Modeling of Magnetization and Intrinsic Properties of Ideal Type II Superconductor in External Magnetic Field

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Abstract

- **A COMSOL Multiphysics® model is developed for ideal type II superconductor with Ginzburg-Landau (GL) parameter κ~50 typical for practical high field, high current superconductors**
- **The model is validated by computing magnetization of such superconductor for every value of external magnetic field and comparing it to that of Brandt [2]**
- **Excellent agreement between these through the entire magnetic field range validates our model: thus flux lattice, interior of a single vortex, etc. are computed with confidence.**

Keywords: Type II superconductor, magnetic field, magnetization, time-dependent GL equations, order parameter, vector potential, flux lattice, vortex, pinning

1. Introduction

- Approach of Sørensen [1] to solve two dimensional time-dependent GL equations in COMSOL Multiphysics® is improved by introducing a hexagonal unit cell with periodic boundary conditions and size linked to the amplitude of magnetic field
- Evolution of a single vortex (nucleated from a small pinhole) is modeled and magnetization is computed for each value of external magnetic field using the virial theorem

2. Equations

The time evolution of the order parameter ψ in the presence of a magnetic field $\boldsymbol{B}=\nabla\times\mathbf{A}$ is given by the time-dependent Ginzburg-Landau equations:

$$\frac{\hbar^2}{2mD}\left(\frac{\partial}{\partial t}+i\frac{q}{\hbar}\phi\right)\psi = -\frac{1}{2m}\left(\frac{\hbar}{i}\nabla-q\mathbf{A}\right)^2\psi+\alpha\psi-\beta|\psi|^2\psi \qquad \text{(1a)}$$

$$\sigma\left(\frac{\partial\mathbf{A}}{\partial t}+\nabla\phi\right) = \frac{q\hbar}{2mi}(\psi^*\nabla\psi-\psi\nabla\psi^*)-\frac{q^2}{m}|\psi|^2\mathbf{A}-\frac{1}{\mu_0}\nabla\times\nabla\times\mathbf{A} \qquad \text{(1b)}$$

The magnetization of a material is: $\mathbf{M}=(\langle\boldsymbol{B}\rangle-\boldsymbol{B}_a)/\mu_0$. Due to the introduction of vortices at finite $|\boldsymbol{B}_a|>B_{c1}$ the magnetization curve $\boldsymbol{M}(\boldsymbol{B}_a)$ of a type-II has a non-trivial $\boldsymbol{B}_a$-dependence. A logarithmic function proposed in [2]) can be used as fit to the curve at high values of κ:

$$-m=\frac{1}{4\kappa^2}ln\left(1+\frac{1-b}{b}f_2(b)\right)$$

$$f_2(b)=0.357+2.890b-1.581b^2 \qquad \text{(2)}$$

$$m=\mu_0|\boldsymbol{M}|/B_{c2} \qquad b=|\boldsymbol{B}_a|/B_{c2}$$

A periodic geometry has no outer boundary, and an external magnetic field can therefore not be specified. The virial theorem can be used to compute the external magnetic field $\mathbf{B}_a$:

$$|\boldsymbol{B}_a|/\mu_0=\langle|\psi|^2-|\psi|^4+2|\boldsymbol{B}|^2\rangle/\langle2|\boldsymbol{B}|\rangle \qquad \text{(3)}$$

3. Method

Considering that $\kappa=\lambda/\xi=50$, where λ is the London penetration depth and ξ is the coherence length, the resolution of the mesh needs to be fine for Ψ (thus ξ) and the total domain must be large because of λ.

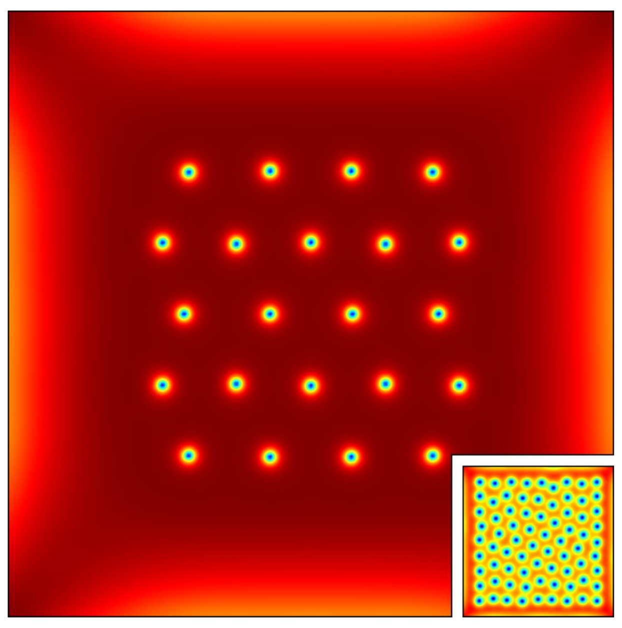

Fig. 1 Vortex distribution at κ=2 (small inset) and κ=50 (large section). At κ=50, the edge effects are dominant on the whole modeling domain.

Illustration of the scaling problem at high k values. The main figure shows the distribution of |Ψ| for κ = 2, the inset the distribution of |Ψ| for κ = 50. It is seen that the ratio between the radius of a single vortex and the distance between vortices decreases rapidly with increased κ. As a consequence, the number of DOFs required to simulate a given number of vortices increases dramatically with increased κ.

To overcome this complication, we implement the concept of a triangular Abrikosov lattice by modeling a single hexagonal unit cell (figure 2) i.e. a domain with periodic boundary conditions on all boundaries To let vortices in a superconductor, we use a small pinhole at the center.

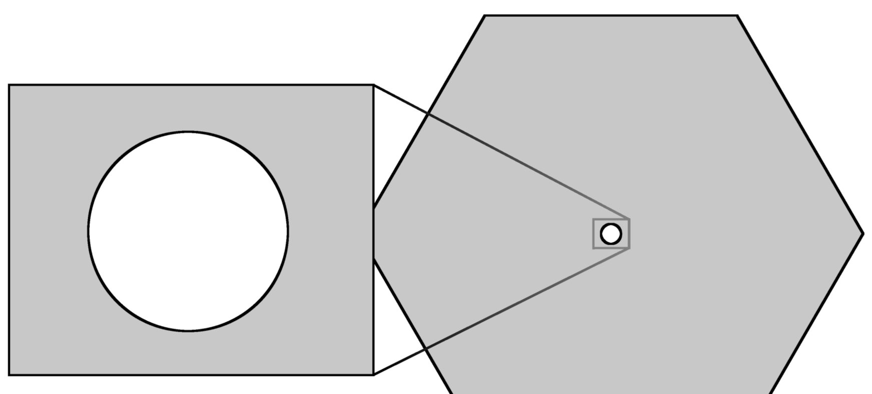

Fig. 2 Hexagonal geometry with a pinhole in the center (left) where pinhole diameter r<< ξ (right). The pinhole interior is not active in the simulation.

Validation of this approach will be done by comparing modeling results with analytical derivation of the vortex field geometry for a single isolated vortex. To validate, we use different hexagon radii 20λ < r <200λ.

Brandt's results are reproduced in our model by changing the unit cell size ***r*** so that it contains exactly one flux quantum at all times.

4. Results

Order parameter of isolated vortex

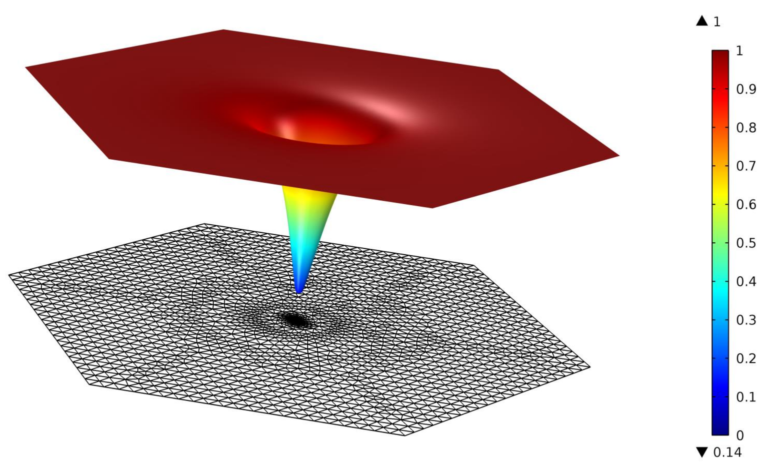

Fig. 3 Ψ-field for k=50 at external field B_a~B_{c1}

Distribution of the order parameter |Ψ| in the unit cell. The initial size of the unit cell is large enough so that vortex-vortex interactions can be neglected. In the center of the vortex, |Ψ|->0 as expected. This is due to the fact that the induced current around the vortex center equals to the critical current of the superconductor. Far away from the vortex center |Ψ|->1 and the uniform superconducting state is restored, as long as the unit cell is large enough that vortex core interaction can be neglected.

Magnetic field and current of a vortex

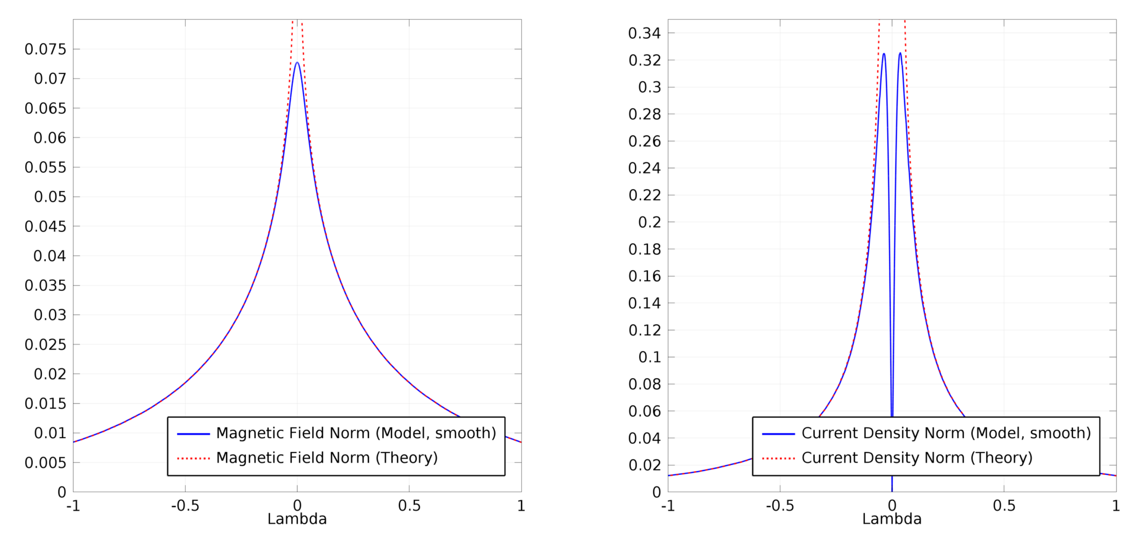

Fig. 4: Comparison of analytic (dashed line) and numeric (solid line) values of the magnetic field B and the induced current J along a line through a single hexagonal unit cell. The analytic values are the results for a single, isolated vortex, and are only valid for distances r > ξ. The numeric results were obtained using the periodic approach discussed above. The unit cell was chosen large enough that the vortex is effectively isolated. It is seen that the results show excellent agreement in the area where the analytic results are valid.

Order parameter of vortex in a lattice

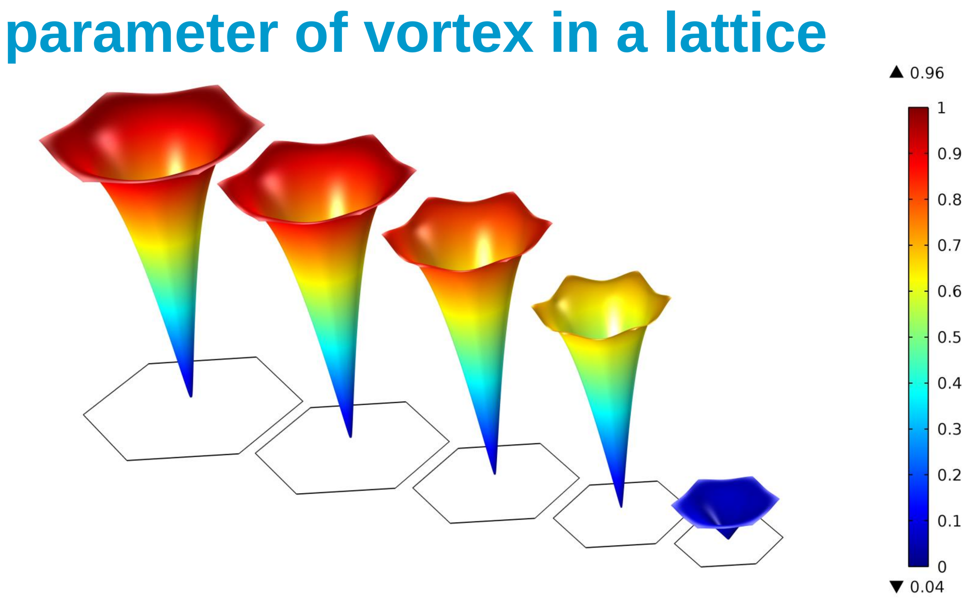

Fig. 5: Distribution of the order parameter |Ψ| for different sizes of the hexagonal unit cell. The size of the unit cell decreases from left to right. It is seen that, as the unit cell shrinks, the order parameter no longer extends up to 1 near the edge of the unit cell. This indicates that vortex-vortex interactions increase with decreasing unit-cell size, which eventually leads to the destruction of the superconducting state at the second critical field Bc2.

Magnetization of ideal type II superconductor

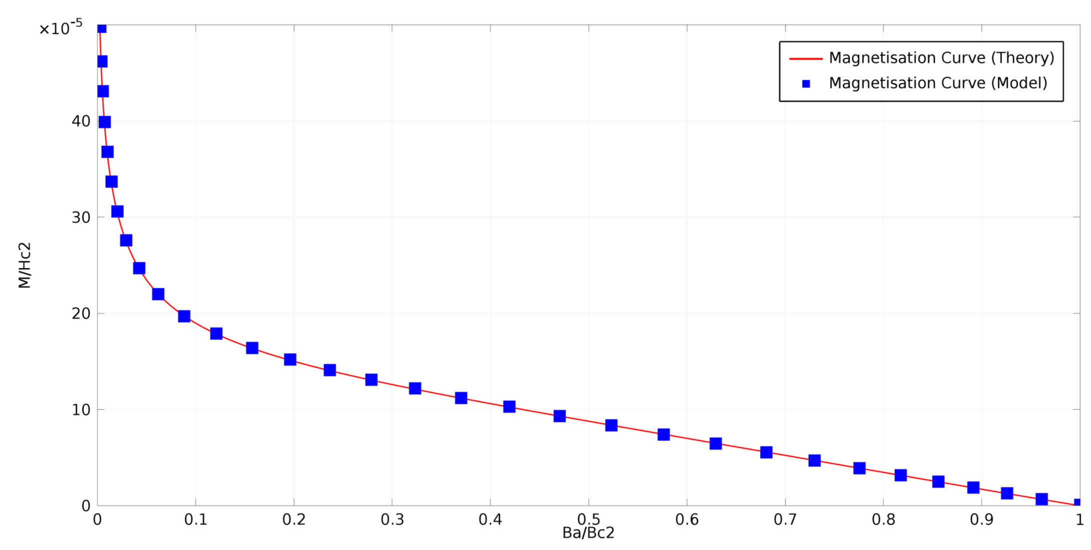

Fig. 6: Magnetization of ideal type II superconductor: the computed values (points) agree with the theory (solid line, Eq. 2). This result validates our model.

5. Conclusions

1) **Excellent agreement of the model with theory**
2) **Automated process**
3) **Ready to investigate effect of pinning**

References

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