Solution of Poroelastic Equations with Different Base Variables Using Equation-based Modeling

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Introduction

Poroelasticity equations describe the interaction between fluid flow and solids deformation within a porous medium. Modeling of poroelasticity is coupling between elastic deformation of porous materials and Darcy's law. We have the following quasi-static poroelastic equations for modeling with unknown variables displacement and pressure[1].

$$-(\lambda+\mu)\nabla(\nabla.u)-\mu\Delta u+\alpha\nabla p=f$$

$$\frac{\partial}{\partial t}\left[c_o p+\alpha\nabla.u\right]-k\nabla^2 p=h$$

The equations are independently transformed into two different formulations: stress-displacement-pressure formulation and increment of fluid content-rotation variables-pressure gradients formulation. The main advantage of using different formulations is to enforce different boundary conditions and hence gives us greater flexibility to handle real life applications. For each case, different admissible boundary conditions are provided knowing its well-posedness [2] associated with the boundary conditions. Because of non-standard formulation, equation-based modeling is used for simulation.

Computational Methods

Formulation A: Unknown Variables $\sigma-u-p$

$$\sigma-\lambda(\nabla.u)1_d-\mu(\nabla u+\nabla u^T)+\alpha(1_d)p=0$$

$$-\frac{1}{2}\nabla.(\sigma+\sigma^T)=f$$

$$\frac{\partial}{\partial t}\left[c_o p+\alpha\nabla.u\right]-k\nabla^2 p=h$$

$$\text{IC \& BC}: u_1=u_2=p=0$$

Formulation B: Unknown Variables $\eta-w-\nabla p$

$$\frac{\partial\eta}{\partial t}-k\nabla\cdot\nabla p=h$$

$$\nabla\times\nabla p=0$$

$$-k\nabla\eta+\alpha_2\nabla\times w+\alpha_3\nabla p=f^*$$

where $\eta=c_o p+\alpha\nabla\cdot u$ $\quad \alpha_2=\frac{2\alpha\mu k}{\lambda+2\mu}$

$w=\frac{1}{2}(\nabla\times u)$ $\quad w=\frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}-\frac{\partial u_2}{\partial x_1}\right)$ (2D)

$f^*=\frac{f\alpha k}{\lambda+2\mu}$ $\quad \alpha_3=c_o p+\frac{\alpha^2 k}{\lambda+2\mu}$

$$\text{IC \& BC}: \eta=w=0$$

Simulation features:

- COMSOL 4.3 coefficient form PDE used
- Domain: Circle with radius 1 with time=[0,1]
- Units: SI units

Results

- Formulation A: $c_o=10^{-8},\lambda=9\times10^9,\mu=9\times10^9$
  $\alpha=0.6,k=1\times10^{-9}$

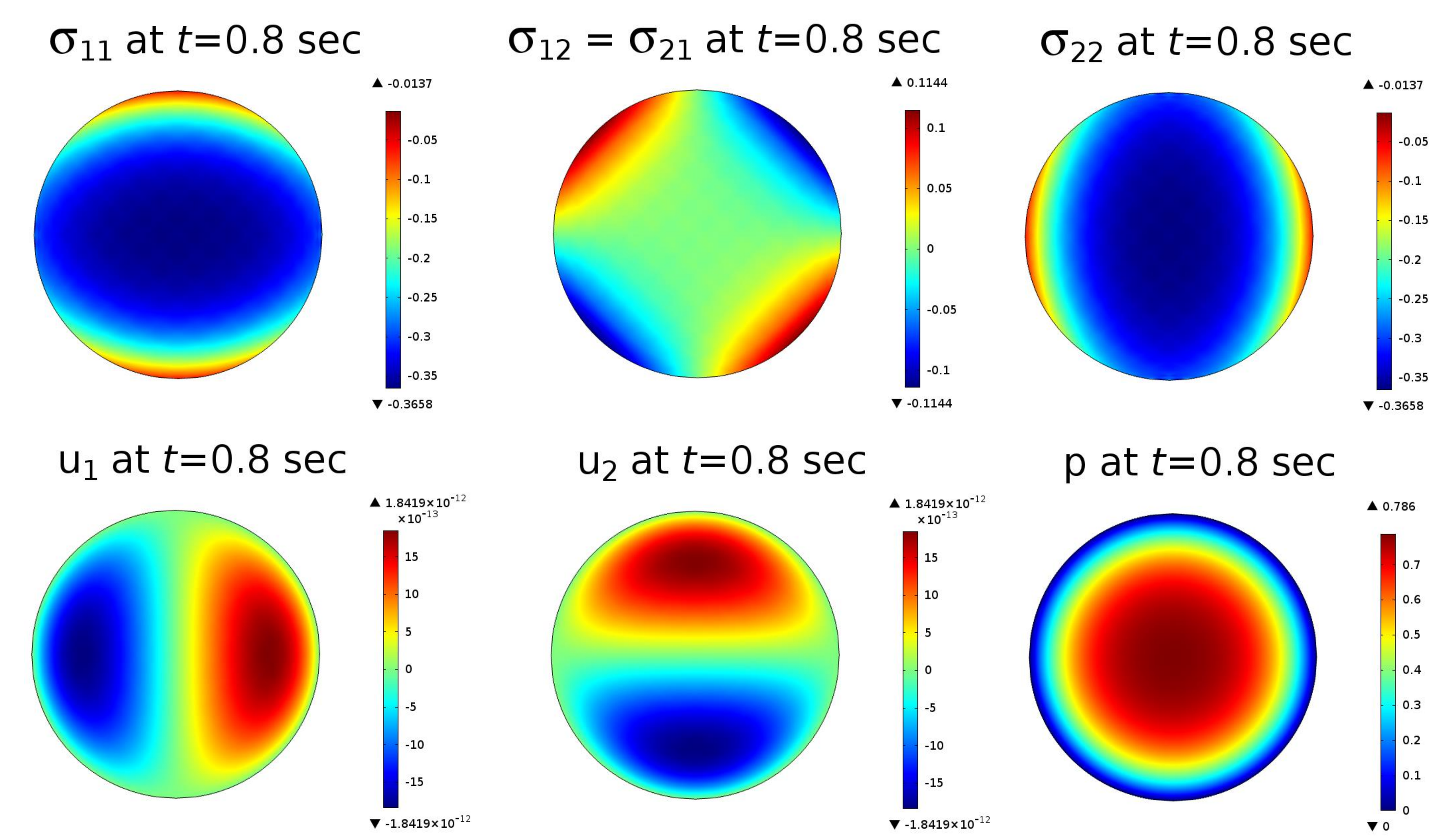

Figure 1: Simulation results of formulation A

- Formulation B: $c_o=10^{-3},\lambda=4\times10^9,\mu=9\times10^9$
  $\alpha=0.5,k=4\times10^{-2}$

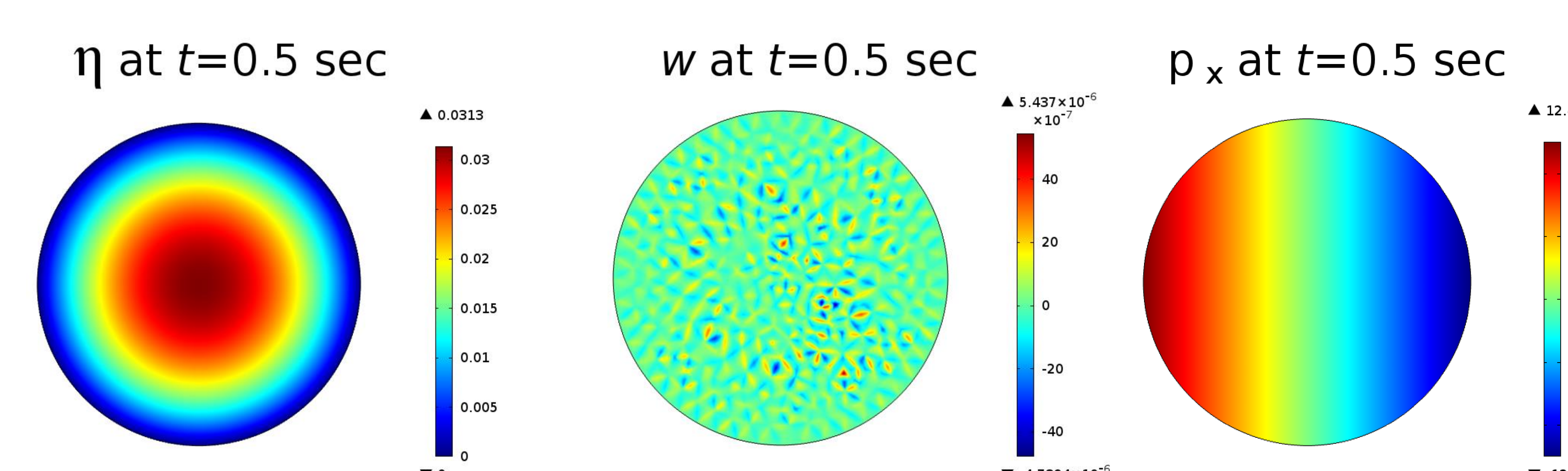

Figure 2: Simulation results of formulation B

Conclusions

- Two different formulations of poroelastic equations have been devised and simulated.
- It allows us to model problems based on the boundary data available.

References

1. R.E. Showalter, Diffusion in poro-elastic media, J. Math. Anal. & Appl., 251:310-340(2000)
2. M. H. Akanda, Y. Cao, A.J. Meir, A few model problems as symmetric positive systems, SIAM-SEAS 2015

Appendix: Notation

η : Increment of fluid content $(m^4/N.s)$

λ,μ : Lame constants (N/m^2)

c_o : Constrained specific storage (m^2/N)

k : Mobility $(m^4/N.s)$

α : Biot-willis coefficient $(-)$

σ : Stress (N/m^2), u : Displacement (m)

p : Pore pressure (N/m^2)