The Spherical Design Algorithm in the Numerical Simulation of Fiber-Reinforced Biological Tissues

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Abstract

The numerical results of the unconfined compression test on a sample of Articular Cartilage (AC) are discussed. AC is modelled as a load-bearing, deformable, fiber-reinforced material filled with an interstitial fluid and comprising statistically oriented collagen fibers, chondrocytes, and a matrix of proteoglycans. The fluid has the role of transporting nutrients to the chondrocytes, and absorbing the stresses in the joint. A strain energy is assigned to characterize the mechanical behavior of the system. The collagen fibers contribute to determine both the elastic properties of the matrix and the pathways taken by the fluid, thereby having a non-trivial influence on the tissue's permeability.

To account for the role that each fiber, or family of families, plays on the overall properties of the tissue, an additive strain energy, often referred to as "ensemble potential", and an additional permeability should be accounted for in the model [1]. AC shows depth and deformation dependent properties [2]. In particular, fibers in the tissue are statistically arranged as in Figure 1. Thus, a probability density distribution is needed as a weight for each fiber direction in the strain energy and in the permeability. Consequently, a suitable averaging procedure should be implemented to compute these quantities, which, in fact, have to be integrated over all the possible directions that fibers could have at each point of the tissue.

Moreover, both these additional quantities are strain-dependent, and have to be computed and updated for each time instant of the related FEM simulation. To numerically account for the presence of collagen fibers, we implemented the Spherical Design Algorithm (SDA) [3] into COMSOL Multiphysics®. As the convergence of the SDA is sensitive to its possible settings (t-Design), some available sets [4] for t-Design have been compared with each other, taking into consideration their numerical performances, and their accuracy in capturing the solution of the considered benchmark. As a basis for comparison, the numerical outcomes of a different quadrature scheme have been used. The latter, in turn, has been plugged into the FEM model through the possibility of a coupling between COMSOL and MATLAB® routines. The presented implementation of the SDA into the FEM software could represent a useful tool in describing more accurately heterogeneous deformable tissues having complex internal structure, as a fiber reinforcement, and whose description is challenging both from the mathematical and the numerical viewpoint.

Reference

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Figures used in the abstract

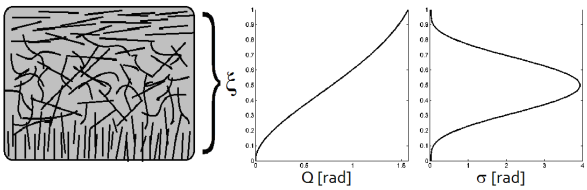

Figure 1: Distribution of fibers in AC. The mean angle (formed with the longitudinal axis) and the variance are represented in function of the depth.