A Systematic Method for Producing Simulated Scattered Field Data from Known Structures

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Abstract: One of the greatest challenges in developing algorithms for imaging from scattered fields is the lack of suitable scattered field data from known targets. It has proved quite difficult to acquire this data from measurements in the laboratory due the fact it can be costly, time consuming, and limited by available target physical characteristics. It would be ideal to develop a structured way to virtually model various known targets to produce simulated scattered field data for use in working with imaging algorithms. In this paper, such a method is proposed and it is demonstrated that the data obtained from this method is valid and comparable to measured data from physical test.

Keywords: scattered fields, COMSOL, finite element method, inverse imaging

1. Inverse Scattering Problem Overview

For some time now, there has been a great deal of effort to develop an algorithm or method for reconstructing a target's image (and its intrinsic properties such as a quantitative map of its permittivity or refractive index) from scattered field data. This is an enormous challenge since the general inverse problem in itself is ill-posed, and can be mathematically difficult and computationally challenging. A general model or approach for this type of problem, that is quite widely used, is called the diffraction tomography method. It is generally restricted to so-called weakly scattering targets. In this method a target is illuminated by a known quasi-monochromatic incident wave, and the scattered field(s) is then measured around the target by some number of receivers. The challenge is then to take the measured field data from all of the receivers and use this in conjunction with knowledge of the incident field to reconstruct the original target. The need for this algorithm has many modern day applications including target identification, biomedical imaging, remote sensing, geophysical imaging, structure synthesis and other non-destructive testing. An illustration of a typical general experimental setup for this method is shown in Figure 1.

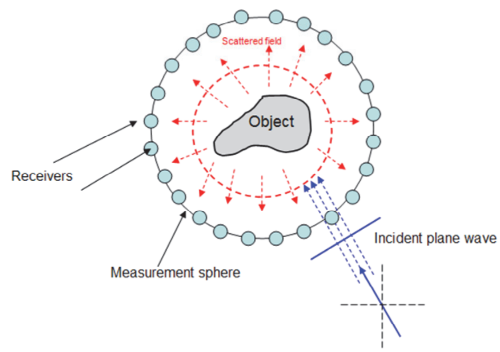


Figure 1. Typical experimental setup for diffraction tomography. The target or object is illuminated with monochromatic plane wave and the scattered waves, after the interaction of the incident plane wave with the scattering object, are measured by receivers placed either in the near or far field, all around the object.

The diffraction tomography method utilizes the knowledge that if one deals with weak scatterers, the measured fields are altered by a target having some complex permittivity distribution, in such a way that the data can be processed using an inverse Fourier transform and other processing methods to reconstruct the original target. There has been limited success in implementing and utilizing this approach because it is limited to weak scatterers, and most targets of practical interest would more likely be deemed as strong scatters.

2. The Need for Valid Input Data

Given the dearth of accurate data sets from strongly scattering targets, it can be difficult to evaluate imaging algorithms. Even with the data that are available through various institutions such as US Air Force Research Laboratory's (AFRL) [1]-[4] and Institut Fresnel [5] among others is that these data are for a very limited and discrete set of measurement conditions. These data are useful in the testing of algorithms, but in order to fully test and evaluate the accuracy of algorithms one needs the ability to vary or sweep various parameters over a wide range of values to observe the effects on the algorithm.

While this can be quite a challenge in the laboratory, it can be investigated much more easily and quickly in the virtual modeling realm by using (hopefully reliable) computer models to generate data sets to be used in conjunction with imaging algorithms. In addition to the advantages mentioned above, these models can also be used to control and vary numerous parameters in ways that would be difficult, if not impossible, to accomplish in the laboratory. This gives the ability to thoroughly examine the performance of an algorithm and its abilities, and limitations to the fullest extent in order to make adjustments for improved performance. The purpose of this effort is to gain the ability to generate simulated data that can be used to test imaging algorithms utilizing currently available modeling software and techniques.

3. Target Modeling and Data Generation

The task of 2-dimensional target modeling and image data generation from scattered fields is not a trivial one. The problem lies in that there is no general solution for analytically determining scattered fields for an arbitrary target. There are some analytical solutions only for a few very simple targets, but in general, no solution exists. This means that the analytical solution would have to be derived each time for a new target, if in fact the analytical solution does exist at all in a closed empirical form, which is unlikely. A common numerical solution to this type of problem or modeling is to use the technique of Finite Element Analysis [6] [7]. In this method, the differential equations involved in calculating these scattered fields are solved numerically in an iterative process. The basic model setup for this procedure is much like the general model shown in Figure 1, with the exception that there is an artificial boundary that defines the extent that the iterative calculations are performed for since this is a finite method as shown in Figure 2. At this boundary the properties of the boundary are defined such that there are no reflections and it gives the "appearance" that the model space goes on forever.

The general solution for an E_z polarized field in the model space satisfies the scalar Helmholtz equation as follows:

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu_r}\frac{\partial E_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu_r}\frac{\partial E_z}{\partial y}\right)+k_0^2\varepsilon_r E_z = jk_0 Z_0 J_z$$

Where μ_r is the relative permeability, ε_r is the relative permittivity, k_0 is the fundamental wave number, and J_z is the z-component of the current density and Z_0 is the intrinsic impedance. This is the basic general equation that is used for the finite element method in the model space that is solved iteratively along the finite element mesh. The bounded area is enclosed using perfectly matched layers that utilize the general relationship along the mesh of

$$\frac{\partial \phi}{\partial n}+\left(jk_0+\frac{\kappa(s)}{2}\right)\phi = \frac{\partial \phi^{inc}}{\partial n}+\left(jk_0+\frac{\kappa(s)}{2}\right)\phi^{inc}$$

where ϕ is the total field which is the scattered field plus the incident field, n is the outward unit vector normal to the artificial boundary, and s is the arc length measured along the boundary and $\kappa(s)$ is the curvature of the boundary at s.

This method is implemented using the commercially available finite element software COMSOL® in this effort. This method and software are used to calculate the total field at each receiver location which greatly reduces the complexity of the approach to these types of problems, but can be computationally costly. This software allows the user to create the target graphically, modify and/or sweep virtually any and all parameters, then the program applies the finite element process to the model and returns both a graphical and numerical solution for the total field in the defined space. The only challenge then is to process the data into a format that can be used by imaging algorithms implemented in MATLAB®, which can easily done in a commercially available spreadsheet such as Microsoft® Excel.

The basic COMSOL® model is similar to that shown in the theory section. The model has been set up as shown in Figure 3 with a fixed number of receivers equally spaced at a fixed and common distance from the target origin and

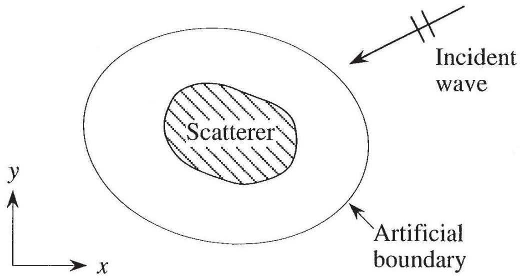


Figure 2. The general two-dimensional finite element scattering model.

a fixed number of sources equally spaced around the target. This will produce some high quality data files to be used to test new and existing imaging algorithms.

For the purpose of evaluating the data generated using this method, the imaging technique utilizing the Ewald circle [8] along with the Born approximation is used on the data to produce Fourier coefficients of the target so that a reconstructed image can be displayed and also compared to measured data for identical targets used in experiments. This method is discussed at length in [9] [10].

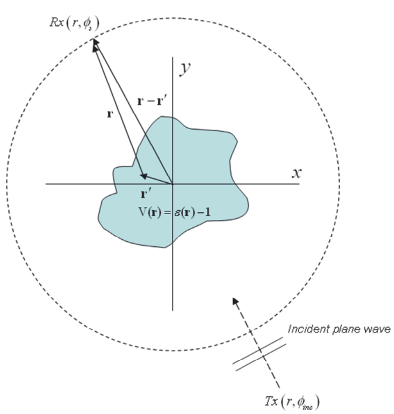


Figure 3. A typical 2D inverse scattering experimental setup. Transmitter *Tx* transmits incident quasi-monochromatic plane wave to the scattering object V(r). The receivers Rx are located all around the target which collects scattered field data after the interaction of incident wave with scattering object.

4. Target Modeling Environment

A number of basic models for suitable targets have already been successfully built in COMSOL® for this research and methodology. The typical model consists of a target area located at the origin. Receiver points, which can be used to "measure" or obtain the calculated complex scattered field values at these points, are located on an imaginary circle centered at the origin with a fixed radius of 760mm. There are 360 data points equally spaced along this circle which basically give the ability to measure the total field on 1 degree increments around the target. The illuminating source is cycled around the target in 36 equally spaced locations which basically gives the ability to view the scattered data from a source rotated on 10 degree increments around the target. This translates to having the capability of creating 36 separate Ewald circles as previously mentioned and discussed in [8]. As already mentioned, this basic model environment has been successfully implemented and data has been successfully gathered from the simulations and formatted for use in already developed MATLAB® algorithms used in [9]. This will be demonstrated in the next section.

The implementation of this environment in the COMSOL® software for a cylindrical target is shown in Figures 4, 5, 6 and 7. In this figure, the basic model, mesh, z–component of the E field which is orthogonal to the plane of propagation, and the normalized E field are shown to illustrate the capabilities of the software and model.

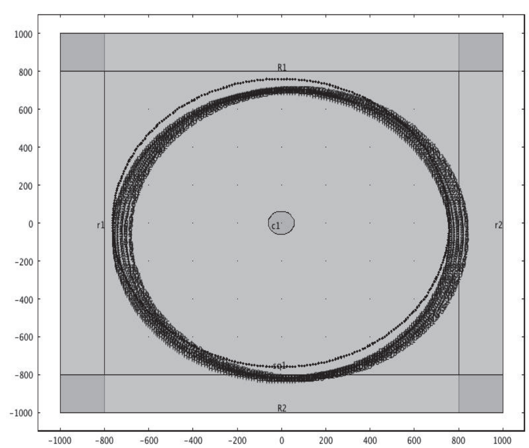


Figure 4. Basic model of a circular cross section target (i.e. a cylinder in 3D) with a radius of 60mm. The larger circle is the location of the 360 receiver points and their respective identification tags. The square and rectangular sections around the border of the space are Perfectly Matched Boundary Layers that eliminate reflections in the model.

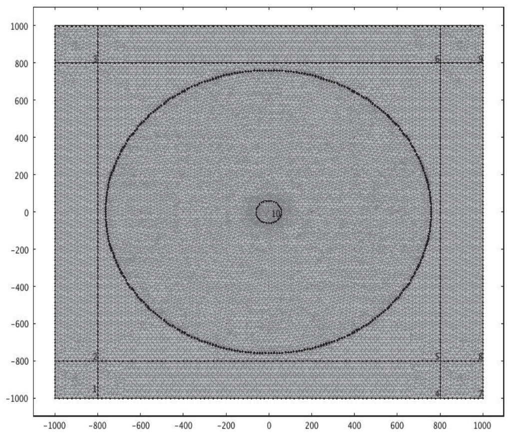


Figure 5. Basic target model from Figure 4 with Finite Element Mesh Applied.

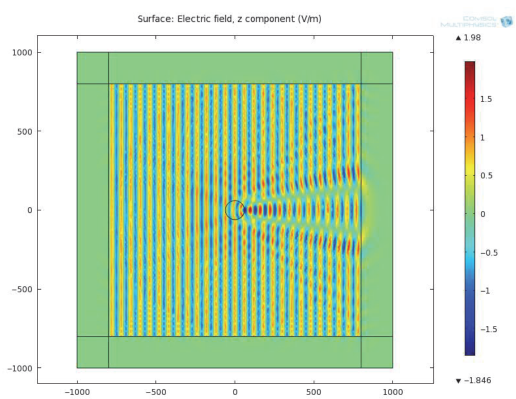


Figure 6. Graphical representation of E_z with incident source frequency of 5GHz for a target in Figure 4 with a relative permittivity of 1.5.

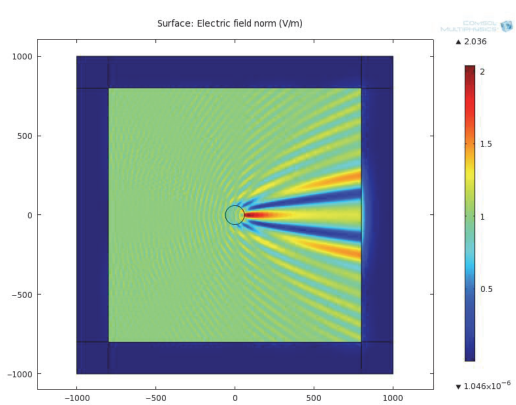


Figure 7. Graphical representation of normalized E_z for conditions in Figure 6.

5. Imaging Algorithm Results

In order to verify the validity of this modeling process, the data obtained from the COMSOL® model was compared to measured data obtained for the Institut Fresnel [5] website for a range of targets. These data were similarly processed using the same algorithm described in [9] that maps the data onto Ewald circles and applies the Born Approximation algorithm to the data to produce a Born reconstructed image. To demonstrate that the data from the COMSOL® modeling process is valid, the two images for each target, one from the measured data, and one from the simulated data respectively are shown to be comparable in appearance. The target definitions, and outputs from both data sets is presented in Figure 8 for comparison below. There are some obvious differences in appearance which is to be expected to some degree. One major difference in the data sets for at least the first two targets is that the simulated data were constructed using nine incident sources equally spaced around the target and the measured data were constructed using eight incident sources equally spaced around the target. Otherwise, the setups are almost identical for each target sets.

6. Conclusions

In this paper, a fundamental challenge to developing imaging algorithms was identified in that there are a limited amount of data to test these algorithms. A structured method to produce simulated scattered field data for known targets is proposed using the finite element method utilized through the commercially available software package of COMSOL®. In this paper it has been demonstrated how these models can be constructed and the pertinent data gathered to use for algorithm evaluation. It should be evident from the images in Figure 8 that while the images produced from the simulated data and the images produced from the measured data are not identical, they are quite similar in appearance and demonstrate that the simulated data process is producing valid data to be used in testing current and future imaging algorithms.

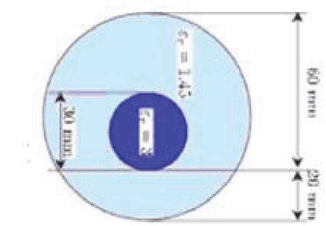
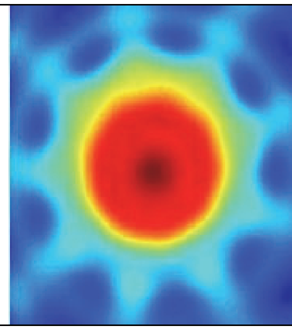
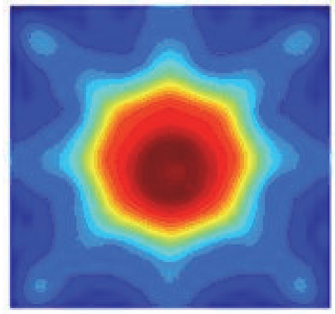
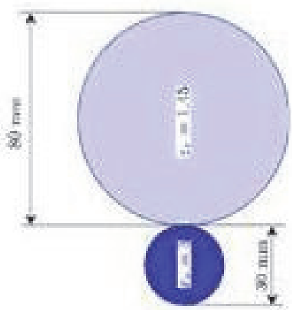
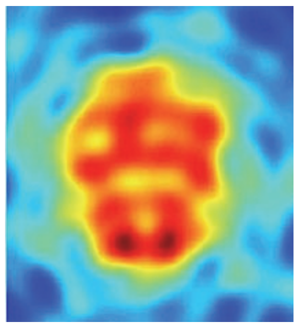
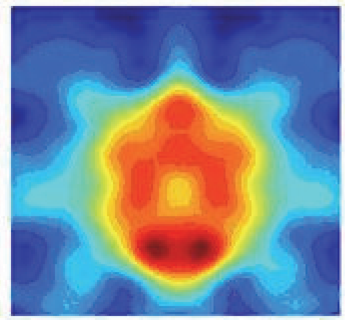
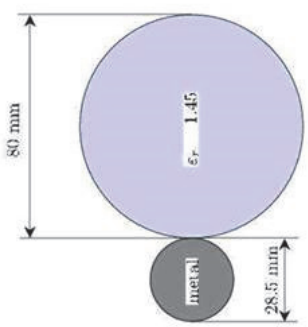
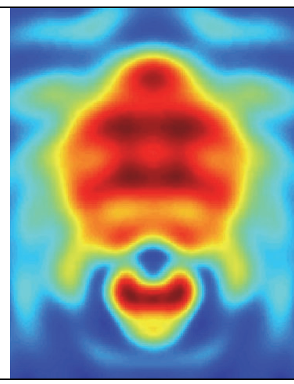
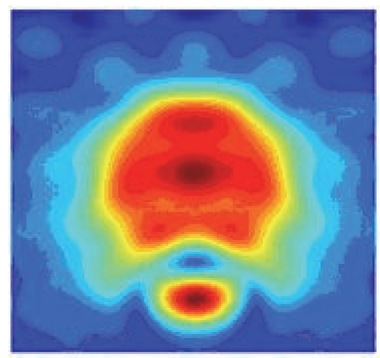
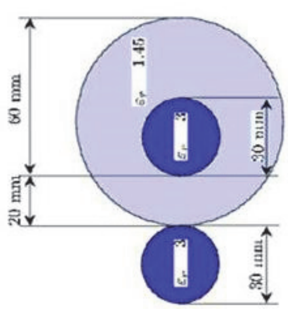
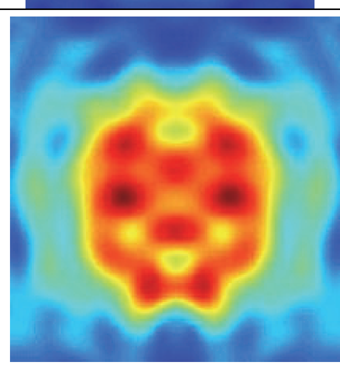
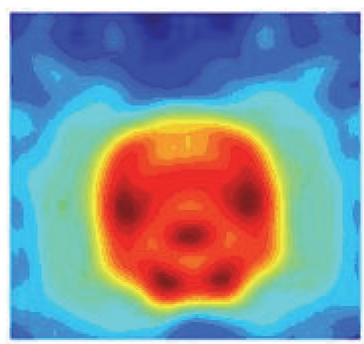
	Institut Fresnel Target Setup	Born Image from Simulated Data	Born Image from Institut Fresnel Measured Data
FoamDielInt			
FoamDielExt			
FoamMetExt			
FoamTwinDiel			

Figure 8. Institut Fesnel Target definitions [5], Simulated Processed Data Output, and Measured Data output [9].

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