Analytical Method to Calculate EMF Induced in Ionic Liquid by Magnetic Field

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Abstract: Simplistic, 1 dimensional (ID) analytical calculations in electro-magneto-hydrodynamics are attractive to technologists and researchers given the computational resources and time required by 3 dimensional F.E. (Finite Element) tools. However, such analytical calculations need to be checked against 3D tools like COMSOL Multiphysics which are more realistic. In this paper EMF (Electro-motive Force) induced in an ionic liquid flowing past a magnetic field (generated by coils carrying current) was calculated using analytical expressions. Results of the calculations were compared with results of a F.E. model built in COMSOL. Calculations were performed at varying coil current levels and for pipes of insulating and conducting materials. The calculated EMF values show minor deviations (~1%) from those computed using the F.E. model, in simple cases (insulated pipe). For complicated cases (electrically conducting pipe), deviation from F.E. modeling results intensifies (~10%). Hence 3D F.E. models are necessary for an accurate evaluation of the outcome in complicated cases. However, 1D calculations are still useful for an initial and quick understanding of electro-magneto-hydrodynamic processes in the interest of time and resources.

Keywords: Finite Element, Electro-Magnetic, Induced EMF

1. Introduction

Electro-Magneto-hydrodynamic phenomena involve interactions between fluid flow, electric and magnetic fields and are important to the working of devices like Micro Electro Mechanical Systems (MEMS), magnetic flowmeters, magnetic separators etc. For a proper understanding of these processes, analytical expressions derived from first principles are often used. For example, the Helmholtz-Smoluchowski equation [1] is popularly used in understanding electroosmotic flow in MEMS devices. In the present study, EMF induced in an ionic liquid (Faraday's law of electromagnetic induction) by a magnetic field traverse to the flow direction (Fig 1) was calculated using analytical expressions [2]. The calculations were performed at varying current levels (producing varying magnetic flux densities) and also for cases where the pipe is made of a conducting material. Subsequently, a 3D F.E. multiphysics model was built in COMSOL, to simulate the interaction between the magnetic field and the flow field to yield the induced EMF. The analytical results were then checked against F.E. modelling results for closeness to reality.

2. Analytical Method

Analytical calculations assume uniform magnetic and flow fields within the pipe (Fig 1). If B is the magnetic flux density, V is the liquid velocity along a pipe of diameter D, the induced EMF ($\Phi = \Phi1-\Phi2$) is calculated as:

$$\Phi = BVD \qquad\qquad 1.$$

The calculation was repeated for varying current levels, I, generating varying B values. The calculated values were then compared with the values obtained from the F.E. model built in COMSOL Multiphysics and described in the next section.
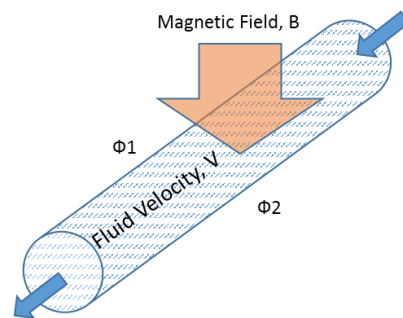

Fig 1: Magnetic Field, B, across pipe with liquid flowing at velocity, V. EMF induced is $\Phi1-\Phi2$.

3. Use of COMSOL Multiphysics

In order to check accuracy of the analytical calculations, a F.E. model in COMSOL was built. First, the geometry, shown in figure 2 below, was created using the design modeler. The magnetic field across the pipe was induced using circular electromagnetic (EM) coils powered by a DC source. The pipe carried an ionic liquid and was surrounded by an air domain. The geometry was meshed using the meshing module, keeping in mind the physics to be captured. A boundary layer was incorporated at the pipe walls to solve the near wall physics accurately, given that fluidic and magnetic field gradients are highest near the walls.
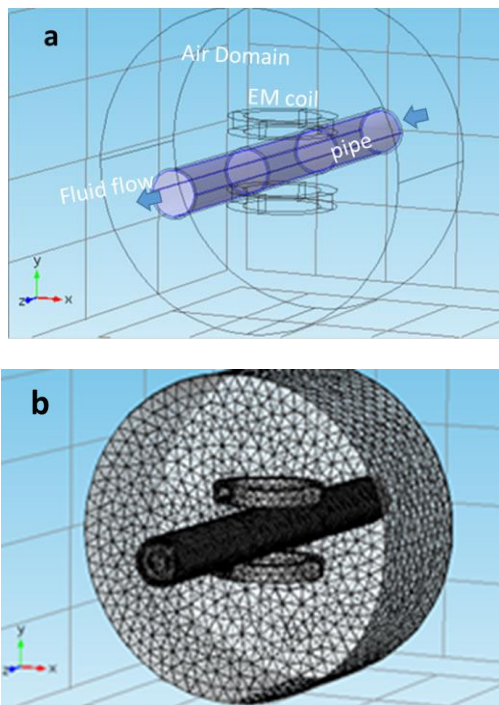

Fig2: a. Computational Domain, b. Mesh created in COMSOL Multiphysics

3.1 Governing Equations

The "Laminar fluid flow" and "Electric and Magnetic Field" modules in COMSOL were used to simulate the phenomenon. The fluid flow module simulated fluid flow through the pipe using the mass and momentum conservation equations:

$$\nabla . u = 0 \qquad\qquad\qquad\qquad 2.$$
$$\rho u \nabla u = -\nabla . p + \mu \nabla^2 u \qquad 3.$$

Where, u is velocity vector, p is pressure, ρ is density and μ is dynamic viscosity. Laminar flow was considered in the analysis. The electric and magnetic field module first calculated the magnetic field generated by the circular coils each on either side of the pipe. A DC current was supplied to both the coils in the same direction, and the magnetic field, B(x,y,z) within the pipe was calculated using the Ampere's law

$$\nabla \times \mu_0^{-1} \mu_r^{-1} B = J \qquad\qquad 4.$$

The Electric field induced in the fluid was calculated using

$$J_i = \sigma E + \sigma u \times B \qquad\qquad 5.$$

Where E is the induced electric field generated by the Lorentz force acting on the fluid, u×B. J is the current induced internally in the fluid and σ is fluid electrical conductivity. Velocity, u, was obtained from the fluid flow equations, 2 and 3. Electric potential difference, or EMF, induced within the fluid was found by integrating equation 5 over the fluid domain. Hence using COMSOL Multiphysics, EMF induced by the interaction of flow and magnetic fields was computed.

3.2 Boundary Conditions

For the fluid flow domain, a uniform inlet velocity was imposed across the pipe inlet and ambient pressure was imposed at the outlet. No slip boundary condition (u = 0) was imposed at the pipe walls. The velocity value chosen ensured that the fluid flow within the pipe is laminar. The central portion of the pipe, where we are interested, was enclosed in an air domain (Fig 2a). A magnetically insulated boundary condition was imposed at the wall of the air domain. The pipe wall was considered electrically insulated.

3.3 Computational Method

The magnetic field induced by the powered coils was simulated using the "Multi-turn Coil" option specifying the coil type as being "circular" which

requires the current loop to be specified. For non- circular coils the "numeric" type is used for accuracy. A stationary or steady state analysis was performed using the segregated solver in 2 steps: segregated step 1 for the electromagnetic fields and step 2 for the fluid field. The AMS (Auxiliary Maxwell Solver) was used to solve the electromagnetic equations.

4. Results
4.1 Electrically Insulated Pipe:
Figures 3 a and b below show contours of the velocity and magnetic fields induced across the pipe cross section. Figure 3c shows contours of the electrical potential field calculated using equation 5. The potential difference or EMF is calculated across the pipe diameter (Fig. 3d) as the difference of the potentials at either end ($\Phi1$-$\Phi2$).
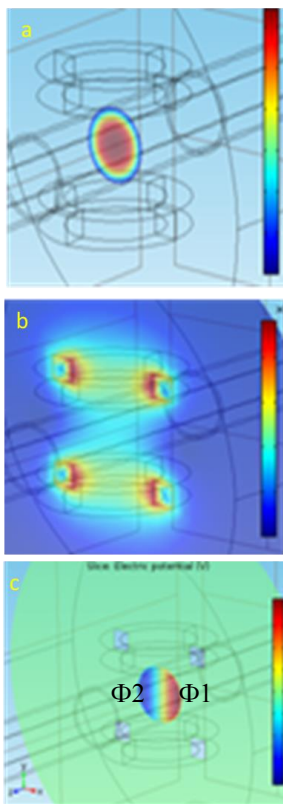

Fig 3: a. Velocity contour, b. Induced magnetic flux density, c. Induced electric potential across pipe.

While the velocity profile is almost parabolic as expected in laminar flows, the magnetic field is almost uniform (Fig. 3a and b) across the pipe

cross section. The induced electric potential, Φ, (Fig 3 c) is positive at the right and negative at the left of the pipe cross section. This potential difference $\Phi1$-$\Phi2$ is important to us in quantifying the effect of the interaction between the flow and magnetic fields. Figure 4 a below shows variation of Φ along the pipe diameter perpendicular to the flow direction. Potential values are normalized with respect to $\Phi1$. Henceforth, in this paper only normalized values will be discussed. Next, the analytically calculated value of the EMF or $\Phi1$-$\Phi2$ was compared with the simulated values in COMSOL for varying current values (Fig. 4b). The current levels are normalized with respect to the maximum current.
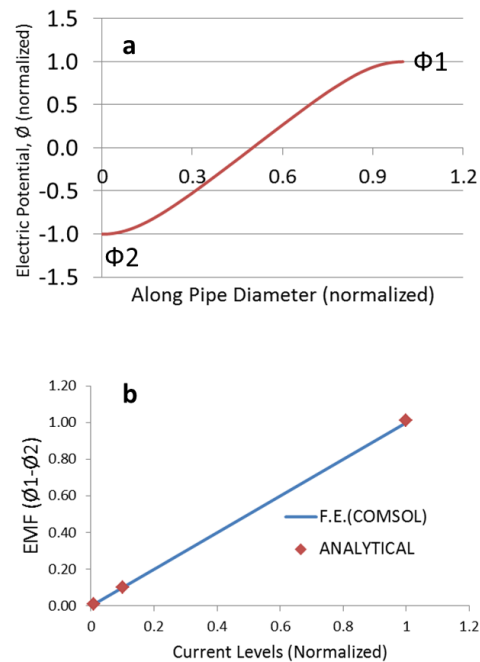

Fig 4: a. Electric potential variation across pipe diameter, b. Comparison of EMFs obtained by analytical and F.E. method (COMSOL Multiphysics).

It is seen that there is an almost 1% deviation between the analytical and F.E. method values.

4.2 Electrically Conductive Pipe

For an electrically conductive pipe following is the analytical expression for the induced EMF [2]:

$$\varphi = BVD \left(1 + \frac{t\tau}{r\sigma}\right)^{-1} \qquad 6.$$

Where t is pipe thickness, τ is pipe electrical conductivity, r is radius and σ is fluid conductivity. Analytical calculations were carried out at 2 coil current levels. Subsequently, F.E. simulations were performed. Figure 5a and b shows potential induced across the pipe cross section when the pipe material is electrically conducting. It is seen that there is a leakage of electric field across the pipe boundary (Fig 5a).
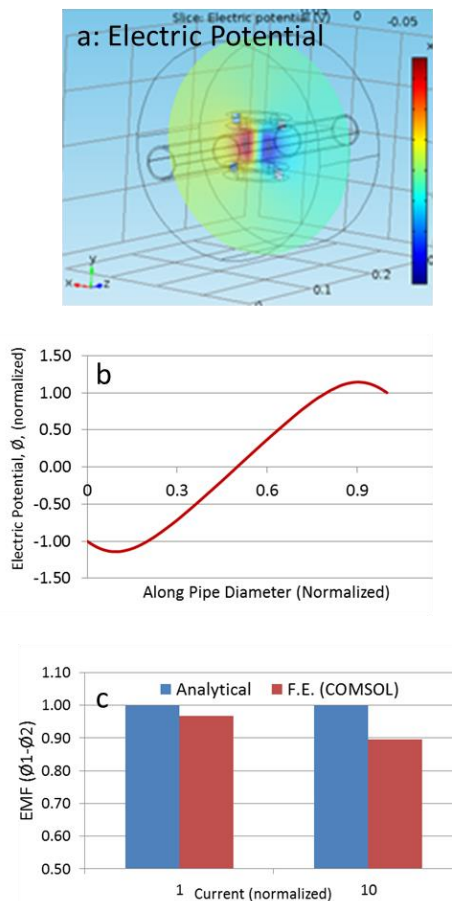

Fig5: a. Electric potential contours, b. Potential distribution across pipe diameter, c. Comparison between analytical and F.E. method at varying current levels (electrically conducting pipe)

Fig. 5b when compared to Fig 4a, shows a marked deviation of the potential distribution from a linear trend. When compared with F.E. simulations (Fig 5c) the analytical calculations reveal an over prediction of the EMF by as much as 10% depending on the current level. The current level is normalized with respect to the lowest value.

5. Discussion

The above findings reveal that analytical calculations are capable of accurately predicting EMF induced in a mobile fluid under the influence of a magnetic field across a pipe, with respect to F.E. computations. However, this is true in case of an insulated pipe. In case of a conductive pipe, the error intensifies to as much as 10%. This is due to 3 dimensional (3D) effects which are not accounted for in the 1D analytical calculations and which could be more prominent in case of conductive pipes (comparing Fig. 4a and 5b). Overall, analytical calculations are useful for a preliminary understanding of the physics behind a phenomena like electromagnetic induction in a flowing liquid, which is the topic of interest in this paper.

6. Conclusion

1 D analytical calculations can be confidently used for a basic and initial understanding of the phenomenon studied in this paper, like the effect of important parameters on the outcome of the process. However for closeness to reality in predictions 3D F.E. models like COMSOL is necessary. This is particularly true in complicated cases with pronounced 3D effects. Future studies could be focused on efforts to improve analytical predictions with the help of F.E. simulations by incorporation of correcting factors.

7. References

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