

# Modeling and Simulation of the Consolidation Behaviour of Cemented Paste Backfill

Liang Cui<sup>1</sup>, and Mamadou Fall\*<sup>1</sup>

<sup>1</sup> University of Ottawa, Department of Civil Engineering

\*Corresponding author: 161 Louis Pasteur, Ottawa, Ontario, Canada K1N 6N5, [mfall@uottawa.ca](mailto:mfall@uottawa.ca)

**Abstract:** In underground mining operations, the mined-out spaces (called stopes) need to be backfilled to maintain the stability of surrounding rock mass and increase the ore recovery. Cemented paste backfill (CPB), a mixture of water, binder, and tailings, has been intensively utilized in underground mining operations to fill the stopes. After preparation, the fresh CPB is transported into stopes via gravity and/or pumping. The CPB in stopes is subjected to thermal, hydraulic, mechanical and chemical loads. As a result, the consolidation behavior of CPB is dominated by complex multiphysics processes that occur within the CPB. However, the conventional consolidation theory mainly focuses on the coupled mechanical and hydraulic processes. Therefore, it is necessary to develop a multiphysics model for the assessment and prediction of the consolidation behavior of CPB. In the study, a 3D coupled multiphysics model is proposed and implemented into Comsol Multiphysics. The predictive capability of the proposed model is validated against experimental data collected from well-controlled laboratory experiments. Three different coupled modules including geomechanics, subsurface flow and heat transfer modules are adopted. The prediction results show a good agreement with the measured data, which verifies the good prediction capability of the developed model.

**Keywords:** coupled processes, multiphysics, consolidation, cemented paste backfill, tailings

## 1. Introduction

As an alternative to surface tailings storage, cemented paste backfill (CPB), a mixture of tailings, binder and water, is widely used for underground mining operation and mine waste management [1-4]. For the optimal design of CPB structure, it is necessary to investigate the CPB performance and controlling mechanisms. After preparation and placement into stopes (i.e., mined-out underground space), strongly coupled multiphysics including thermal (T), hydraulic

(H), mechanical (M) and chemical (C) processes occur inside CPB mass [5, 6]. Correspondingly, the consolidation process of CPB show more complex characteristics compared with common geomaterials, such as soil. Specifically, binder hydration takes place immediately after mixing with water. During the chemical reaction, the capillary water will be gradually consumed and converted into parts of hydrates (i.e., self-desiccation). Due to the fact that the specific volume of chemically combined water is smaller than the one of capillary water [7], volume change of CPB will occur (i.e., chemical shrinkage). Moreover, due to the temperature dependence of hydration rate [8], the surrounding rock temperature, initial temperature of mix components, and the heat released by binder hydration will have significant impacts on the chemical reaction. As a result, the volume change induced by chemical shrinkage will evolve with temperature of CPB. Additionally, backfilling operation is a dynamic process, namely, the height of CPB mass gradually increase during filling period. Hence, the self-weight effect will directly affect the consolidation behavior of CPB. In practice, retaining structure (called “barricade”) should be constructed to retained fluid backfill within stope. However, the capillary water can be drained out from barricade, which can result in consolidation as well. Therefore, the consolidation process of CPB is closely related to the coupled thermo-hydro-mechanical-chemical (THMC) processes within CPB, which means that the conventional consolidation theory (i.e., Terzaghi’s theory for coupled HM processes) is unsuitable for consolidation analysis of CPB. Therefore, the thorough understanding of the effects of coupled multiphysics on consolidation behavior of CPB will significantly facilitate the optimum design of CPB in practice.

## 2. Governing Equations

CPB materials can be considered as a multiphase (i.e., solid, liquid and gas phase)

porous medium. In order to quantitatively assess the volume changes of the CPB, the continuity of pore space is adopted to develop the consolidation model, namely, the pore space changes related to the solid skeleton of the CPB and solid phase must equal the total volume change of the capillary water and pore air. Therefore, the consolidation equation can be derived as follows:

$$\begin{aligned} & \left\{ \left[ SP_w + (1-S)P_a \right] \frac{1-2\nu}{E} \frac{\partial \alpha_{Biot}}{\partial \xi} \right. \\ & \left. - \frac{[\sigma + \alpha_{Biot} [SP_w + (1-S)P_a]]}{E} \left\{ \frac{9[1-2\nu]}{E} \frac{\partial E}{\partial \xi} + 18 \frac{\partial \nu}{\partial \xi} \right\} \right. \\ & - \alpha_{Biot} (P_a - P_w) \frac{1-2\nu}{E} \left\{ [1-S_e(P_w, P_a, \xi)] \frac{\partial \theta_r}{\partial \xi} + \left( \frac{e}{1+e} - \theta_r \right) \frac{\partial S_e}{\partial \xi} \right\} \\ & - \frac{(1+e)(v_n + v_{ab-w})R_{n-w/hc}}{(w/c)v_w + v_c + (1/C_m - 1)v_{tailings}} - e\alpha_c \left\{ \frac{\partial \xi}{\partial t} \right. \\ & + \alpha_{Biot} \frac{1-2\nu}{E} \left\{ S - (P_a - P_w) \left( \frac{e}{1+e} - \theta_r \right) \frac{\partial S_e}{\partial P_w} \right\} \frac{\partial P_w}{\partial t} \\ & + \frac{1-2\nu}{E} \frac{\partial \sigma}{\partial t} + \frac{\partial \lambda}{\partial t} \frac{\partial g}{\partial I_1} + \alpha_{T_s} \frac{\partial T}{\partial t} \\ & + \alpha_{Biot} \frac{1-2\nu}{E} \left\{ (1-S) - (P_a - P_w) \left( \frac{e}{1+e} - \theta_r \right) \frac{\partial S_e}{\partial P_a} \right\} \frac{\partial P_a}{\partial t} \\ & = - \left\{ \frac{1}{1+e} + \alpha_{Biot} (P_a - P_w) \frac{1-2\nu}{e^2(1+e)^2 E} [S_e e + (1+e)^2(1-S_e)\theta_r] \right\} \frac{\partial e}{\partial t} \quad (1) \end{aligned}$$

where  $w/c$  and  $C_m$  respectively denote the water to cement ratio and binder content,  $v_w$ ,  $v_n$ ,  $v_{ab-w}$ ,  $v_c$  and  $v_{tailings}$  represent the specific volume of the capillary water, chemically combined water, physically absorbed water, cement and tailings, respectively,  $R_{n-w/hc}$  is the mass ratio of the chemically combined water and hydrated cement,  $\xi$  is the hydration degree,  $\alpha_{T_s}$  is the coefficients of the thermal expansion (CTE) of the CPB solid phase,  $\alpha_{Biot}$  is the Biot's effective stress coefficient,  $P_w$  and  $P_a$  respectively denote the pore liquid water pressure and pore air pressure,  $S$  accounts for the degree of saturation,  $S_e$  refers to the effective saturation degree,  $\lambda$  is a non-negative plastic multiplier and  $g$  is a plastic potential function,  $I_1$  is the first stress invariant of the deviatoric stress,  $E$  and  $\nu$  denote elastic modulus and poisson's ratio.

In order to solve the consolidation Eq. (1), conservation equations (water and air mass, energy and momentum balance equations) are needed:

$$\begin{aligned} & \frac{e}{1+e} S \frac{\partial \rho_w}{\partial t} + \frac{e}{1+e} \rho_w \frac{\partial S}{\partial t} + S \rho_w \left[ \frac{\partial \varepsilon_v}{\partial t} + \frac{1}{(1+e)\rho_s} \frac{\partial \rho_s}{\partial t} \right] \quad (2) \\ & - \frac{e}{1+e} S \dot{m}_h \left( \frac{\rho_w}{\rho_s} S - 1 \right) = -\nabla \cdot \left( \frac{eS\rho_w}{1+e} \mathbf{v}^{rw} \right) \end{aligned}$$

$$\begin{aligned} & (1-S)\rho_a \left[ \frac{1}{(1+e)\rho_s} \frac{\partial \rho_s}{\partial t} + \frac{\partial \varepsilon_v}{\partial t} - \frac{eS}{(1+e)\rho_s} \dot{m}_h \right] \quad (3) \\ & + \frac{e}{1+e} (1-S) \frac{\partial \rho_a}{\partial t} - \frac{e}{1+e} \rho_a \frac{\partial S}{\partial t} = -\nabla \cdot \left[ \frac{e(1-S)\rho_a}{1+e} \mathbf{v}^{ra} \right] \end{aligned}$$

$$\begin{aligned} & \left[ \left( \frac{1}{1+e} \right) \rho_s C_s + \frac{e}{1+e} S \rho_w C_w + \frac{e}{1+e} (1-S) \rho_a C_a \right] \frac{\partial T}{\partial t} \quad (4) \\ & + \nabla \cdot (-k_{eff} \nabla T) + (\rho_w C_w \mathbf{v}^{rw} + \rho_a C_a \mathbf{v}^{ra}) \cdot \nabla T = Q_h \end{aligned}$$

$$\nabla \cdot \left( \frac{\partial \sigma}{\partial t} \right) + \frac{\partial \left[ \left( \frac{1}{1+e} \right) \rho_s + \frac{e}{1+e} S \rho_w + \frac{e}{1+e} (1-S) \rho_a \right]}{\partial t} \mathbf{g} = 0 \quad (5)$$

where  $\rho_i$  is the density ( $i$  refers to air, water and solid),  $e$  is void ratio,  $S$  is the saturation degree of liquid phase,  $\dot{m}_h$  is the mass source term,  $\mathbf{v}^{ri}$  denote the phase velocity with respect to fixed spatial axes,  $Q_h$  represents the heat generation by binder hydration,  $C_i$  is the specific heat capacity,  $\sigma$  is the (macroscopic) total stress tensor, and  $\mathbf{g}$  is the acceleration of gravity.

In this study, the prediction model of binder hydration degree proposed by Schindler and Folliard [9] is adopted:

$$\xi = \left( \frac{1.031 \cdot w/c}{0.194 + w/c} + 0.5 \cdot X_{FA} + 0.30 \cdot X_{slag} \right) \quad (6)$$

$$\times \exp \left\{ - \left\{ \tau / \left[ \int_0^{\xi} \exp \left[ - \frac{E_a}{R} \left( \frac{1}{T_c} - \frac{1}{T_r} \right) dt \right] \right]^{\beta} \right\} \right\}$$

where  $\tau$  is the hydration time parameter (hours),  $\beta$  represents hydration shape parameters,  $t_e$  is the equivalent age at the reference temperature  $T_r$ ,  $T_c$  is the temperature of CPB,  $E_a$  is activation energy (J/mol) and  $R$  equals the natural gas constant (8.314 J/mol/K),  $w/c$  is the water-cement ratio,  $X_{FA}$  and  $X_{slag}$  respectively represent the weight fraction of fly ash and blast furnace slag with respect to total weight of binder.

For the sink term,  $\dot{m}_h$ , in water mass balance equation and the heat source term,  $Q_h$ , in the energy balance equation, the following equations are derived based on previous research performed by Schindler and Folliard [7] and Powers and Brownyard [10]:

$$\dot{m}_h = 2m_{hc0} (0.187x_{C_3S} + 0.158x_{C_2S} + 0.665x_{C_3A} + 0.2130x_{C_4AF}) \quad (7)$$

$$\times \left\{ \left( \frac{\tau}{t_e} \right)^{\beta} \left( \frac{\beta}{t_e} \right) \xi \exp \left[ \frac{E}{R} \left( \frac{1}{273+T_r} - \frac{1}{273+T} \right) \right] \right\}$$

$$Q_h = (H_c \cdot X_c + 461 \cdot X_{slag} + 1800 \cdot x_{CaO/FA} \cdot X_{FA}) C_b \quad (8)$$

$$\times \left( \frac{\tau}{t_e} \right)^{\beta} \cdot \left( \frac{\beta}{t_e} \right) \cdot \xi(t_e) \cdot \exp \left[ \frac{E}{R} \left( \frac{1}{273+T_r} - \frac{1}{273+T} \right) \right]$$

where  $m_{hc0}$  is the initial cement mass,  $T_c$  is the temperature of the cement-based materials,  $H_c$  is

total heat of hydration of the cement,  $C_b$  is the apparent binder density with respect to total volume of CPB mixture,  $X_i$  is weight ratio of corresponding minerals in terms of total binder content.

### 3. Model Validation and Simulation

In order to simulate the consolidation process of CPB, pressure cell test was conducted in this study. The monitoring data were used to validate the developed consolidation model via Comsol Multiphysics.

#### 3.1 Pressure cell test

In order to quantitatively investigate the influence of coupled THMC processes on CPB consolidation, a laboratory pressure cell apparatus (CUS cell) originally developed by Ghirian and Fall [11] is adopted in this study. A schematic diagram of the pressure cell apparatus is shown in Fig. 1. The apparatus is able to apply up to 600 kPa of air pressure to the CPB sample which can be used to mimic the self-weight effect induced by the dynamic backfilling processes. During the test, various sensors were installed to monitor coupled processes within CPB, which includes LVDT, temperature sensor (model: 5TE) and suction sensor (model: MPS-2).

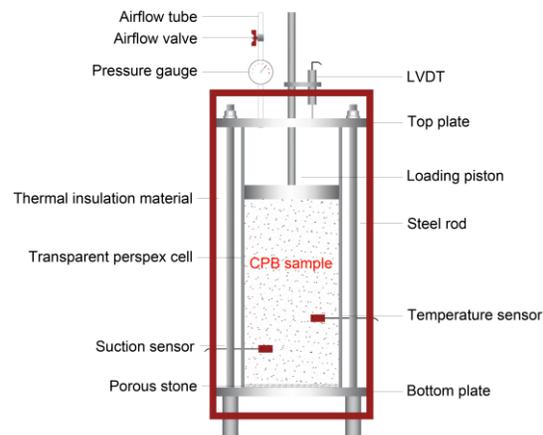


Fig. 1. Schematic diagram of pressure cell apparatus

For the mix recipe of CPB in this study, artificial silica tailings, tap water ( $w/c=7.6$ ) and PCI (4.5 wt%) are employed. The initial

temperature of mix components was set to 25 °C. After mixing in a food mixer for 7 minutes, the fresh CPB was cast into the curing cell. Afterwards, the pressure cell was covered by thermal insulation glass wool blanket and cured in a temperature (22.1 °C) and humidity (45% RH) controlled room.

In order to simulate the dynamic backfilling process (i.e., the self-weight effect), the scheme of applied air pressure presented in Fig. 2 is employed in this study. In particular, immediately after placement, an initial contact pressure of 20 kPa was applied onto the loading piston. During the testing, pressures of 35, 55, 75 and 150 kPa were applied to the sample every 3 hours for the first 12 hours. Then, pressures of 300, 450 and 600 kPa were applied every 24 hours.

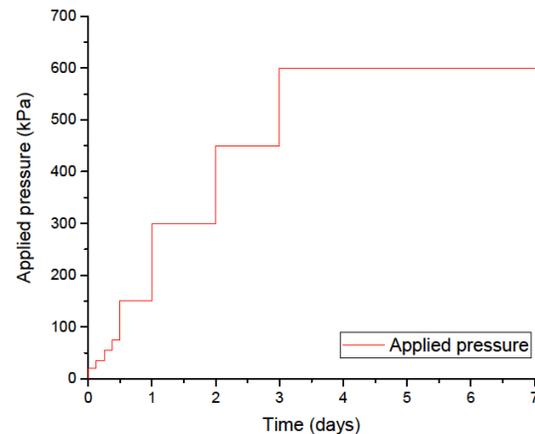


Fig. 2. The scheme of applied air pressure

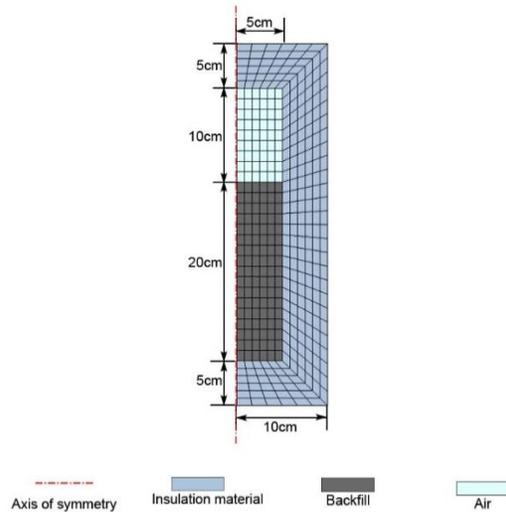
#### 3.2 Experimental results and model validation

Due to geometrically characteristic of pressure cell, the 2D axisymmetric model is utilized in this study. The geometry model and mesh are demonstrated in Fig. 3.

Based on the experimental set-up and curing conditions of pressure cell test, initial values and boundary conditions adopted for the numerical simulation of each module are tabulated in Table 1.

As shown in Fig. 4, it can be observed that the vertical settlement immediately increases after the air pressure is applied on CPB, which demonstrates the impact of self-weight pressure.

In addition, the settlement gradually varies during the loading interval, which is caused by the effect of chemical shrinkage. Through the comparison between measured data and predicted results, a good agreement is obtained. Hence, the developed consolidation model is able to capture the influence of self-weight effect and chemical shrinkage on the consolidation.



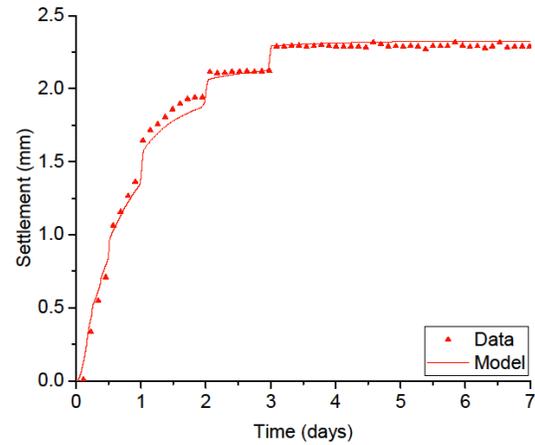
**Fig. 3.** Geometry and mesh of simulated model

**Table 1.** Initial values and boundary conditions

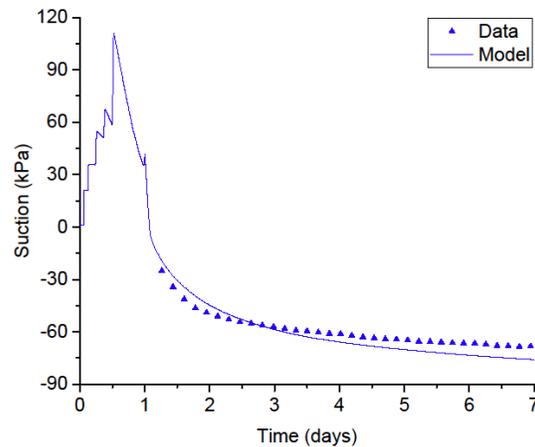
Mechanical module	
Top surface	Boundary load
Lateral sides	Roller
Bottom side	Fixed
Volume force	Gravity
Hydraulic module	
Top surface	Insulated
Lateral sides	Insulated
Bottom side	Insulated
Volume force	Gravity
Initial value	Hydraulic head=0
Thermal module	
Top side ( °C )	22.1
Lateral sides ( °C )	22.1
Bottom side ( °C )	22.1
Initial temperature ( °C )	25

For the evolution of suction within CPB (Fig. 5), it can be seen that the detectable suction value was collected at around the end of the 1<sup>st</sup> day. With curing time, the rate of change of suction gradually reduces. As expected, the

numerical result is in very good agreement with experimental data.



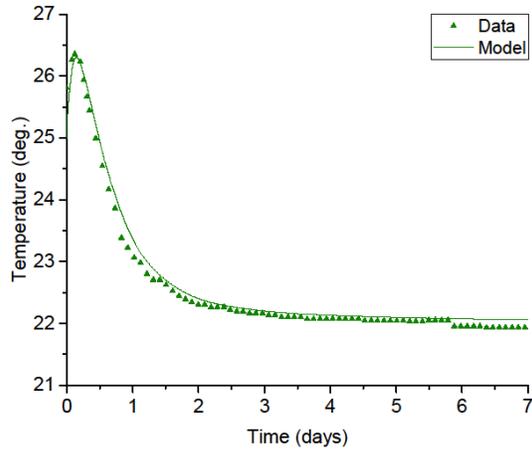
**Fig. 4.** Comparison of measured data and model results of vertical settlement



**Fig. 5.** Comparison of measured data and model results of suction

As mentioned in Section 1, the released heat by the binder hydration will affect the evolution of temperature inside CPB. Due to the temperature dependence of hydration rate, the evolved temperature will directly influence the hydration degree and correspondingly the chemical shrinkage. Therefore, it is of great interest to investigate the temperature evolution within CPB with curing time. As can be seen in Fig. 6, the temperature quickly reaches the peak value after about 3 hours, then, gradually approaches the room temperature (22.1 °C). According to the comparison between

experimental data and predicted result presented in Fig. 6, the numerical simulation show remarkable agreement with the measured data.



**Fig. 6.** Comparison of measured data and model results of temperature

#### 4. Conclusions

From the results obtained in this study, the following conclusions can be drawn:

- (1) Based on the experimental results of pressure cell test, the consolidation behavior of CPB is closely related to the strongly coupled multiphysics including thermal (T), hydraulic (H), mechanical (M) and chemical (C) process within CPB. Therefore, the conventional consolidation analysis technique (i.e., analysis of coupled H-M process) is unsuitable to predict the deformation of CPB.
- (2) Compared with the measured data of pressure cell test, the developed consolidation model is able to capture the evolution of vertical settlement, suction and temperature within CPB. Hence, the consolidation model possesses the capability to mimic the effects of coupled THMC processes on consolidation behavior.

#### 5. References

[1] Cui L, Fall M. Multiphysics modelling of the behaviour of cemented tailings backfill materials. International Conference on Civil,

Structural and Transportation Engineering. Ottawa, Canada. (330),1-7. (2015).

[2] Fall M, Nasir O. Mechanical behaviour of the interface between cemented tailings backfill and retaining structures under shear loads. Geotechnical and Geological Engineering; 28(6):779-90.(2010)

[3] Cui L, Fall M. An evolutive elasto-plastic model for cemented paste backfill. Computer and Geotechnics (In press).(2015)

[4] Fall M, Nasir O, Cui L. Coupled modeling of the strength development and distribution within cemented paste backfill structure. 49th US Rock Mechanics/Geomechanics Symposium. San Francisco, CA, USA: American Rock Mechanics Association; p. Paper 587. (2015).

[5] Cui L, Fall M. Multiphysics model for consolidation behaviour of cemented paste backfill. Journal of Geotechnical and Geoenvironmental Engineering (submitted).(2015)

[6] Cui L, Fall M. A coupled thermo-hydro-mechano-chemical model for underground cemented tailings backfill. Tunnelling and Underground Space Technology; 50:396-414.(2015)

[7] Powers TC, Brownyard TL. Studies of the physical properties of hardened Portland cement paste. ACI Journal Proceedings: ACI. (1946).

[8] Schindler AK, Folliard KJ. Influence of supplementary cementing materials on the heat of hydration of concrete. Advances in Cement and Concrete IX Conference, Copper Mountain Conference Resort in Colorado. (2003).

[9] Schindler AK, Folliard KJ. Influence of supplementary cementing materials on the heat of hydration of concrete. Advances in Cement and Concrete IX Conference. Copper Mountain Conference Resort in Colorado. 17-26. (2003).

[10] Powers TC, Brownyard TL. Studies of the physical properties of hardened Portland cement paste. ACI Journal Proceedings: ACI; p. 249-336. (1946).

[11] Ghirian A, Fall M. Coupled Behavior of Cemented Paste Backfill at Early Ages. Geotechnical and Geological Engineering:1-26.