Oct. 8, 2015, 1:00 p.m. Boston Conf. Session on Optimization & Simulation Methods

*Uncertainty of **FEM** Solutions using a Nonlinear Least Squares Fit Method and a Design of Experiments Approach* **

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COMSOL CONFERENCE
2015 BOSTON

***Co-authored by Fong, Heckert, Filliben, Marcal, and Rainsberger.*

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Outline of Talk

(23 slides)

- *(5)* Why Accuracy in FEM Stress Estimates are Important ?
- *(5)* COMSOL Solutions for a Wrench at different **mesh densities**.
- *(1)* What is a logistic distribution ?
- *(5)* Uncertainty of COMSOL Wrench Solutions using NL-LSQ.
- *(7)* Stresses in a Cantilever Beam for different **element types.**

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(1) Why Accuracy In FEM Stress Estimates Are Important ?

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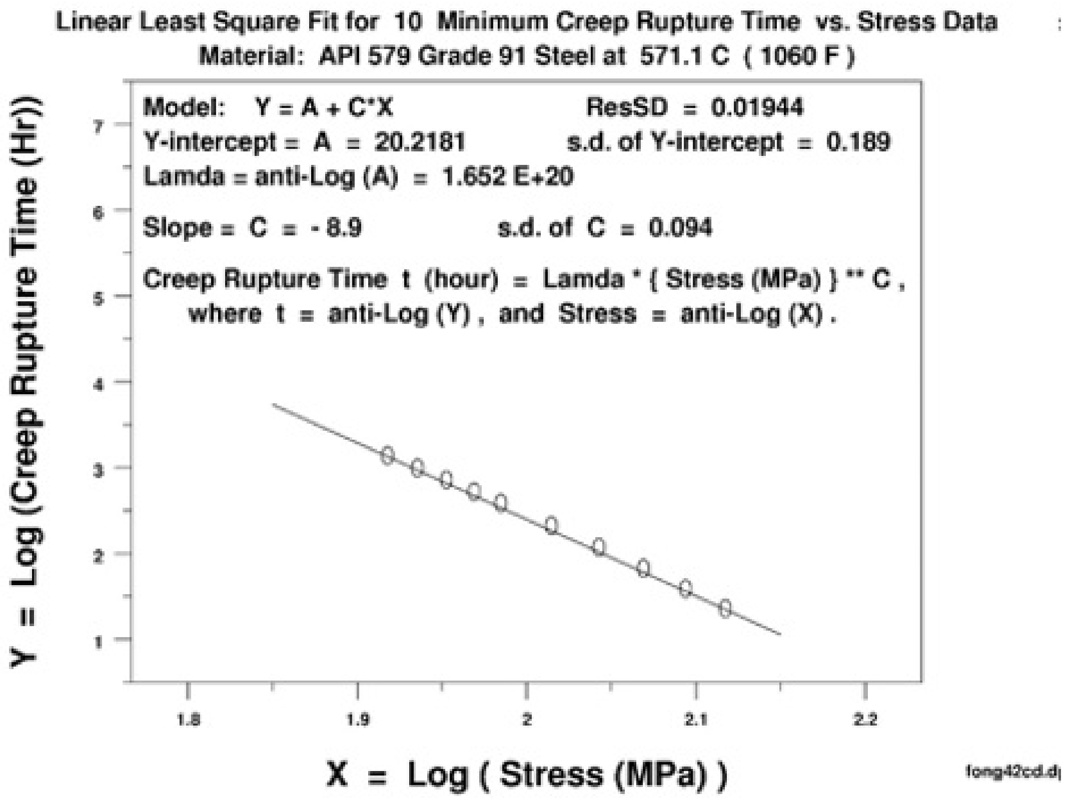
Fig 1. plot of Log(Creep Rupture Time) vs Log (Stress)

slope C = - 8.9



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- Recent experimental results on Creep Rupture in Fig.1
- Let cv = coefficient of variation = s.d. / estimated mean.
- Following Fong et al [2015] considering classical laws of error propagation, we establish Eq. (2) for the coefficient of variation (*cv*) of the creep rupture time, *t*, as a linear function of the *cv* of stress, σ :

$$cv(t) = |C| * cv(\sigma). \qquad (2)$$

Need to assess carefully the uncertainty of the stress estimate,

1% variation in stress = 9% var. in creep rupture time

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- There are at least four sources of uncertainty in FEM:
- (1) Uncertainty due to Element Type (2015).
- (2) Uncertainty due to Mesh Density (2015).
- (3) Uncertainty due to **Model Parameters (2014).**
- (4) Uncertainty due to **Solution Platform (2016).**

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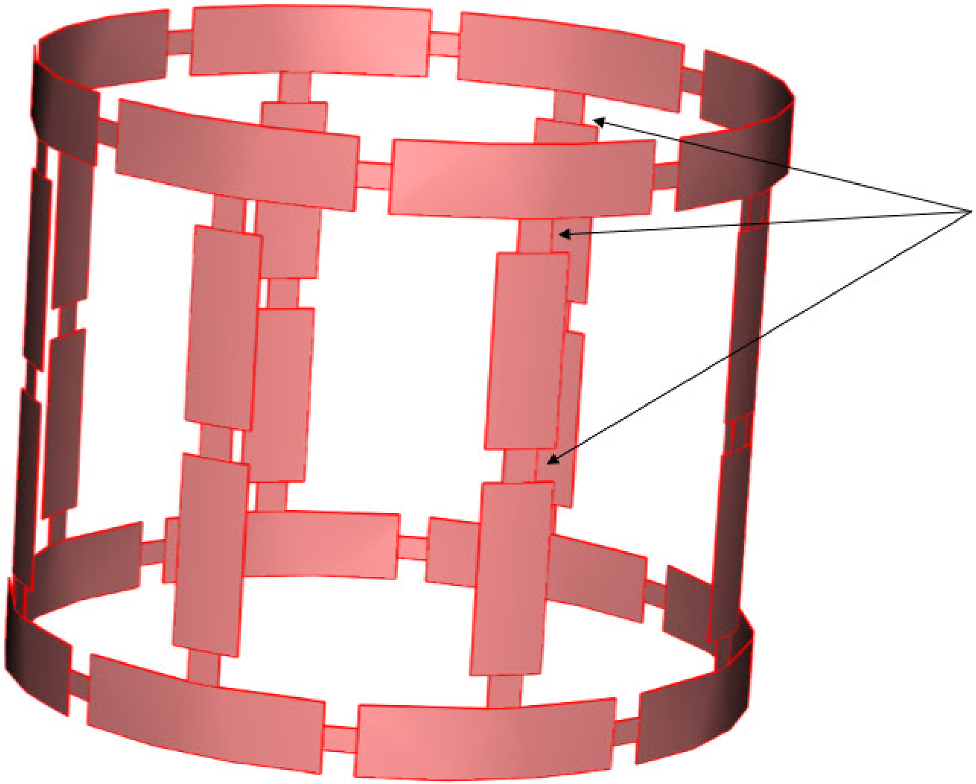
- MRI



Each flap represents a simple circuit or port element such as a 50 Ohm feed or a tuning capacitor.

(Standard finite element modeling approach for RF and Microwave components)

$$\nabla \times \mu_r^{-1}(\nabla \times \mathbf{E}) - k_0^2(\epsilon_r - \frac{j\sigma}{\omega\epsilon_0})\mathbf{E} = 0$$

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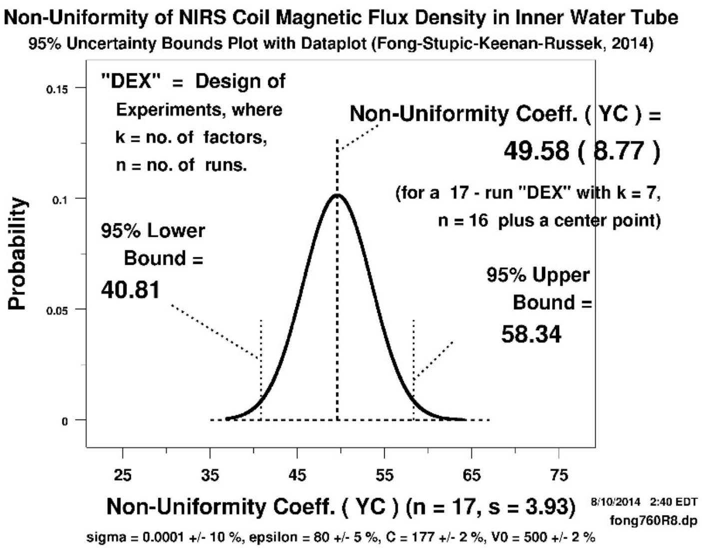
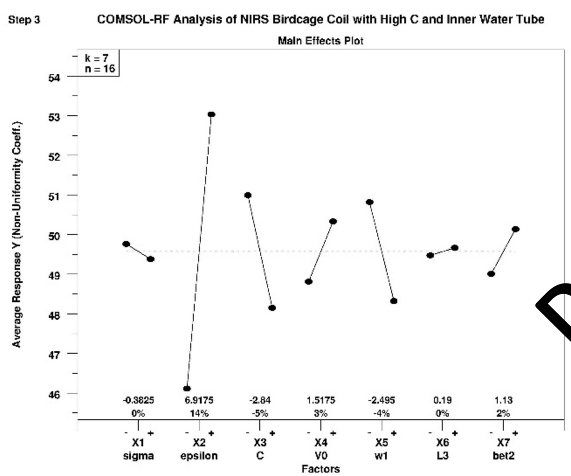
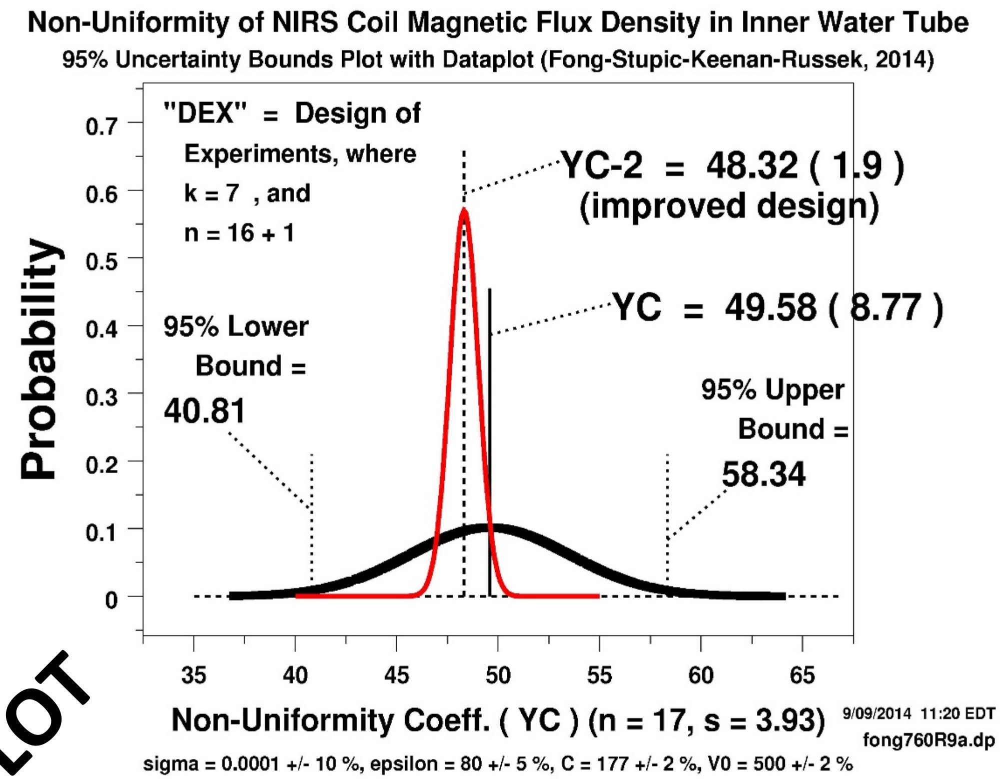




DATAPLOT



Before:

NUC = 49.58 (8.77)

After:

NUC = 48.32 (1.90)

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(2) COMSOL Solutions for a Wrench at different **mesh densities.**

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Stresses and Strains in a Wrench

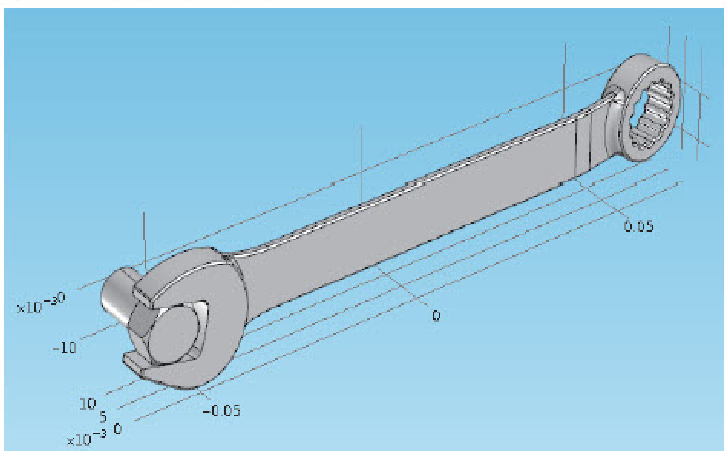
Introduction

This tutorial demonstrates how to set up a simple static structural analysis. The analysis is exemplified on a combination wrench during the application of torque on a bolt.

Despite its simplicity, and the fact that very few engineers would run a structural analysis before trying to turn a bolt, the example provides an excellent overview of structural analysis in COMSOL Multiphysics.

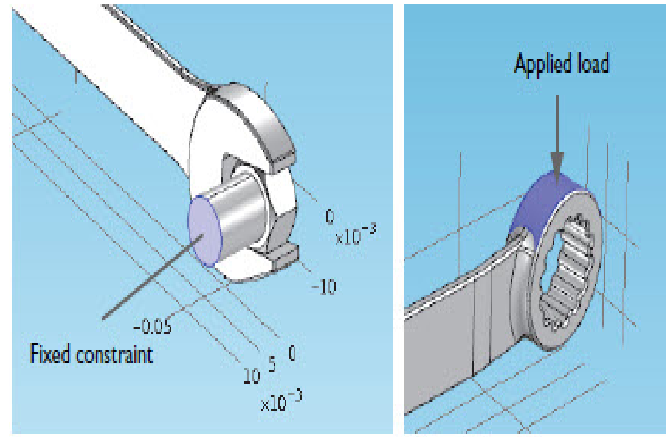
Model Definition

The model geometry is shown below.



Solved with COMSOL Multiphysics 5.0

The bolt's fixed constraint is at the cross section shown below. A load is applied at the box end of the combination wrench.



For the purpose of this model, assume that there is perfect contact between the wrench and the bolt. A possible extension of the model is to apply a contact condition between the wrench and the bolt where the friction and the contact pressure determines the position of the contact surface.

Model Library path: `COMSOL_Multiphysics/Structural_Mechanics/wrench`

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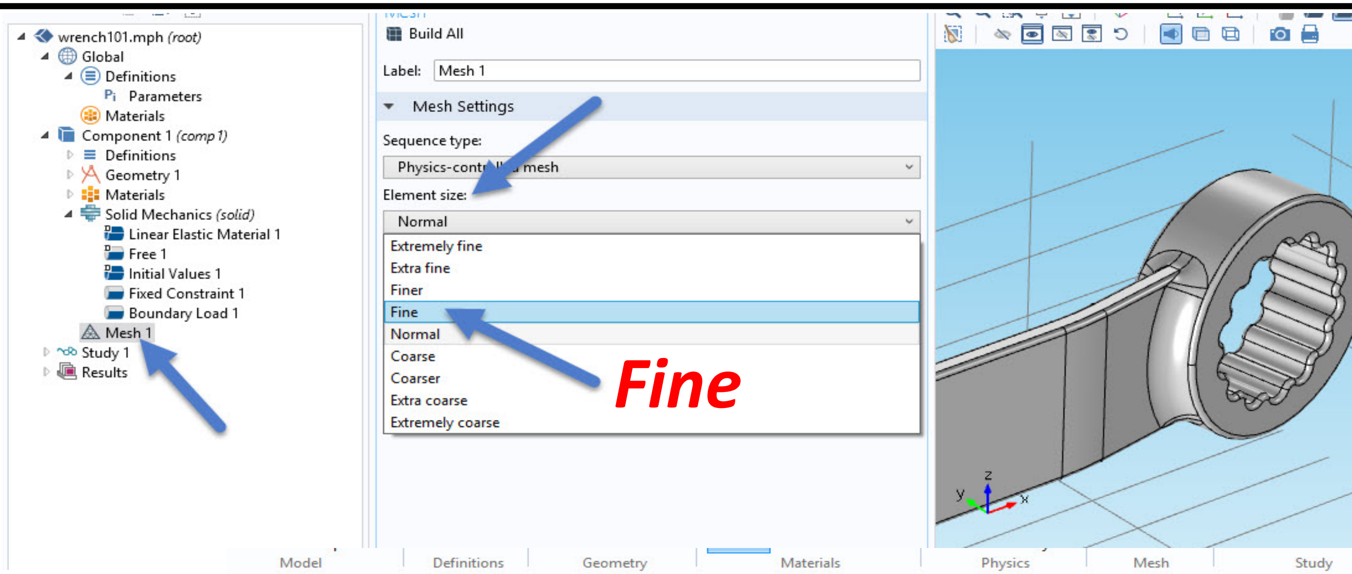

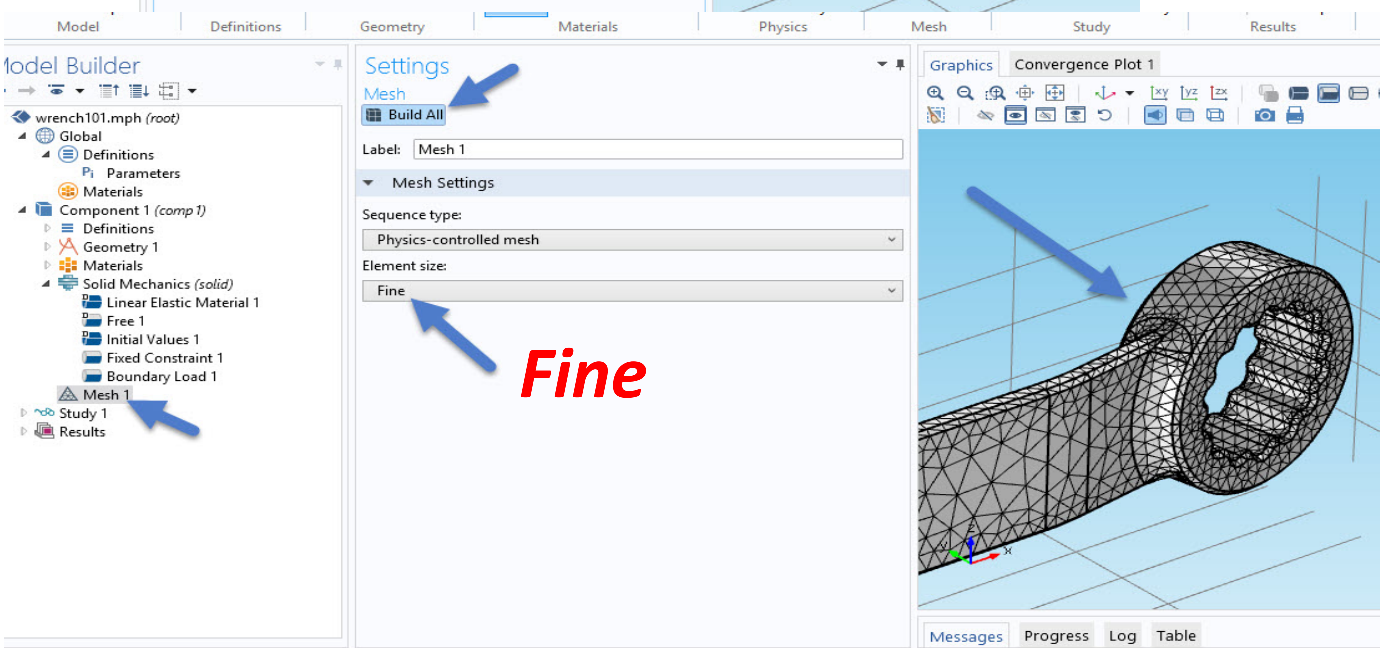

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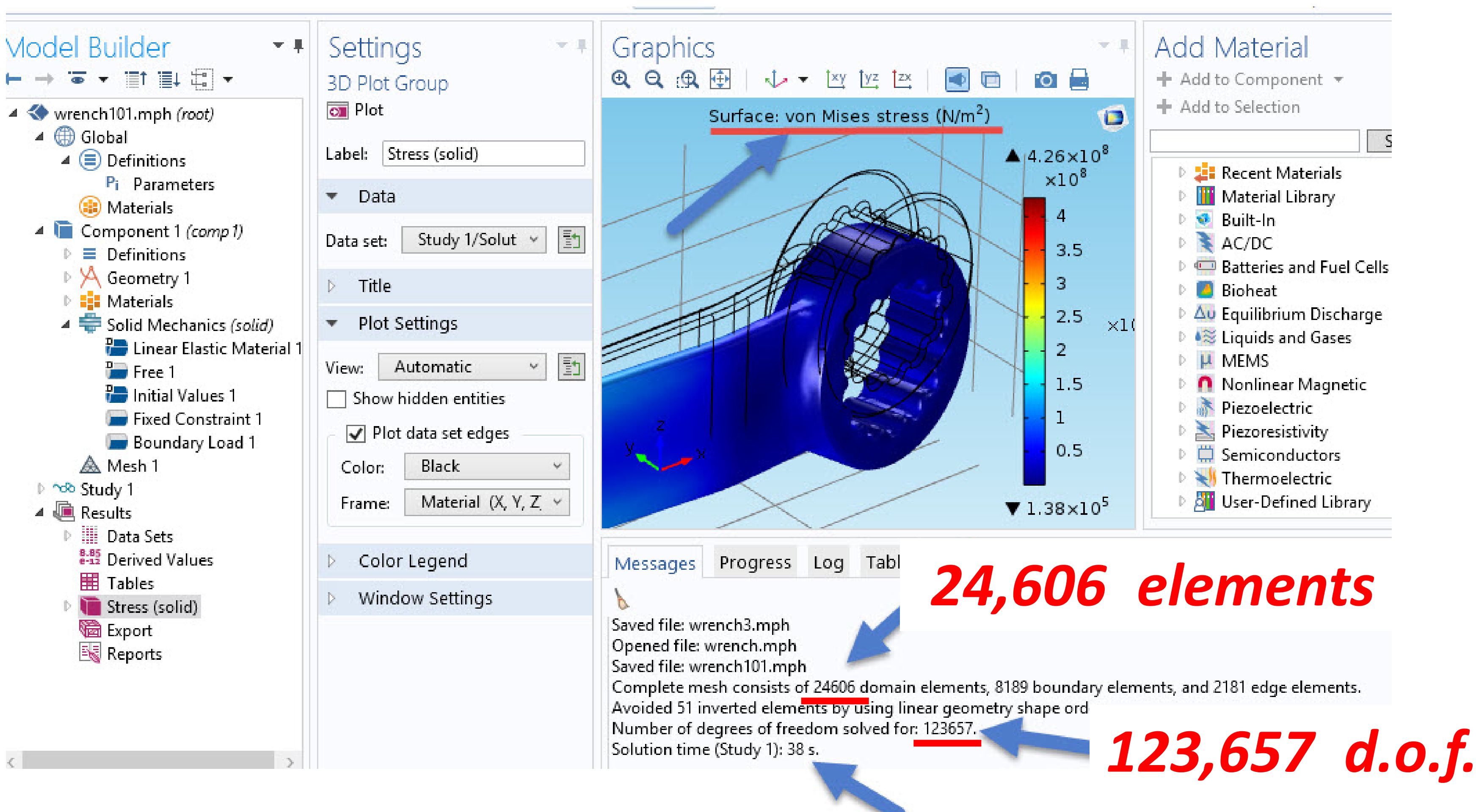
Element Size: Fine



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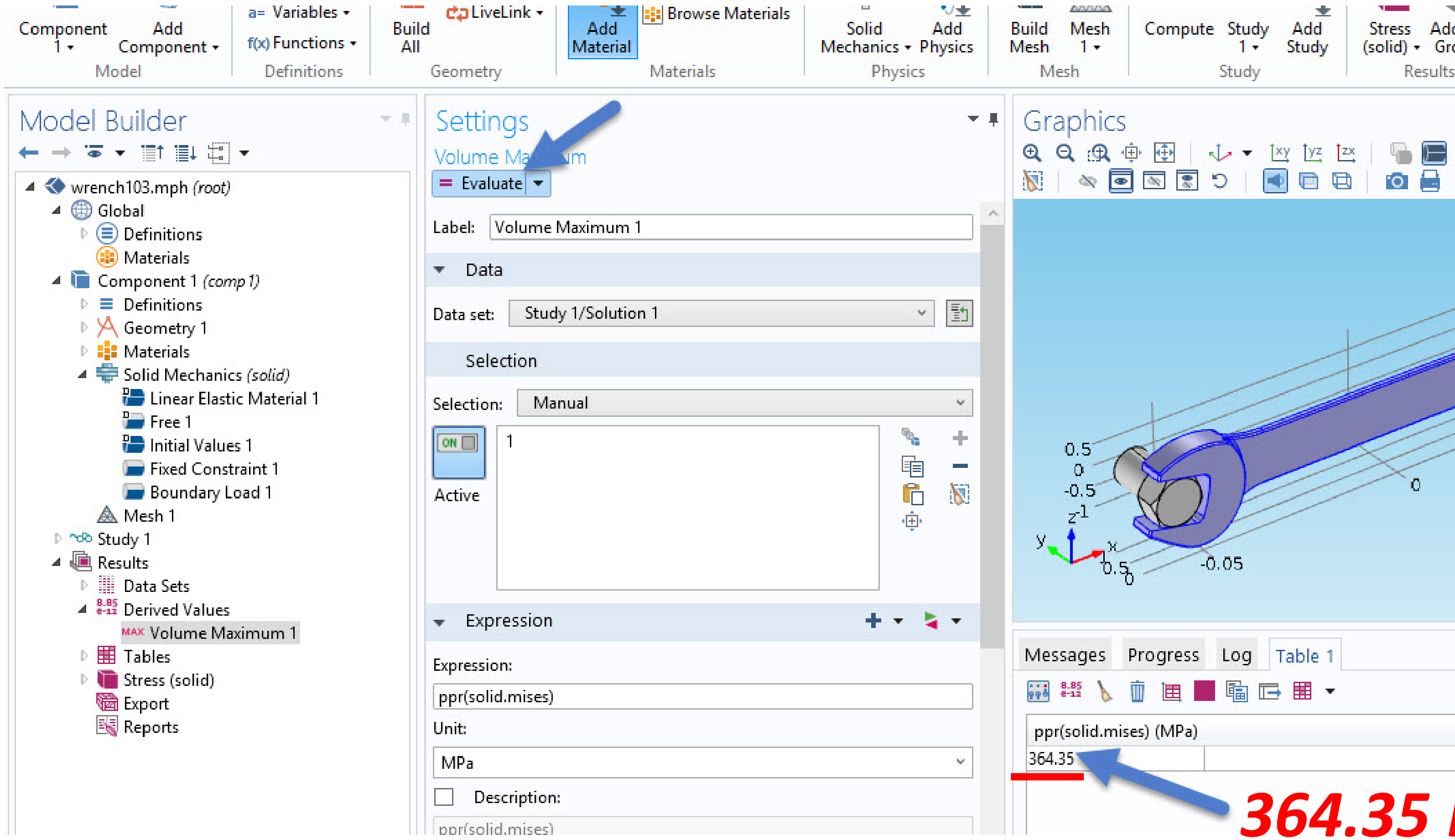
Element Size: Fine



364.35 MPa

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Element Size	Degrees of Freedom (d.o.f.) (Log_10 (dof))	Max. Mises Stress (MPa)	% of Stress (100 for fine)
Fine	**123, 657** (5.0922)	**364.35**	**(100 %)**
Normal	74,226 (4.8706)	355.02	(97.4 %)
Coarse	47,022 (4.6723)	339.37	(93.1 %)
Coarser	31,476 (4.4980)	326.76	(89.7 %)
Extremely Coarse	10743 (4.0311)	322.45	(88.5 %)

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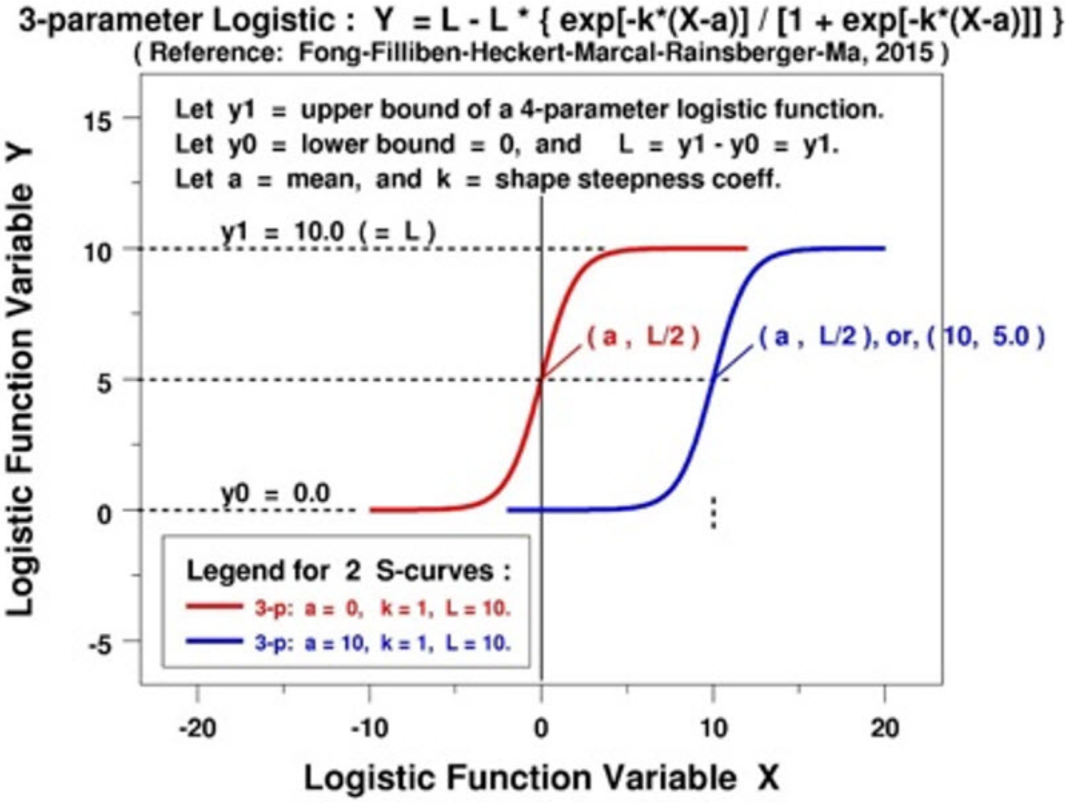
(3) What is a logistic distribution ?

Pierre Francois Verhulst (1845)

$$f(x) = y1 - L / (1 + \exp(-k * (x - a))),$$



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(4) Uncertainty of COMSOL Wrench Solutions Using Nonlinear Least Squares (NL-LSQ) Fit.

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A Non-Linear Least Square Fit using an S-curve Logistic Function:

```
Least Squares Non-Linear Fit for:        COMSOL Wrench Stress Analysis (5 Fine to Coarse Meshes)

4-Parameter Logistic Function Model:     Y05 = Y1 - L*(EXP(-K*(XLOG-X0))/(1 + EXP(-K*(XLOG-X0))))

Sample Size:     5                       No Replication Case:
```

Iteration Number	Convergence Measure	Residual Standard Deviation	*	Parameter Estimates			
1	0.1000000E-01	0.2886132E+02	*	0.3643500E+03	0.4190000E+02	0.1000000E+01	0.5000000E+01
2	0.1139062E+00	0.2782295E+02	*	0.3720484E+03	0.3211696E+02	0.4972718E+01	0.4835044E+01
3	0.5695313E-01	0.1013394E+02	*	0.3644373E+03	0.4212619E+02	0.8006263E+01	0.4635532E+01
4	0.2847656E-01	0.1516070E+01	*	0.3659203E+03	0.4371380E+02	0.8126122E+01	0.4737073E+01
5	0.1423828E-01	0.1218623E+01	*	0.3663495E+03	0.4436871E+02	0.8229342E+01	0.4733777E+01
6	0.7119141E-02	0.1217993E+01	*	0.3663420E+03	0.4436198E+02	0.8250083E+01	0.4733750E+01
7	0.3559570E-02	0.1217981E+01	*	0.3663429E+03	0.4436449E+02	0.8248517E+01	0.4733739E+01

		Final Parameter Estimates	Approximate Standard Deviation	t-Value
1	Y1	366.3426	2.0070	182.5263
2	L	44.3641	2.5945	17.0992
3	K	8.2487	1.0702	7.7076
4	X0	4.7336	0.0194	243.7687

366.34 MPa
s.d. = 2.0 MPa

```
Residual Standard Deviation:              1.2179
Residual Degrees of Freedom:                 1
```


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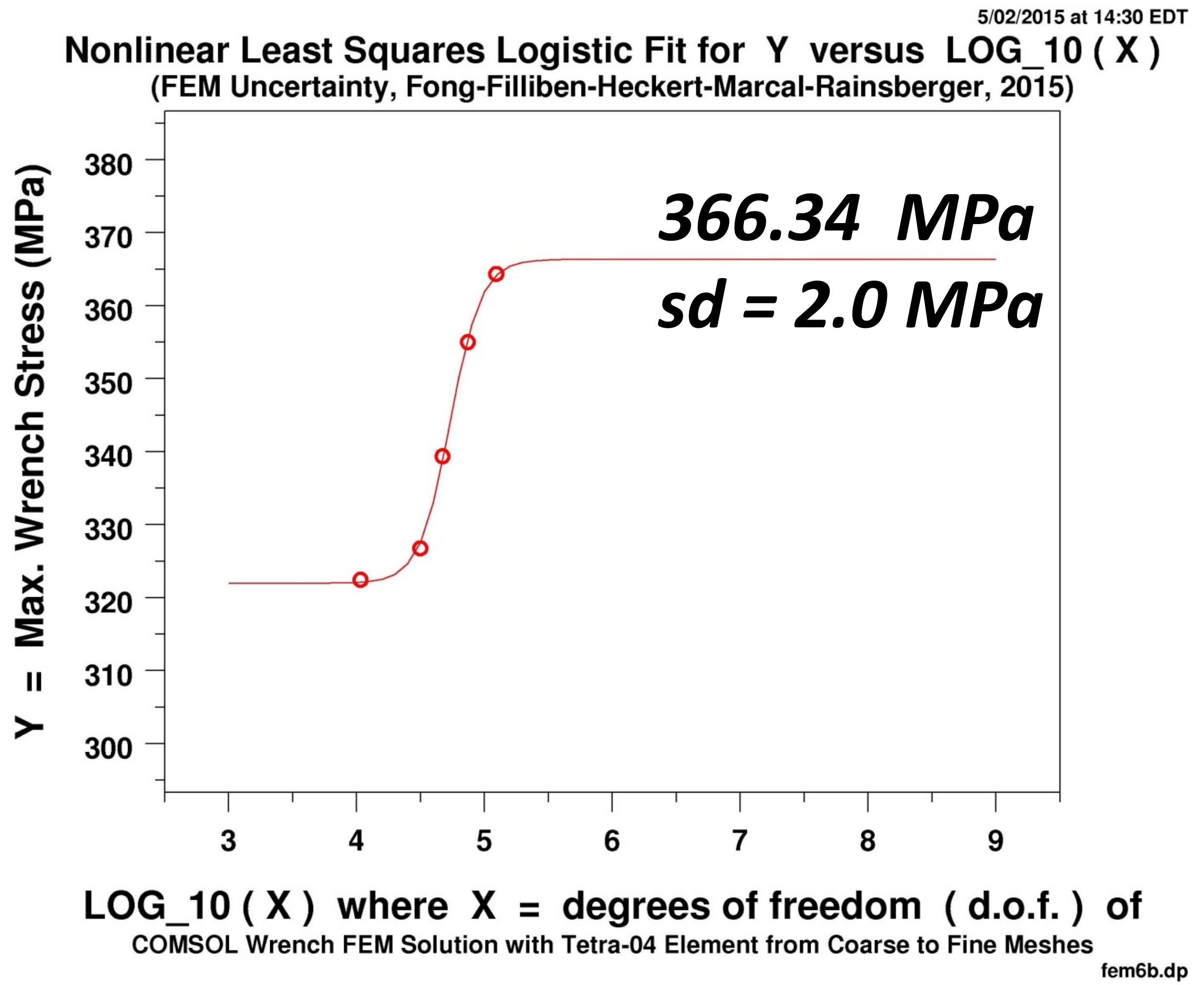

Degrees of Freedom (d.o.f.) (Log_10 (dof))	Max. Mises Stress (Mpa)
123, 657 (5.0922)	**364.35**
74,226 (4.8706)	355.02
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31,476 (4.4980)	326.76
10743 (4.0311)	322.45

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Element Size	Degrees of Freedom (d.o.f.) (Log_10 (dof))	Max. Mises Stress (Mpa)
Fine	**123, 657** (5.0922)	**364.35**
Normal	74,226 (4.8706)	355.02
Coarse	47,022 (4.6723)	339.37
Coarser	31,476 (4.4980)	326.76
Extremely Coarse	10743 (4.0311)	322.45

Predicted Max. Mises Stress = **366.34 MPa, s.d. = 2.0 MPa.**

Question: Is that good enough ?

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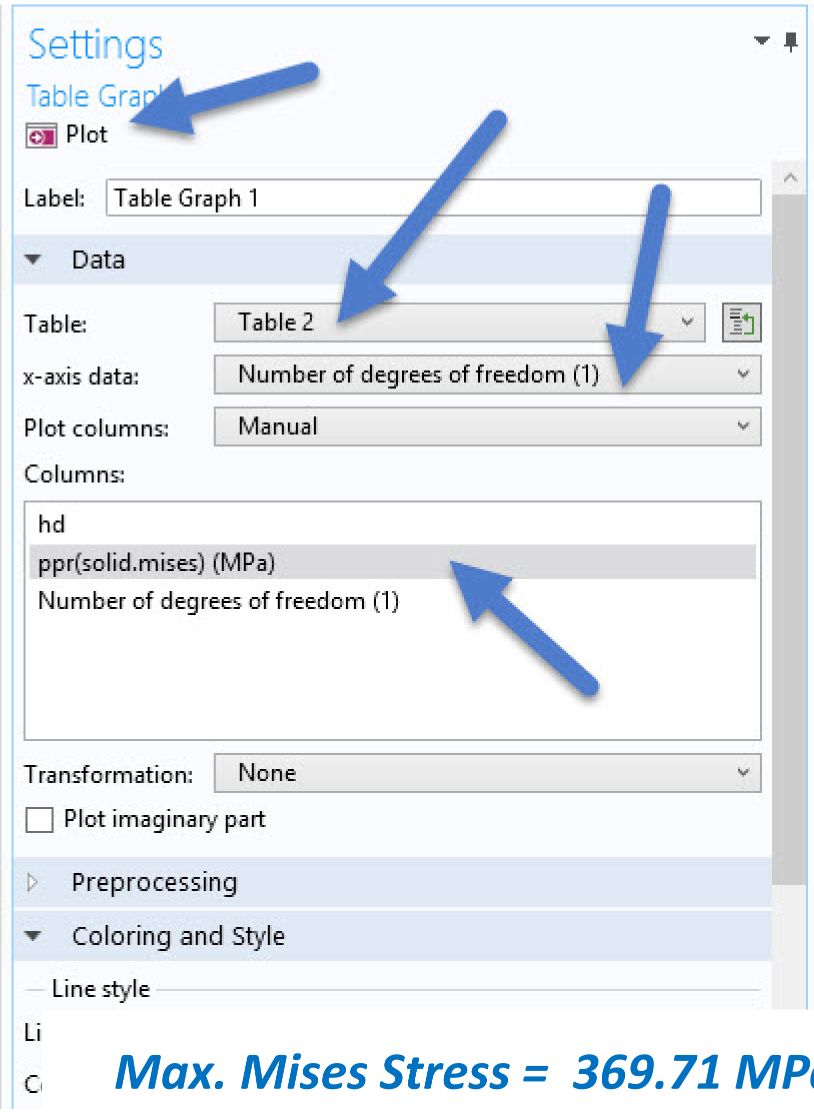

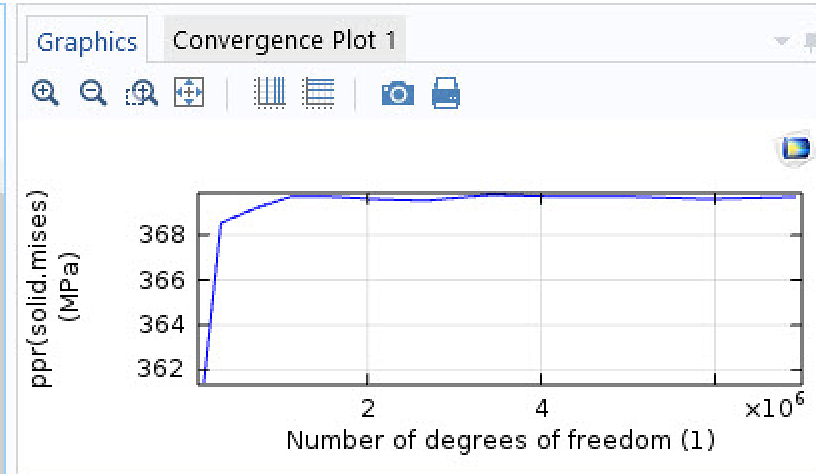

Messages | Progress | Log | Table 2

hd	ppr(solid.mises) (MPa)	Number of degrees of freedom (1)
1.0000	361.40	1.1875E5
2.0000	368.55	3.1397E5
3.0000	369.24	7.3222E5
4.0000	369.72	1.1196E6
5.0000	369.73	1.5178E6
6.0000	369.61	2.1136E6
7.0000	369.54	2.6701E6
8.0000	369.80	3.4118E6
9.0000	369.72	4.1930E6
10.000	369.72	5.0332E6
	369.61	5.9196E6
	369.71	6.9329E6

Max. Mises Stress = 369.71 MPa = 369.71 … 6.9329E6 = 6,932,883 (d.o.f.)

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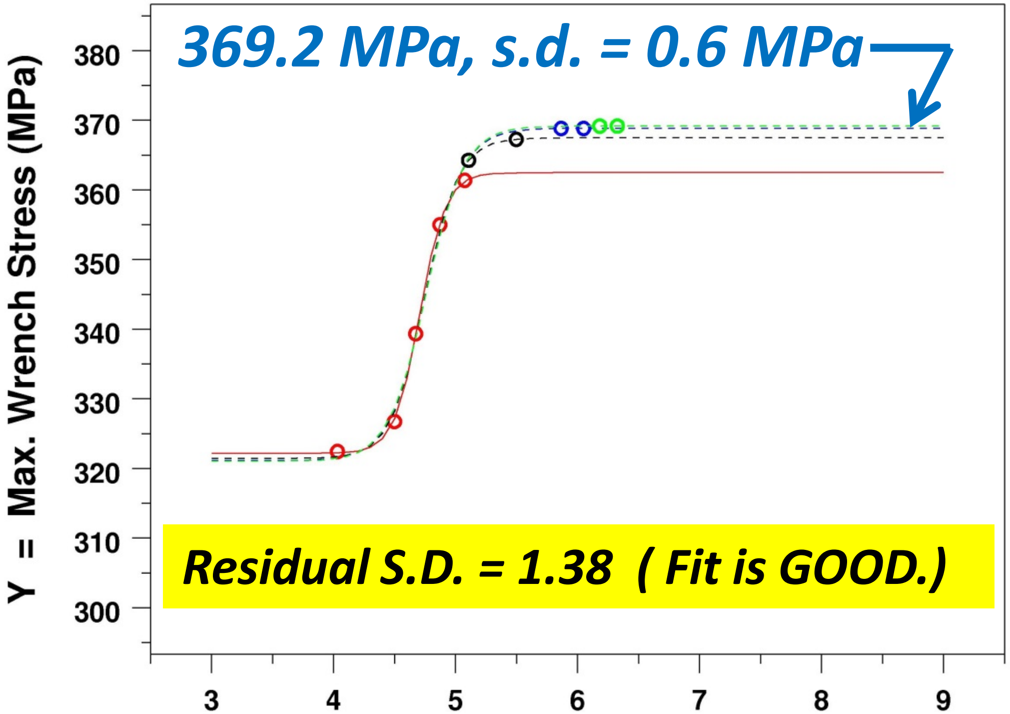
4/30/2015 at 21:30 EDT

Nonlinear Least Squares Logistic Fit for Y versus LOG_10 (X)
(FEM Uncertainty, Fong-Filliben-Heckert-Marcal-Rainsberger, 2015)



LOG_10 (X) where X = degrees of freedom (d.o.f.) of
COMSOL Wrench FEM Solution with Tetra-04 Element from Coarse to Fine Meshes
fem7_cms_0506070809_10_12.dp

FEM Result Y (MPa)	Degrees of Freedom (dof) X
322.45	10743
326.76	31476
339.37	47022
355.02	74226
361.40	118750
364.35	127663
368.55	313970
369.24	732220
369.72	1119600
369.73	1517800
369.61	2113600
369.54	2670100
369.80	3411800
369.72	4193000
369.72	5033200
369.61	5919600
369.71	6,932,883

Ans. Max. Mises Stress at 95 % confidence level = (368.0, ... , 370.4 MPa)

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(5) Stresses in a Cantilever Beam for Different **element types.**

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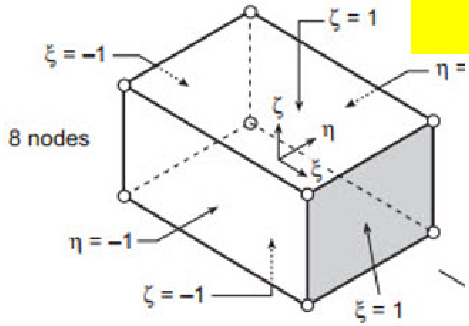
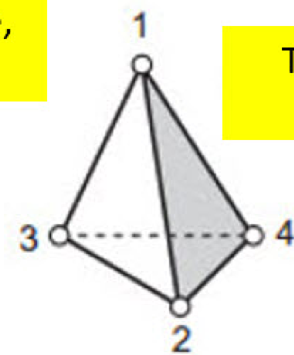
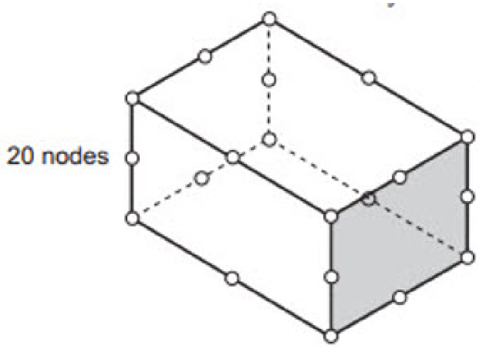


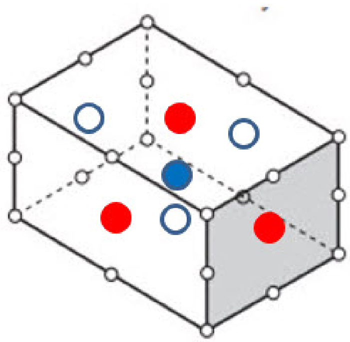
Hexahedron – 8-node,
or, **Hexa-8.**



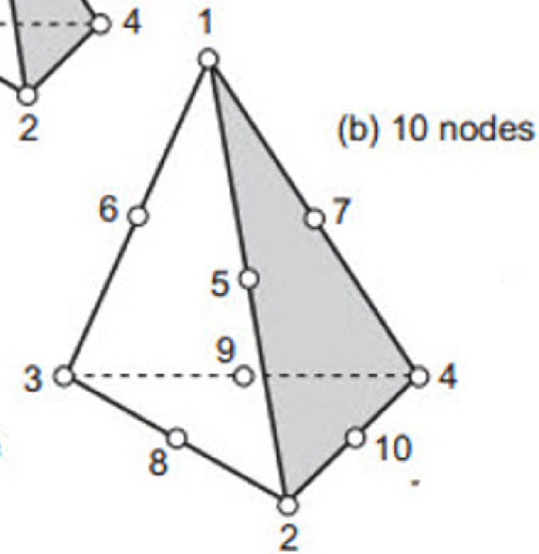
Tetrahedron – 4-node,
or, **Tetra-4.**



Hexahedron- 20 nodes
or, **Hexa-20**



Hexahedron- 27 nodes
or, **Hexa-27**



Tetrahedron – 10-node,
or, **Tetra-10.**

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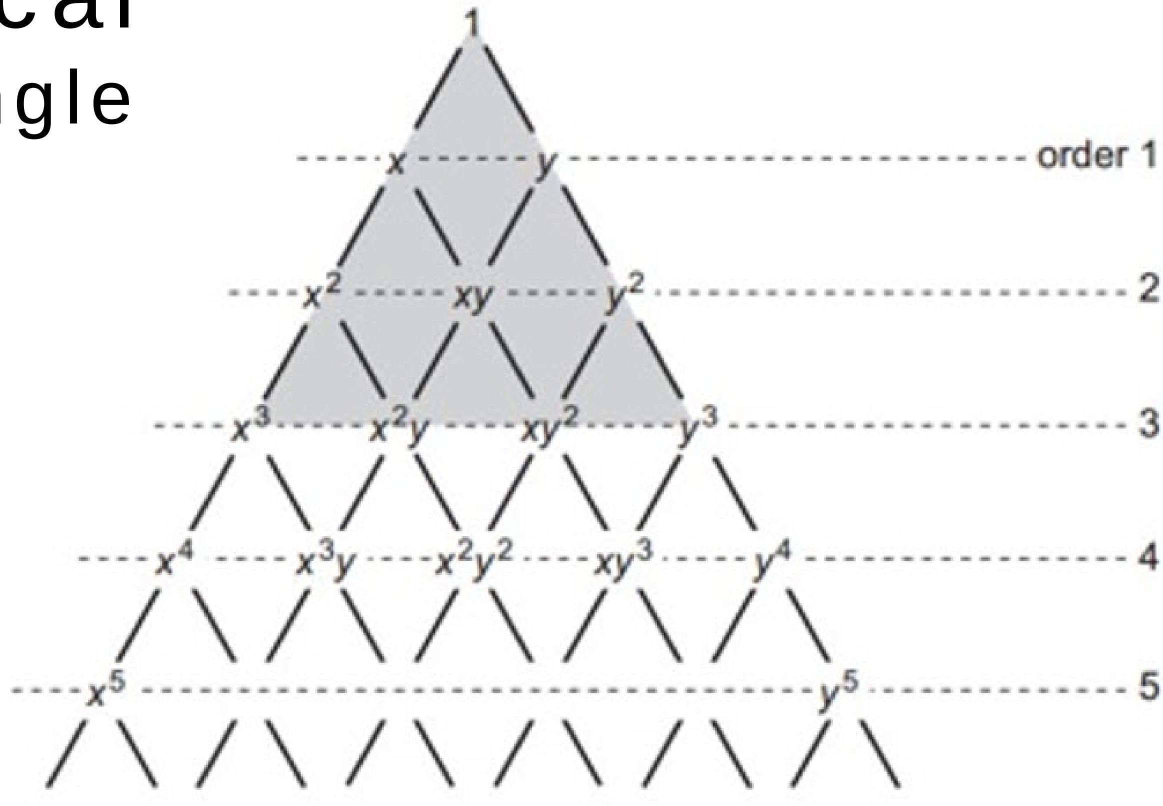
Pascal Triangle



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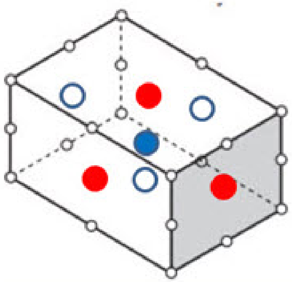
FEM Uncertainty due to Element Type

Truncation error in displacement method in FEA, u is a single d.o.f. and {a} the undetermined coefficients , and $h(0)^n$ the truncation error .



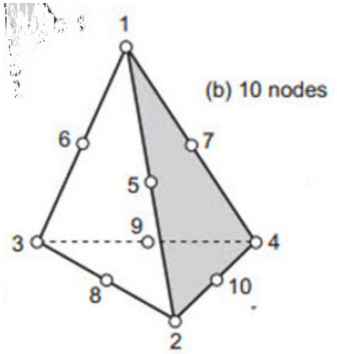
Hexahedron- 27 nodes or, **Hexa-27**

- For quadrilaterals in 2-D with 9 nodes

$$u = [\, 1,\ x,\ xy,\ y,\ x^2,\ x^2y,\ x^2y^2,\ xy^2,\ y^2 \,]\, \{a\} + h(0)^3$$

- For simplex elements in 2-D with 6 nodes

$$u = [\, 1,\ x,\ xy,\ y,\ x^2,\ y^2 \,]\, \{a\} + h(0)^2$$



Tetrahedron – 10-node, or, **Tetra-10.**

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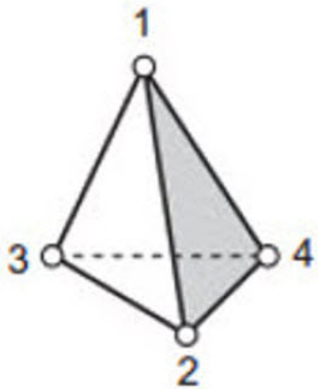
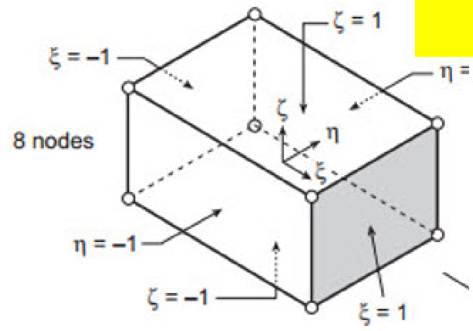
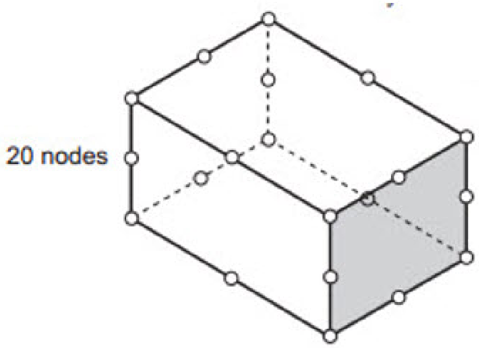
Tetrahedron – 4-node,
or, Tetra-4.



Hexahedron – 8-node,
or, **Hexa-8.**

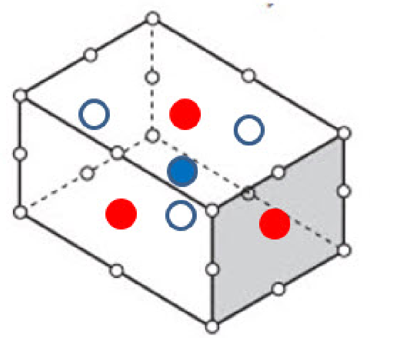


$h(0)^2$



Hexahedron- 20 nodes
or, **Hexa-20**

$h(0)^3$



Hexahedron- 27 nodes
or, **Hexa-27**

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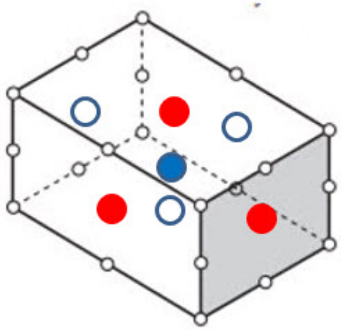
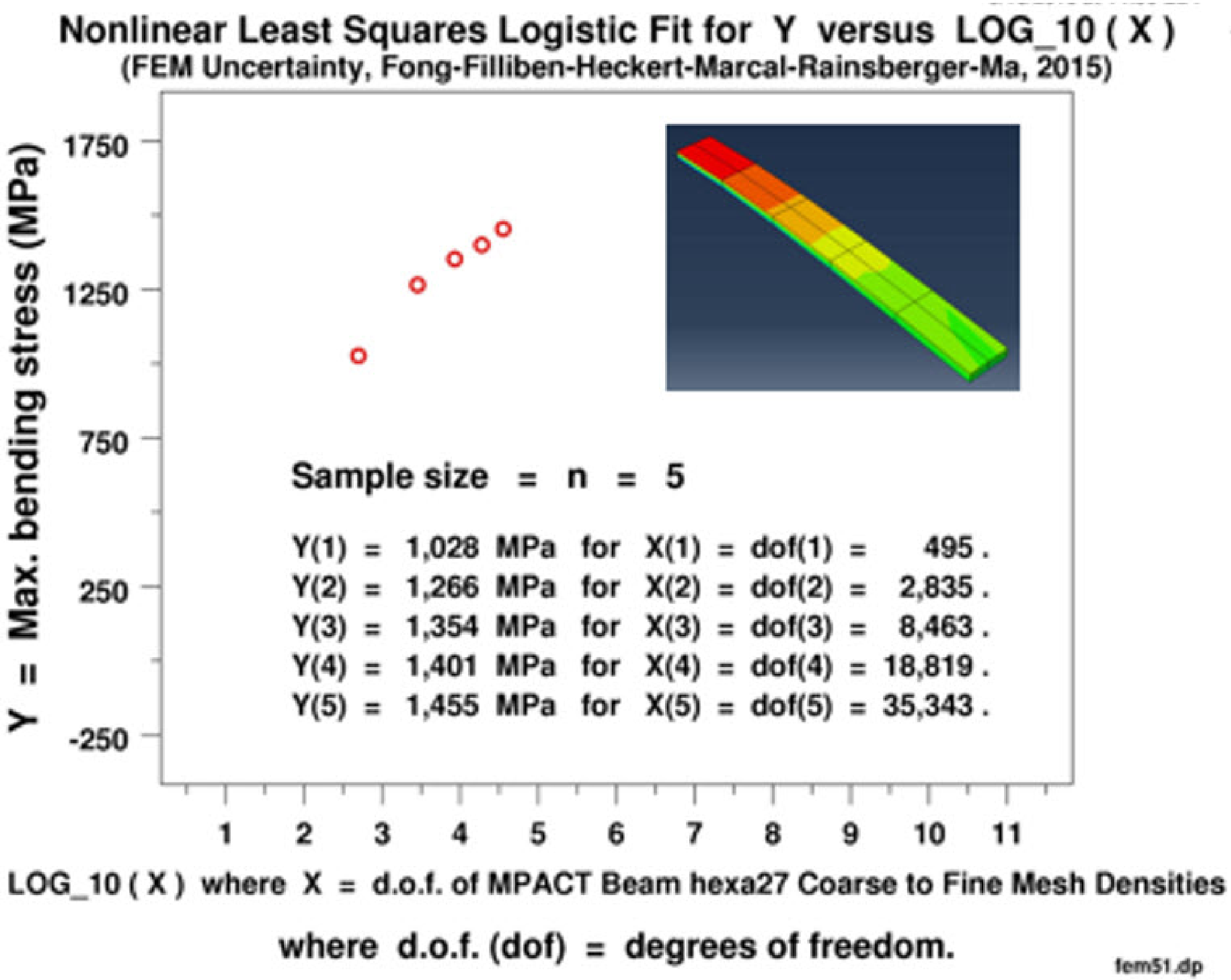


Hexahedron- 27 nodes
or, **Hexa-27**



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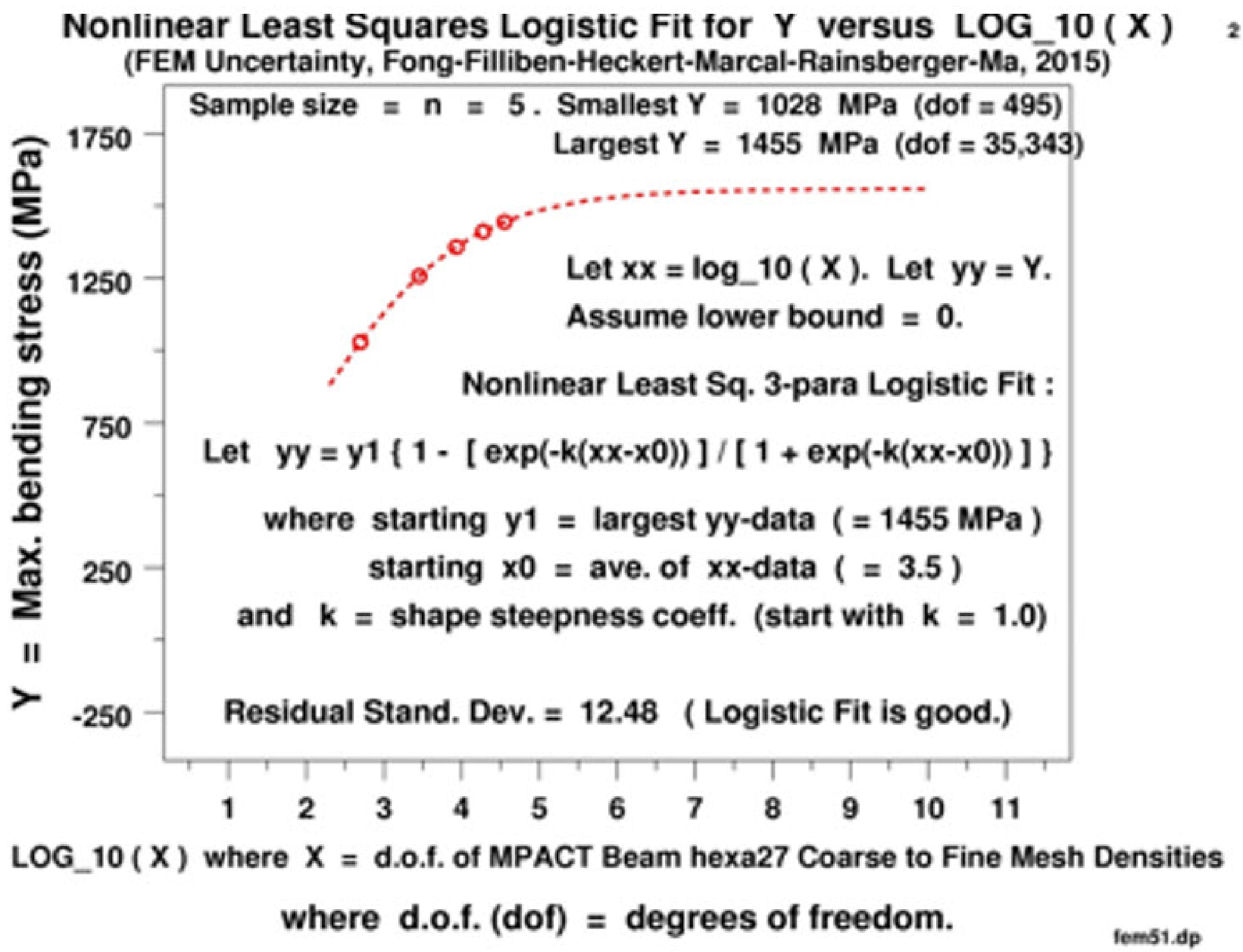

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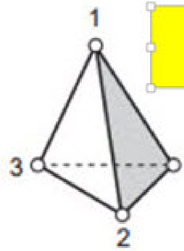
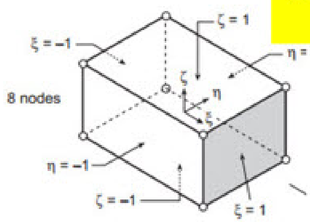
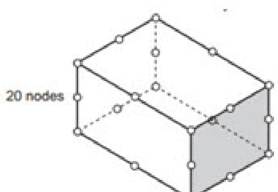


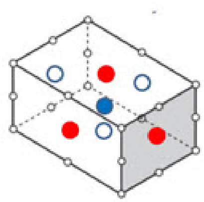
Tetrahedron – 4-node, or, **Tetra-4.**

Hexahedron – 8-node, or, **Hexa-8.**

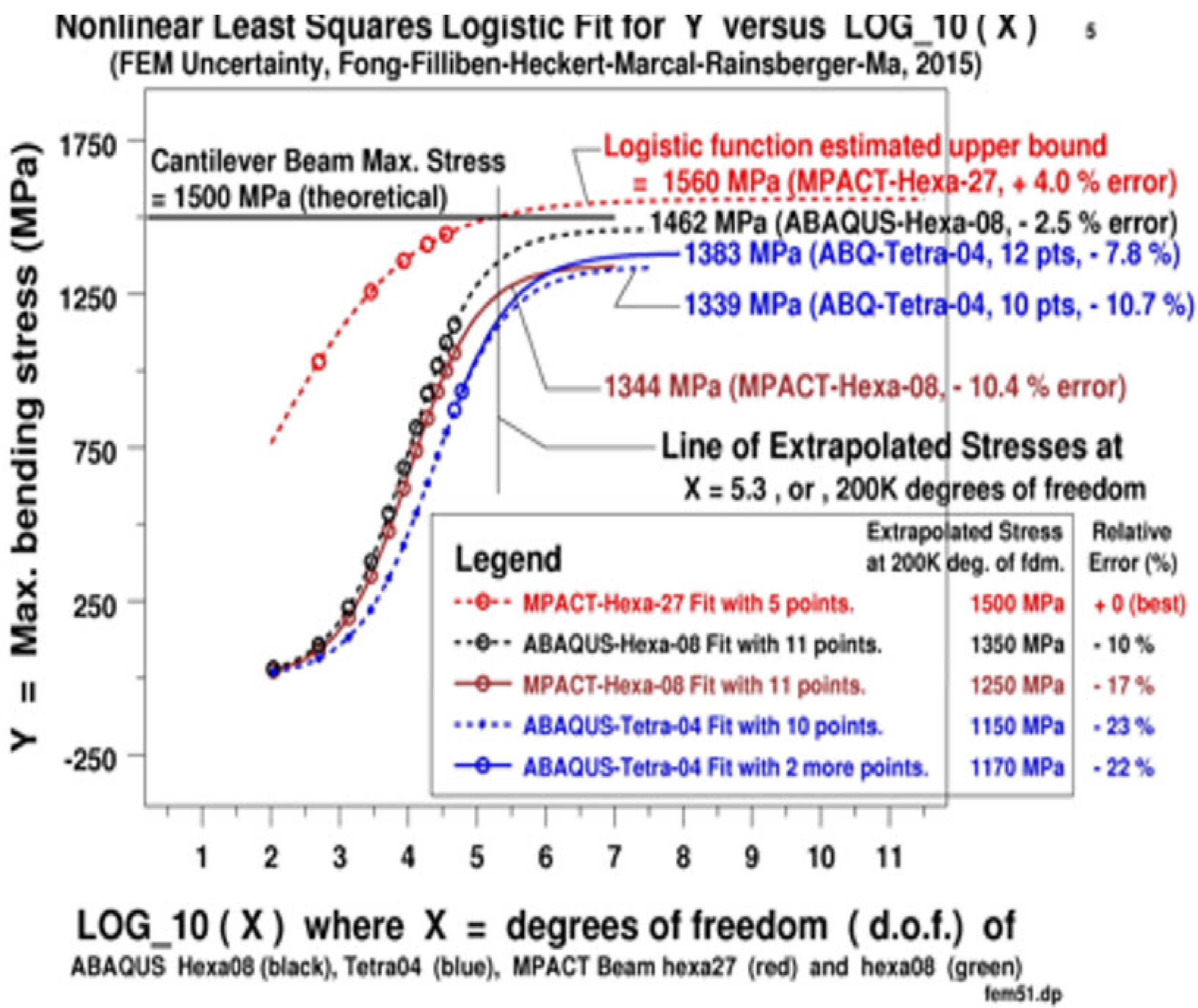




Hexahedron- 20 nodes or, **Hexa-20**



Hexahedron- 27 nodes or, **Hexa-27**



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(6) Concluding Remarks.

- In this talk, we show how one can quantify FEM uncertainties due to the following three sources:
- (1) Uncertainty due to Element Type (2015).
- (2) Uncertainty due to Mesh Density (2015).
- (3) Uncertainty due to **Model Parameters (2014).**

In 2016, we will show that a combination of the NL-LSQ fit method and the **design of experiments** approach can address uncertainty due to the 4th source, namely,

- (4) Uncertainty due to **Solution Platform (2016).**

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Disclaimer

Certain commercial equipment, instruments, materials, or computer software are identified in this talk in order to specify the experimental or computational procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards & Technology, nor is it intended to imply that the materials, equipment, or software identified are necessarily the best available for the purpose.

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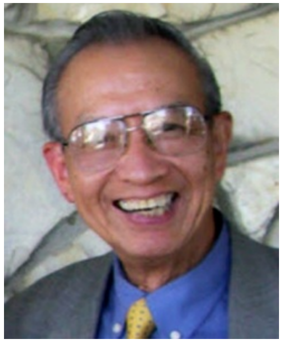
A Biographical Sketch of the Speaker



Dr. Jeffrey T. Fong has been Physicist and Project Manager at the Applied and Computational Mathematics Division, Information Technology Laboratory, National Institute of Standards and Technology (NIST), **Gaithersburg, MD, since 1966.**

He was educated at the University of Hong Kong (B.Sc., Engineering, first class honors, 1955), Columbia University (M.S., Engineering Mechanics, 1961), and Stanford (Ph.D., Applied Mechanics and Mathematics, 1966). Prior to 1966, he worked as a design engineer (1955-63) on numerous power plants (hydro, fossil-fuel, nuclear) at Ebasco Services, Inc., in New York City, and as teaching & research assistant (1963-66) on engineering mechanics at Stanford University.

During his 40+ years at NIST**, he has conducted research, provided consulting services, and taught numerous short courses on mathematical and computational modeling with uncertainty estimation for fatigue, fracture, high-temperature creep, nondestructive evaluation, electromagnetic behavior, and failure analysis of a broad range of materials ranging from paper, ceramics, glass, to polymers, composites, metals, semiconductors, and biological tissues.**

A licensed professional engineer (P.E.) in the State of New York since 1962 and a chartered civil engineer in the United Kingdom and British Commonwealth (A.M.I.C.E.) since 1968, he has authored or co-authored more than 100 technical papers, and edited or co-edited 17 national or international conference proceedings. He was elected Fellow of ASTM in 1982 and Fellow of ASME in 1984. In 1993, he was awarded the prestigious ASME *Pressure Vessels and Piping Medal.* Most recently, he was honored at the 2014 International Conference on Computational & Experimental Engineering & Sciences (ICCES) with a *Lifetime Achievement Medal.*

Since 2006, he has been Adjunct Professor of Mechanical Engineering and Mechanics at Drexel University **and taught a graduate-level 3-credit course on "Finite Element Method Uncertainty Analysis." Since Jan. 2010, he has given every 6 months an on-line 3-hour short course at** Stanford University **on "Reliability and Uncertainty Estimation of FEM Models of Composite Structures." In 2012, he was appointed Adjunct Professor of Nuclear and Risk Engineering at the** City University of Hong Kong**, and Distinguished Guest Professor at the** East China University of Science & Technology**, Shanghai, China, to teach annually a 1-credit 16-hour short course on "Engineering Reliability and Risk Analysis."**

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