

Multiphase Laminar Flow with More Than Two Phases

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Abstract

Introduction

The COMSOL Multiphysics® software and its CFD Module have a physics interface for solving Laminar Two-Phase Flow problems using the Level Set method (1). This paper discusses how the tool may be used with more than two phases when the flow is laminar. It then introduces a second method for solving such problems.

Use of COMSOL Multiphysics

The Laminar Two-Phase Flow, Level Set interface in the COMSOL software uses a smooth auxiliary function $0 \leq \xi(r,t) \leq 1$ with a $\xi=0.5$ level-set dividing the model geometry into regions containing each of two fluid phases. The solver is provided with an original fluid configuration for use in constructing the initial function $\xi(r,t_0)$. The Navier-Stokes equation is then used to compute fluid velocity u and a differential equation is applied to $\xi(r,t)$ so that its $\xi=0.5$ level-set properly tracks the fluid boundaries. An initialization-free scheme (2, 3) has been used for improved stability and mass conservation.

This method can be extended to three or more fluids by using of multiple instances of the two-fluid interface. Each instance corresponds to one of the $N \geq 3$ fluids, though one of the fluids (denoted $j=0$) does not require one. Therefore, $N-1$ interfaces represent N fluids. Each interface models two fluids. The first is the fluid j corresponding to the interface, and tracked by $\xi_j(r,t)$. The other is a composite of all remaining fluids. The density ρ and viscosity ν of the composite fluid are computed using the auxiliary functions from the other interfaces, so that each point in the fluid has the correct properties. The interfaces all share the same pressure and velocity and are coupled by the auxiliary functions.

An alternative method of tracking fluids is to incorporate a concentration c_j of dilute species (initially set at some convenient value such as $c_j=1[M/m^3]$) into $N-1$ of the fluids. These $N-1$ species are provided by a single Transport of Diluted Species interface in the COMSOL software. In this case, one uses a single Navier-Stokes laminar flow model and there is a single composite fluid. This density and viscosity of this fluid is computed from thresholded values of the dilute species concentrations, which reveal which of the fluids is present at each location.

Results

The methods are illustrated by a problem inspired by a COMSOL Application Library model of an oil drop rising in water. In the modified model there are two droplets with $\rho_1=0.6[\text{g}/(\text{cm}^3)]$ and $\rho_2=0.8[\text{g}/(\text{cm}^3)]$ (see Fig 1). The lower droplet rises faster and overtakes the upper droplet before reaching the surface. Surface tension is not included.

Figures 2 and 3 show the results of two simulations after 0.3 seconds. Figure 2 was obtained using two Laminar Two-Phase Flow, Level-Set interfaces, and the graph shows the $\xi(r,t)=0.5$ contours of the level-set functions representing the two droplets. Figure 3 was obtained using two dilute species concentrations c_j and shows the $c_j=0.5[\text{M}/\text{m}^3]$ contours.

Conclusions

We have demonstrated two methods of solving laminar multiphase fluid-dynamic problems with more than two fluids. Both provide similar results, but only the level-set version currently implements surface tension.

Reference

1. Sethian, J. A., Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Computer Vision and Materials Science, Cambridge Monographs on Applied and Computational Mathematics, Cambridge University Press, New York, NY (1999).
2. Chunming Li et al., Level Set Evolution without Re-Initialization: A New Variational Formulation, Proc. Of the 2005 IEEE Computer Society Conf. on Computer Vision and Pattern Recognition, Vol. 1, pp. 430-436 (2005).
3. Chunming Li et al., Distance Regularized Level Set Evolution and its Application to Image Segmentation, IEEE Trans. On Image Processing, Vol 19, No. 12, pp. 3243-3254 (2010).

Figures used in the abstract

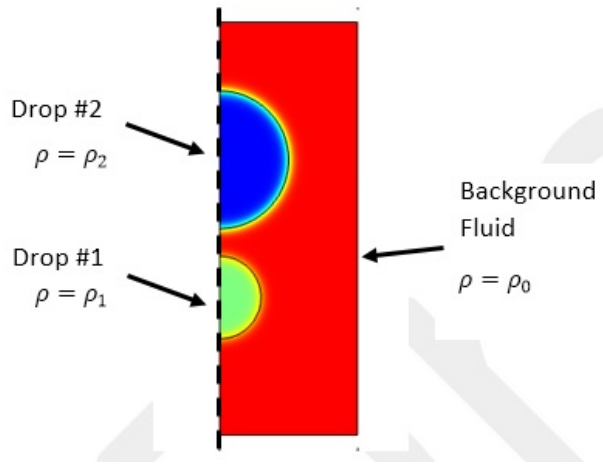


Figure 1: Test problem with two dissimilar droplets suspended in a third fluid.

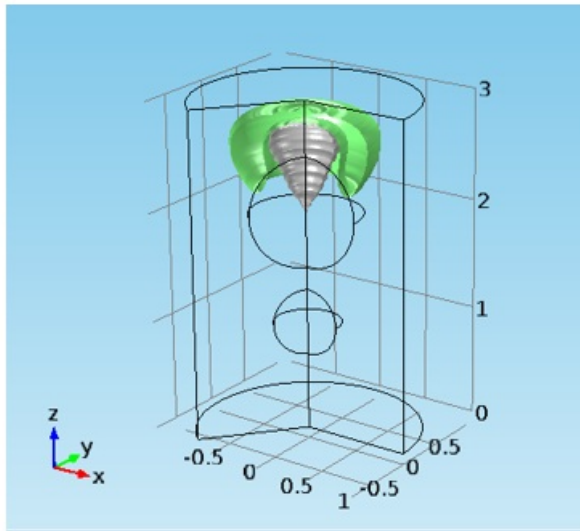


Figure 2: Level Set solution of the test problem at $t=0.3$ sec. Dimensions are in cm.

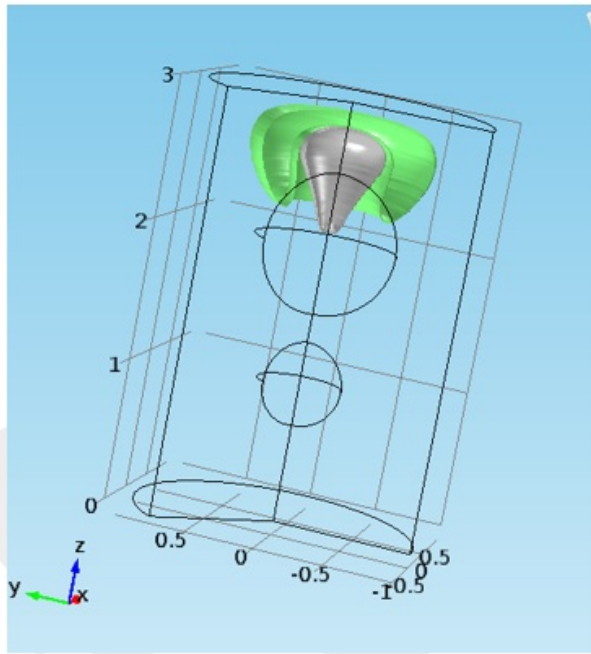


Figure 3: Dilute species solution of the test problem at $t=0.3$ sec. Dimensions are in cm.