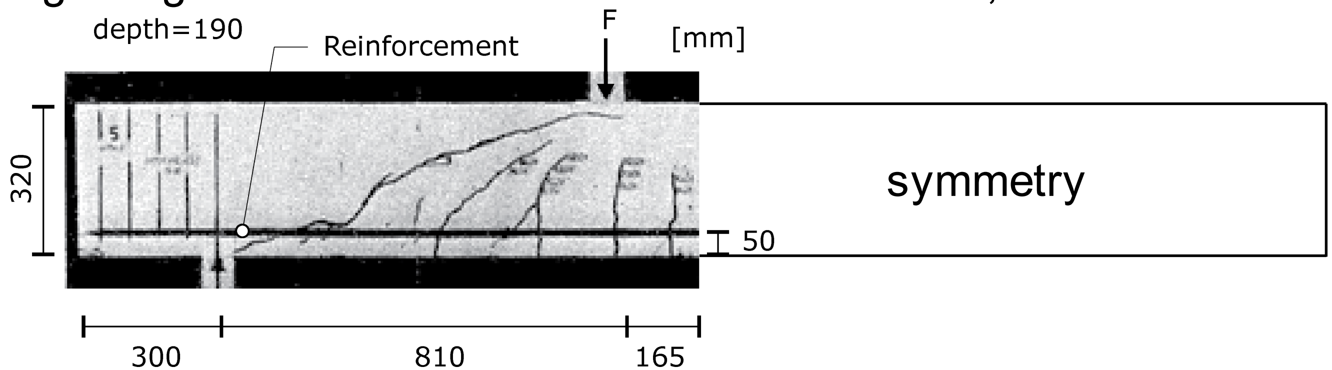
Non-local model

— *Extent of damage*



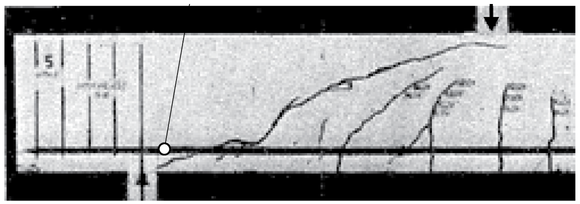
Reinforced concrete – 4-point bending

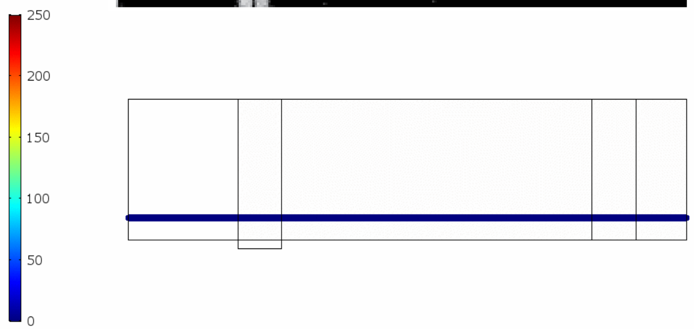
- Application of the implemented model to a more complicated problem
  - Only Local model
  - Both tensile and compressive damage
- Heavily reinforced concrete beam tested by Leonhardt (1972)
  - Failure due to inclined crack from support to load point
- Reinforcement remain elastic, included as truss elements
- Triangular grid to minimize the mesh bias of cracks, ~15 mm

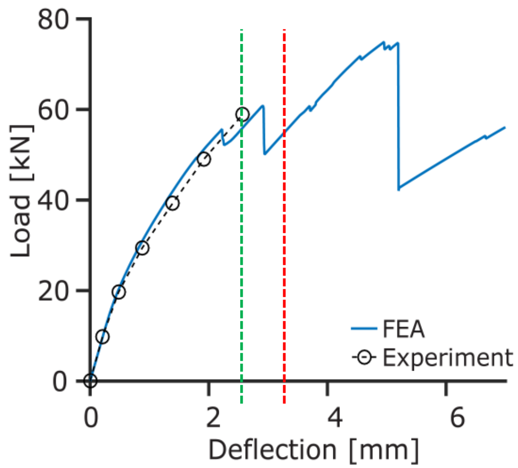


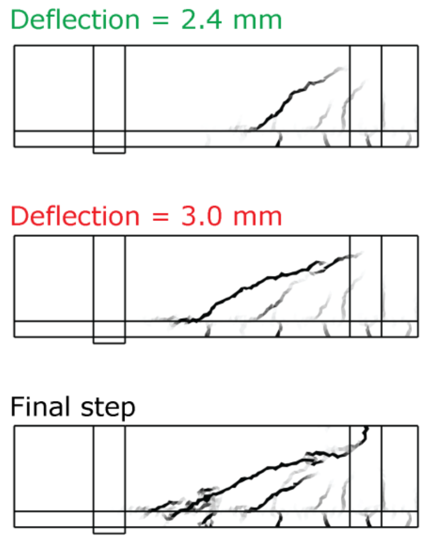
Reinforced concrete – 4-point bending

- First inclined crack agrees with the maximum reported load
- Followed by additional inclined cracks
- “Ultimate” failure due to combination of inclined cracks and chrushing









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Conclusions

- Implementation of an isotropic damage mechanics model to complement the solid mechanics features of Comsol Multiphysics
- Enables efficient analysis of cracking in quasi-brittle materials
- The model is introduced in a custom physics interface
- Two different regularization techniques are studied to ensure mesh objectivity of solutions during strain localization
- The model is applied to both plain and reinforced concrete with good agreement between simulated and experimental results

Thank you for your attention!

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