Iterative Learning Control for Spatio-Temporal Repetitive Processes

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Abstract: The optimal tracking control problem for repeated trials of distributed-parameter process is presented. Also, an adaptive control scheme based on iterative learning control technique is adopted for effective solution of underlying optimization task. The important component of the resulting approach is the efficient modelling and simulation of the distributed system under consideration using COMSOL environment and tools. In such a way, in each replication of the process both the quality of control and robustness with respect to model uncertainty can be significantly improved. Finally, a proposed approach is verified by a computer simulation regarding control of combustion process.

Keywords: iterative learning control, distributed parameter systems, repetitive process.

1. Introduction

Recently, due to the dynamically increasing complexity of modern systems, a strong necessity appears for more systematic approaches to high quality control and process monitoring [9]. Requirements imposed by process control in the area of spatio-temporal physicalsystems also called distributed parameter systems (DPSs) are associated with using very accurate models in which spatial dynamics cannot be neglected and has to beincluded in addition to the temporal one [4,5]. Many classical control approaches dedicated for lumped parameter systems are not able to meet these sophisticated requirements [10]. Therefore, the area of improved and more accurate control methods and strategies constitutes an important domain of engineering research.

A broad class of industrial processes, including numerous industrial production systems are of repetitive nature, that is, they have to replicate some operations in consecutive trials, where the main aim is to gain the system response accurately following a given reference trajectory. Usually, the conventional control system often replicate the same operation numerous times without any improvement [3]. In particular, in the case of repetitive processes it can be observed that in each replicated trial, the system produce at the output the same tracking error, oscillations and overshot, and that characteristic could be used to improvement of tracking performance. For that problems, a promising approach is constituted by technique called Iterative Learning Control (ILC), which since the late 70s [1,2] has established a separate field of control theory.

The main purpose of this work is to show a general control scheme and its effective application to the repetitive spatio-temporal process modelled as DPS. Also a strong evidence that ILC considered to be efficient tool for lumped systems could be successfully adopted for DPSs with restrictions according to control signal, quality of tracking, control restrictions etc.

As an example of repetitive distributed-parameter system, a combustion process has been taken under consideration. To increase the modelling accuracy and obtain more realistic results a three dimensional model of object with high dynamics was created using efficient tools of COMSOL environment. In particular, the Navier-Stokes equations for fluid dynamics, together with diffusion and convection were used to simulate the transport and mixing of the fuel inside the reactor. General feedback control scheme was developed for tracking reference signal in the measurement point inside reactor to obtain the high performance of control process.

2. ILC scheme for distributed-parameter system

Although ILC scheme is a well known technique for linear lumped-parameter systems, it is not widely used for spatio-temporal systems and recently become the field of intensive research with numerous contributions [6,7]. To this end, the guaranteed learning convergence of ILC has been proven for many types of DPSs [6]. In general, ILC algorithms designed for repetitive processes are usually based on previous trial data [1,2] (so-called first order ILC). In the literature, various implementation schemes for ILC exist, but for purpose of this work a typical control architecture was used [8] as an example of the proportional differential scheme

$$u_{k+1}(t) = \mu u_k(t) + \lambda e_k(t)$$

where the integer $k \geq 0$ denotes the trial or cycle number, $u(t)$ is the system input along the trial, μ denote the momentum coefficient, λ is the learning coefficient, $y_k(t)$ is the system output and $e_k(t) = y_d(t) - y_k(t)$ being the tracking error where $y_d(t)$ is a desired system output. In first iteration u_0 can be set as 0 or generated by other feedback controller that leads to bounded tracking error. Previous error is used to develop the control signal in the next iteration of algorithm. It is also assumed that the control signal was designed with input saturation at lower level $u_k(t) > 0$ because the inlet velocity to chamber cannot be less than zero, therefore avoiding a sucking effect. Upper saturation was not taken into account – assumption of input power was sufficiently high to fulfill controller demands.

3. Numerical Model

For the purpose of ILC algorithm a proper model of DPS response is required. Consider a tree dimensional generic model of gas combustion chamber with two inlets of a gas and one outlet. Injected gases (usually fuel and oxygen) have different concentrations and are mixed making possible the effective combustion. Spatial geometry domain of the model is presented in Fig. 1. On the left side is located the input inlet injecting the gas with the lower level of oxygen and controlled velocity. At the top inlet the fuel enters with high content of oxygen and constant velocity. At right side of chamber the outlet is located and the oxygen concentration is measured at some point inside the chamber. The model uses the Navier-Stokes equations to describe the fluid flow and the convection and diffusion application mode for the mass balance. It can be formally described by following equation system:

$$\rho \frac{\partial \mathbf{u}}{\partial t} - \nabla \cdot [\eta(\nabla \mathbf{u} + (\nabla \mathbf{u}^T)] + \rho \mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{u}p = \mathbf{F},$$

$$\nabla \cdot \mathbf{u} = 0,$$

$$\delta_{\mathrm{ts}} \frac{\partial c}{\partial t} + \nabla \cdot (-D \nabla c) = R - \mathbf{u} \cdot \nabla c,$$

where:

- $\rho[\mathrm{kg/m^3}]$ – density,
- $\mathbf{u}[\mathrm{m/s}]$ – velocity vector,
- $\mathbf{F}[\mathrm{N/m^3}]$ – volume force vector,
- $c[\mathrm{mol/m^3}]$ – concentration,
- $\eta[\mathrm{Pa/s}]$ – dynamic viscosity,
- $p[\mathrm{Pa}]$ – pressure at output,
- $R\left[\frac{\mathrm{mol}}{\mathrm{m^3 \cdot s}}\right]$ – reaction rate,
- $D[\mathrm{m^2/s}]$ – diffusion coefficient,
- δ_{ts} – time scaling coefficient.

Under the assumption that concentrations at two inlets are known we can describe boundary conditions as well for the mass balance and fluid flow. Additionally, for internal walls in the tank it is assumed that there is no friction effect on them, bypassing the inlets near the upper and controlled inlet. For these areas it is allowed for a smooth transition to the laminar flow profile.

Furthermore, it is assumed that the convection is a primal mechanism of transport of the reactants at the outlet. It means, that dispersal in the main direction of the convective flow should be ignored. After that boundary conditions for mass balance can be described by

- $c_t[mol/m^3]$ – concentration at upper input,
- $c_c[mol/m^3]$ – concentration at controlled input,
- $\mathbf{n} \cdot (-D\nabla c) = 0$ – output boundary condition,
- $\mathbf{N} \cdot \mathbf{n} = 0$ for walls with molar flux given as $\mathbf{N}[\mathrm{mol/m^2 \cdot s}]$.

Boundary conditions for fluid flow can be separately given as upper constant inlet $\mathbf{u} = (0, -u_{\mathrm{t}}, 0)$, controlled inlet $\mathbf{u} = (u_{\mathrm{c}}, 0, 0)$, pressure at output $p_0 = 0$, inlet sections $\mathbf{n} \cdot \mathbf{u} = 0$ and walls $\mathbf{u} = 0$.

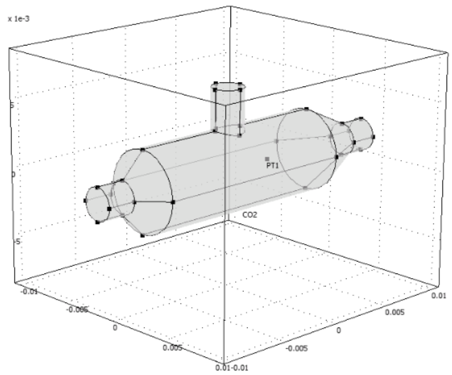

Figure 1. Geometry view of a three dimensional model of chamber with two inlets and one outlet.

The ultimate goal is to control the oxygene concentration inside the chamber to achieve the desired concentration at the output, according to given reference signal. For that purpose a one point inside the tank was chosen as the reference point $p = [0,\ 0.005, 0]$, at which the measurements will be taken providing the information for control feedback loop.

Initial condition for gas mixing chamber was:

- $u_{\mathrm{t}} = 0.01\ [\mathrm{m/s}]$,
- $c_{\mathrm{t}} = 1[\mathrm{mol/m^3}]$,
- $c_{\mathrm{c}} = 0.2[\mathrm{mol/m^3}]$,
- $c_0 = 0.5[\mathrm{mol/m^3}]$ which stand as initial concentration in chamber,
- $\rho = 1.2[\mathrm{kg/m^3}]$,
- $\eta = 3 \cdot 10^{-5}[\mathrm{Pa/s}]$,
- $D = 1 \cdot 10^{-4}[\mathrm{m^2/s}]$.

A suitable process model was created and then simulated in COMOSOL 3.5 Multiphysics® in connection with MATLAB® environment. Such a combination allows for efficient implementation of complex process model together with nontrivial control algorithm offering the high quality of modelling and simulation of real process.

In the simulation experiment, an ILC method was used as a feedforward controller. Control signal was implemented to fulfill the required concentration time profile illustrated in Fig. 3. At the beginning of the cycle the concentration has to smoothly and slowly grow up to the maximum desired level, and then in a same way it have to decrease to the initial level as an assumption is that every cycle starts at the same concentration level. Assuming that the combustion will be made repetitively during some period of time, the experiment consist in updating the input control signal in order to provide the desired profile of combination of gases at output and measured point.

4. Experimental Results

In order to illustrate the developed control scheme two approaches were compared, namely ILC and the classical simple PID controller. For ILC it was assumed arbitrarily that $\lambda = 0.3$. For PID the proportional gain was set to 2, integral gain to 0.1 and differential gain to $3 \cdot 10^{-3}$.

In the first trial of ILC for chamber mixing simulation a simple PID controller was used for stable control (set point). However the same control solution was not enough to satisfy complicated tracking of reference concentration profile (Fig. 3). The fixed point reference control (constant concentration profile) experiment is shown in Fig. 2. Result of using the same PID for tracking time-varying reference profile is presented in Fig. 3 where as first trial the PID is used (black solid line). Thus, the level of improvement in control after 10 trials of experiment becomes clear (blue solid line) especially in the second part of the concentration profile.

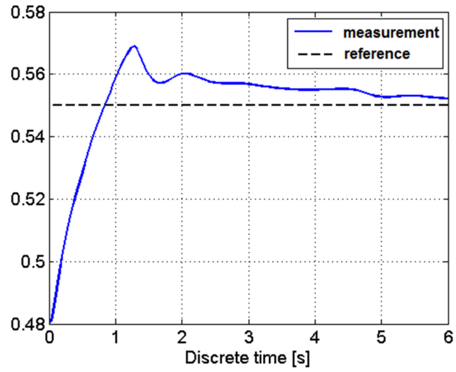

Figure 2. Concentration inside reactor for PID for fixed set point reference signal.

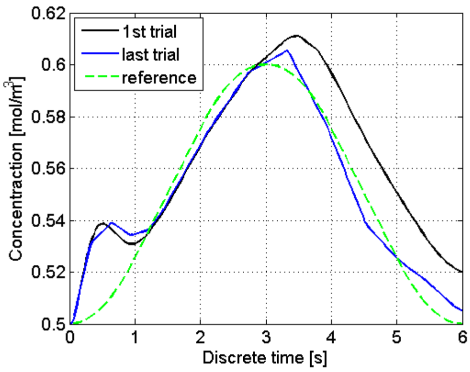

Figure 3. Concentration inside reactor for first (PID) and last trial with reference signal.

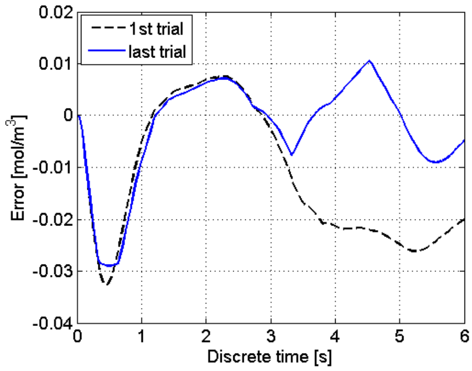

Figure 4. Error in the 1st(PID) and last trial of experiment.

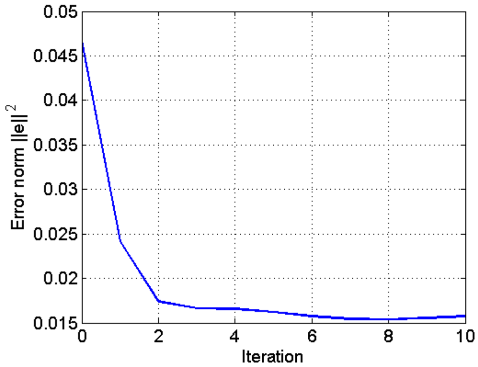

Figure 5. Error norm for ILC approach in each trial.

Differences and improvement can be observed in the concentration (Fig. 3) as well as in error between measurement and reference signal (Fig. 4) for PID (1st trial) and ILC (last trial).

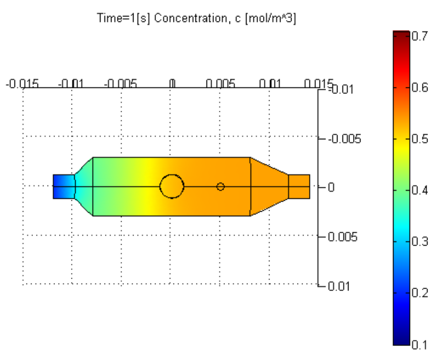

Figure 6. Slice plot of concentration on *xz* plane in 1st sec. (top view).

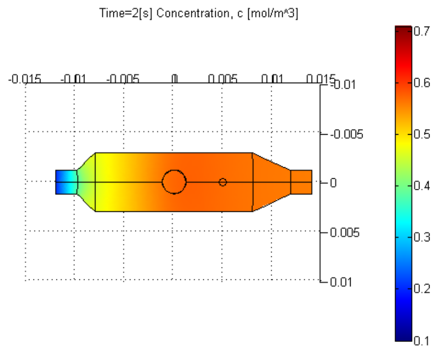

Figure 7. Slice plot of concentration on *xz* plane in 2nd sec.(top view).

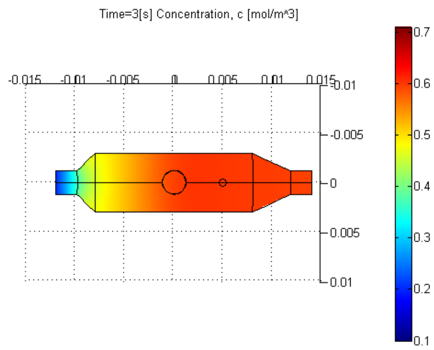

Figure 8. Slice plot of concentration on *xz* plane in 3rd sec.(top view).

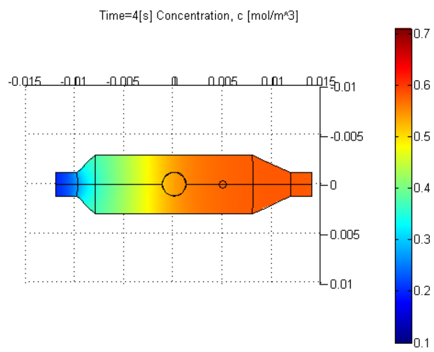

Figure 9. Slice plot of concentration on *xz* plane in 4th sec. (top view).

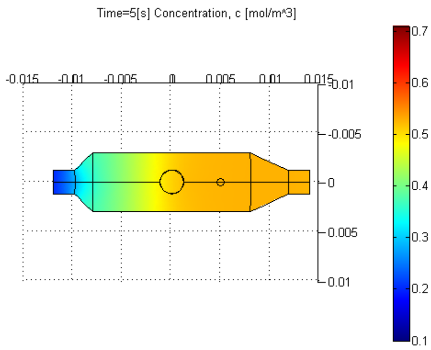

Figure 10. Slice plot of concentration on *xz* plane in 5th sec. (top view).

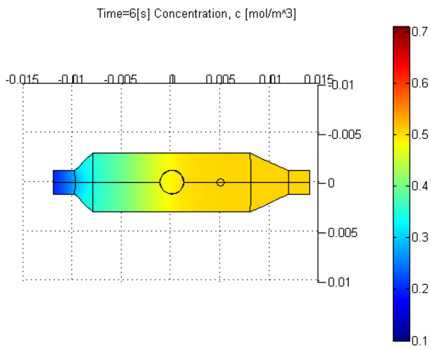

Figure 11. Slice plot of concentration on *xz* plane in 6th sec.(top view).

Also, it has to be elucidated that control of that type of nonlinear object is a highly non-trivial task, because of hard practical restrictions according to the control signal and spatio-temporal dynamics of the system. In fact, increasing the flow rate could lead to both increase or decrease of concentration in the measurement point due to the complex and nonlinear transport process. Also at the time interval $t \in (0,1)$ it can be observed that object is, to some extent, insensitive to control signal – this could be explained by the distance from input inlet to measurement point and time needed for establishing control signal. To show dynamic of object the velocity field in the last second of the last ILC trial are also presented in Fig. 12 where bolded streamline means bigger velocity. It becomes clear that that object behavior is very difficult to predict and control.

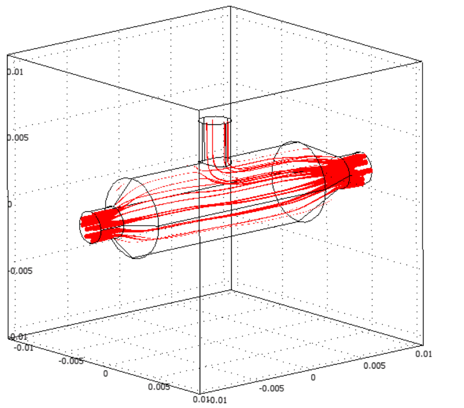

Figure 12. Velocity field result in from last second from last trial of ILC algorithm.

5. Discussion

During the experimental phase, two techniques were applied for tracking reference signal problem, ILC for DPSs and simple PID controller. During the experiments it was shown that simple PID controller can reach satisfactory results in the constant set point tracking task. However, trying to apply such controller to track a time-varying reference signal which requires dynamic adaptation of control, it cannot provide reasonable control accuracy. We observed that with the ILC controller the significant decrease of the error norm $||e_k^2||$ can be achieved and this is presented in Fig 5. Slice plots from last trial are presented in Figs 6-11 showing the spatial dynamics of system in every second.

6. Conclusions

An iterative learning control for distributed parameter system was presented as an promising approach for the improvement of control quality. The potential of the resulting control scheme was illustrated on the application to the fluid dynamics with the mass transport as an example of real chemical process.

This work is part of ongoing research on more general methodology for combining iterative learning control and sequential experimental design to improve control quality for systems with repetitive operations. Since the location of the measurement point can highly influence the quality of control, therefore further research will be also focused on the sensor location for system identification and control.

7. References

1.M. Uchiyama, Formulation of high-speed motion pattern of a mechanicalarm by trial, *Trans. SICE (Soc. Instrum. Contr. Eng.), vol. 14, no. 6*, 706–712 (1978)

2. S. Arimoto, S. Kawamura, and F. Miyazaki, Bettering operation of robots by learning,*Journal of Robotic Systems, vol. 1, no. 2,* 123–140 (1984)

3.W. Paszke, R. Merry, and R. van de Molengraft, Iterative learning control by two-dimensional system theory applied to a motion system, *in Proc. American Control Conference (ACC)*, New York, USA, pp. 5484–5489 (2007)

4.M. Patan,*Sensor Networks Scheduling for Identification of Distributed Systems*, Lecture Notes in Control and Information Sciences. Springer, 2012, vol. 425.

5. M. Patan, Optimal activation policies for continous scanning observations in parameter estimation of distributed systems, *International Journal of Systems Science, vol. 37, no. 11*,. 763–775, (2006)

6. H., Deqing, et al. D-type anticipatory iterative learning control for a class of inhomogeneous heat equations. *Automatica 49.8,* 2397-2408 (2013)

7. K. L. Moore, Y. Chen, and H.-S. Ahn, Iterative learning control:A tutorial and big picture view, *Proceedings of the 45th IEEEConference on Decision and Control*, 2352–2357 (2006)

8. M. Sun, W. Wang. Anticipatory iterative learning control for nonlinear systems with arbitrary relative degree, *Automatic Control, IEEE Transactions on 46.5,* 783-788 (2001)

9. M. Patan. Distributed scheduling of sensor networks for identification of spatio-temporal processes, *International Journal of Applied Mathematics and Computer Science* 22.2 (2012): 299-311.

10. L. Korus, Efficiency analysis of control algorithms in spatially distributed systems with chaotic behavior, *International Journal of Applied Mathematics and Computer Science* 24.4 (2014): 759-770.