A *Design-of-Experiments (DOE)* *approach* to FEM Uncertainty Analysis *for optimizing* *Magnetic Resonance Imaging RF Coil Design*

COMSOL CONFERENCE 2014 BOSTON

Jeffrey T. Fong, *Ph.D., P.E.*
Physicist and Project Manager
Applied & Computational Mathematics Division
National Institute of Standards & Technology
Gaithersburg, MD 20899-8910, U.S.A.
http://www.nist.gov/itl/math/Jeffrey-t-fong.cfm
(301) 975-8217 fong@nist.gov

Outline of Talk

(16 slides)

- *(4)* What is a "Design-of-Experiments" approach ?
- *(4)* A Design Optimization Problem using FEM (via COMSOL-RF).
- *(6)* Solution before optimization.
- *(1)* Solution after optimization.
- *(1)* Future work.

Jeffrey T. Fong, NIST, Gaithersburg, MD
20899 USA. Tel: 1-301-975-8217
Email: fong@nist.gov

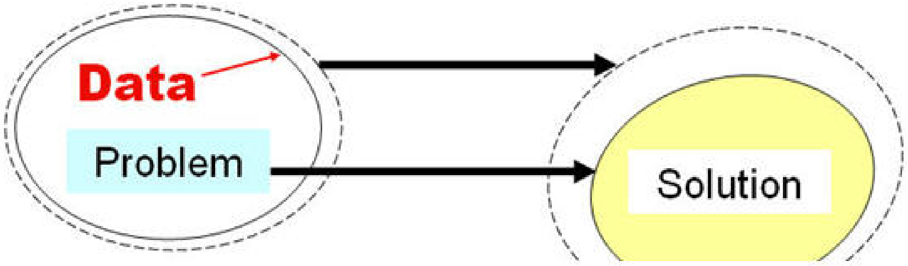
Metrological Approach - "Wiggle each FEM solution"

One "wiggles" an **FEM** solution by varying the input parameters and material property data in a *statistically* rigorous way, i.e., *design of experiments*, to calculate uncertainty within an affordable time frame and computing budget.

Such "wiggle" is the 3rd requirement of a "correct" problem:

To be precise, we define a boundary or initial value problem for a partial differential equation, or for a system of partial differential equations, to be *correctly set in the sense of Hadamard* if and only if its solution exists, is unique, and depends continuously on the data assigned.

Ref.: Garabedian, P. R., 1964, Partial Diff. Equations, page 109. Providence, RI: AMS Chelsea Publishing (1964).



Jeffrey T. Fong, NIST, Gaithersburg, MD
20899 USA. Tel: 1-301-975-8217
Email: fong@nist.gov

Design of Experiments (Full vs. Fractional)

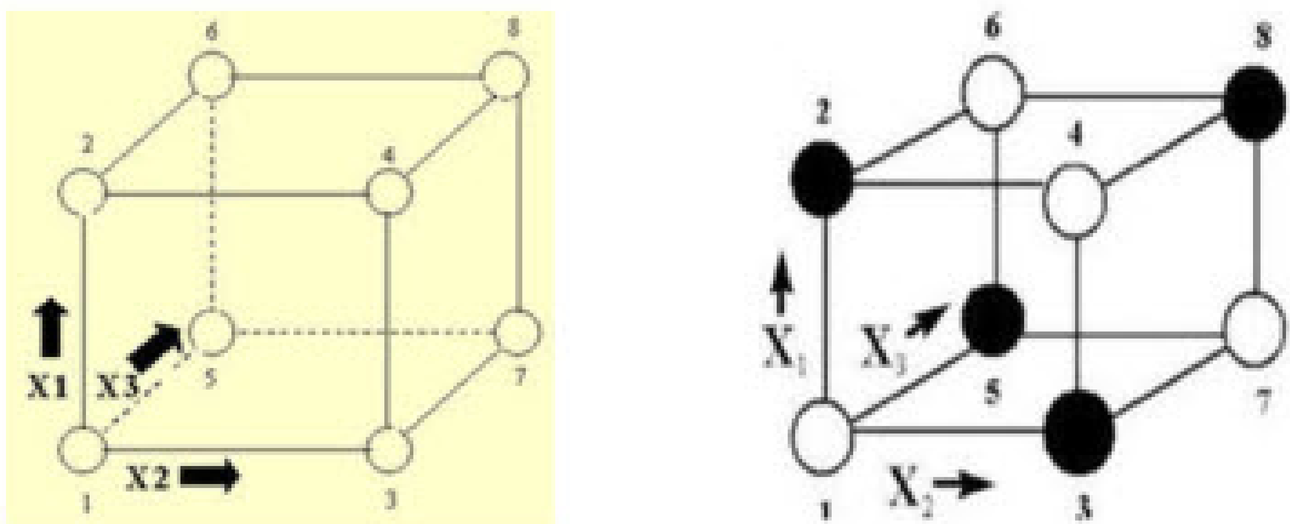


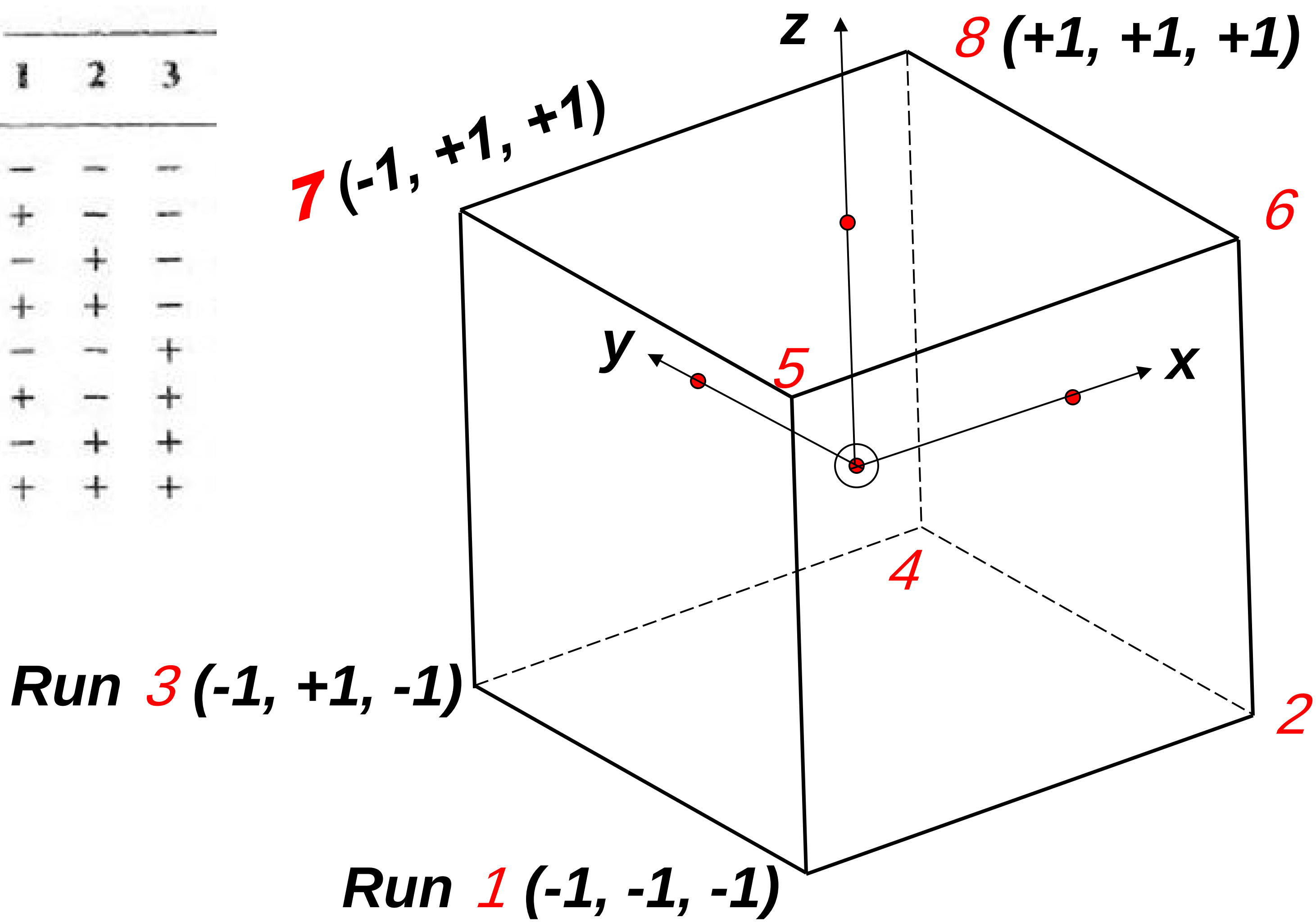
Figure 1. (left) **A full-factorial 8-run orthogonal design for 3 factors.**
(right) **A fractional factorial 4-run orthogonal design for 3 factors.**

Jeffrey T. Fong, NIST, Gaithersburg, MD
20899 USA. Tel: 1-301-975-8217
Email: fong@nist.gov

What is "Design of Experiments" ?

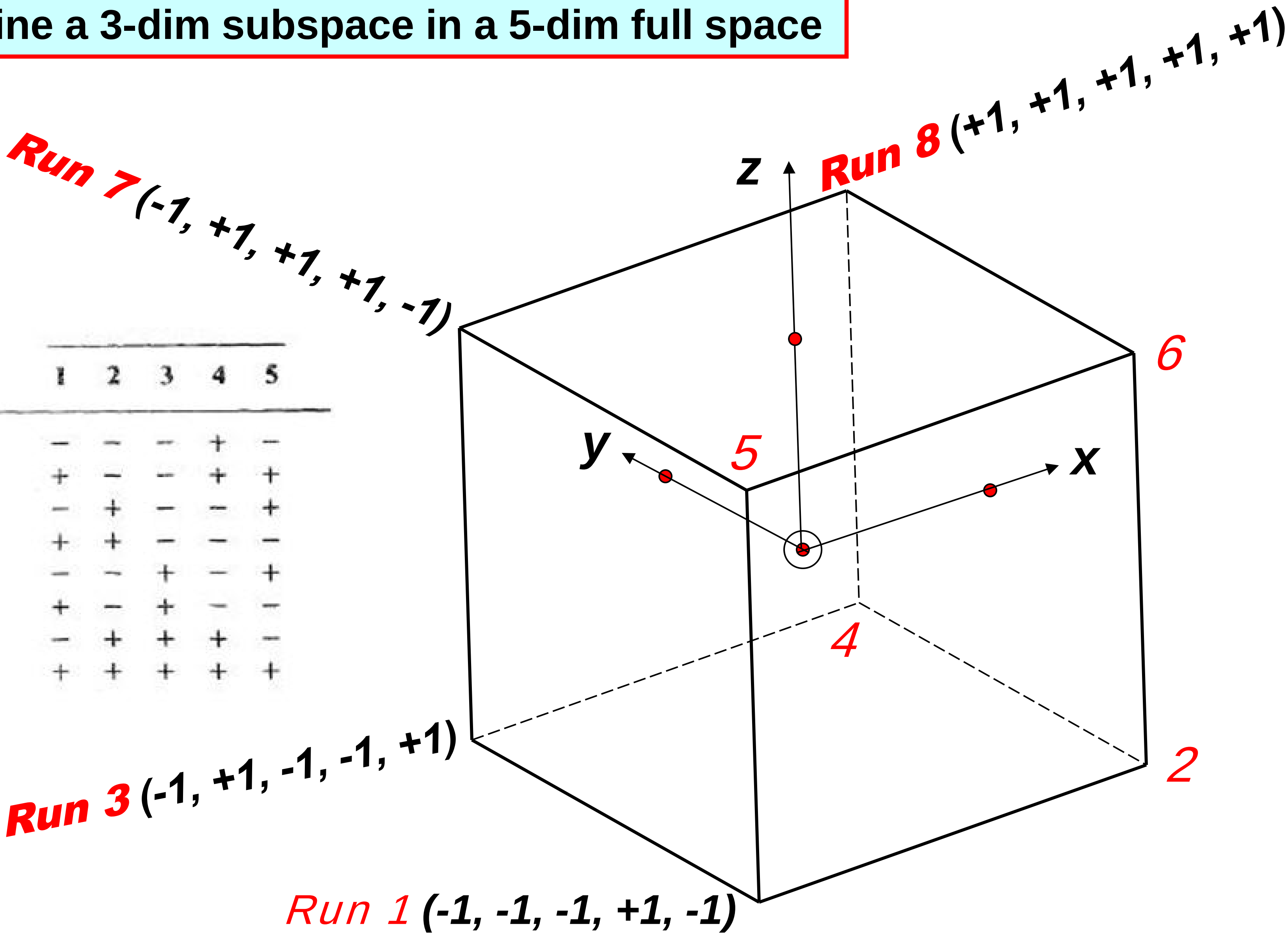
Metrological Approach - "Wiggle each FEM solution"

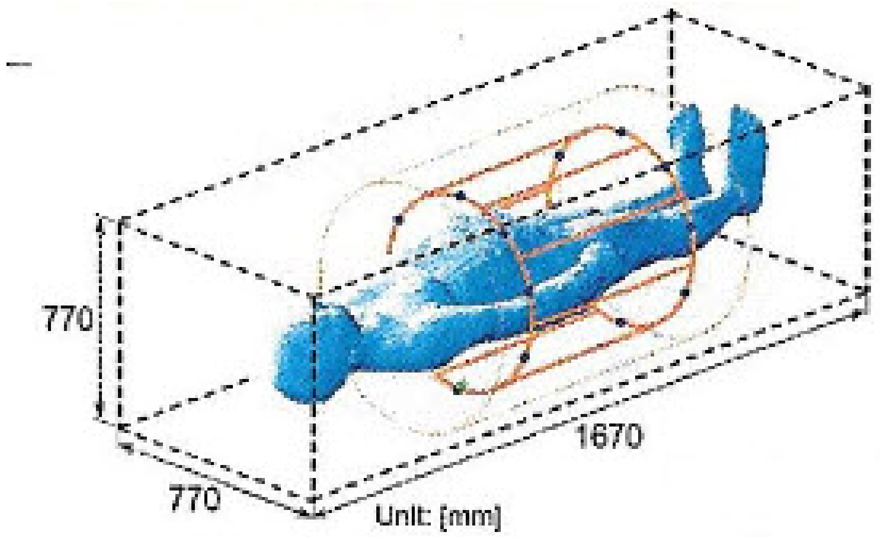
run	1	2	3
1	−	−	−
2	+	−	−
3	−	+	−
4	+	+	−
5	−	−	+
6	+	−	+
7	−	+	+
8	+	+	+

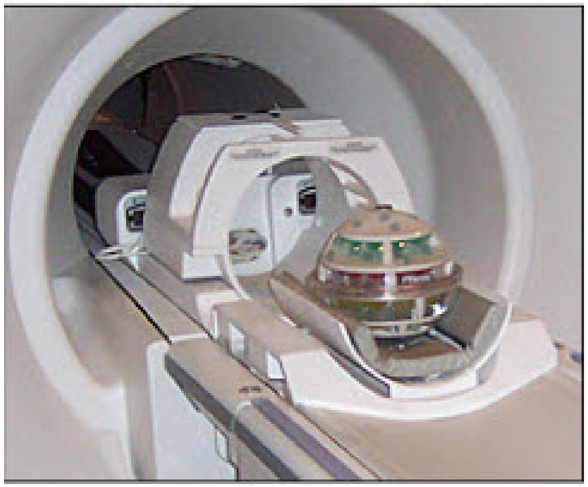


Imagine a 3-dim subspace in a 5-dim full space

run	1	2	3	4	5
1	−	−	−	+	−
2	+	−	−	+	+
3	−	+	−	−	+
4	+	+	−	−	−
5	−	−	+	−	+
6	+	−	+	−	−
7	−	+	+	+	−
8	+	+	+	+	+



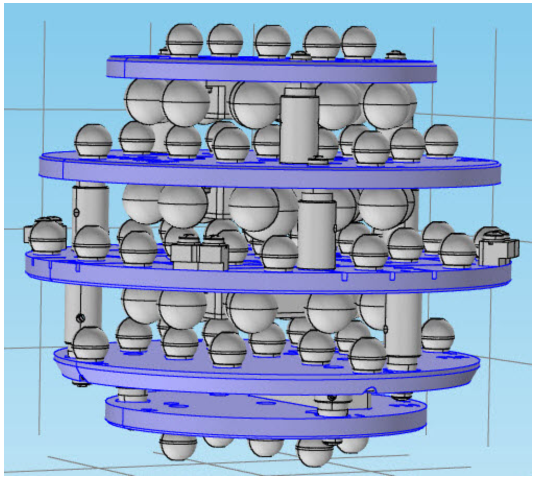


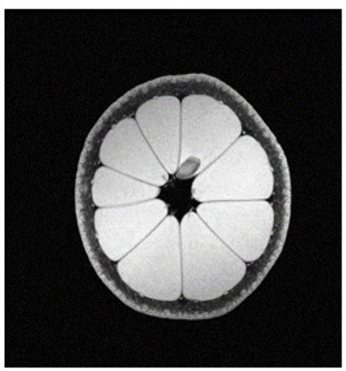


Motivated by a FEM project

Jeffrey T. Fong, NIST, Gaithersburg, MD 20899 USA. Tel: 1-301-975-8217
Email: fong@nist.gov

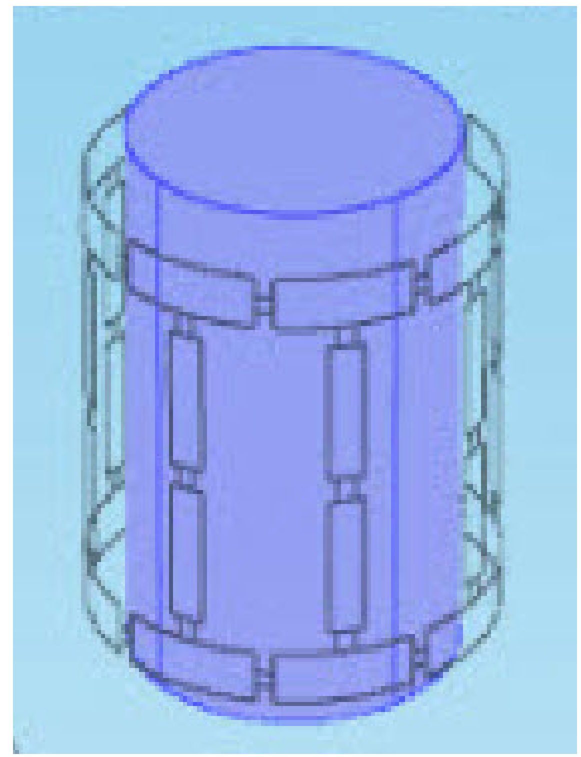
A Model Simplification Approach





MRI image of a lemon
(Courtesy of Dr. Steve Russek NIST, Boulder CO 80305)

MRI image of a banana
(Courtesy of Dr. Steve Russek NIST, Boulder CO 80305)



Jeffrey T. Fong, NIST, Gaithersburg, MD
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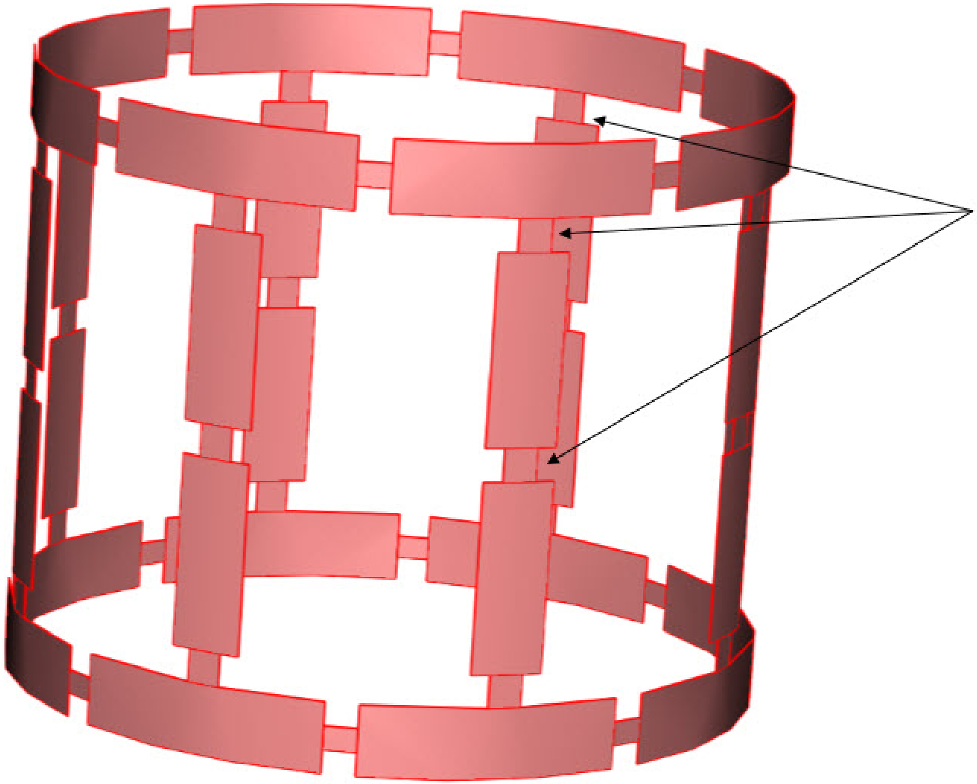
- MRI



Each flap represents a simple circuit or port element such as a 50 Ohm feed or a tuning capacitor.

(Standard finite element modeling approach for RF and Microwave components)

$$\nabla \times \mu_r^{-1}(\nabla \times \mathbf{E}) - k_0^2\left(\epsilon_r - \frac{j\sigma}{\omega\epsilon_0}\right)\mathbf{E} = 0$$

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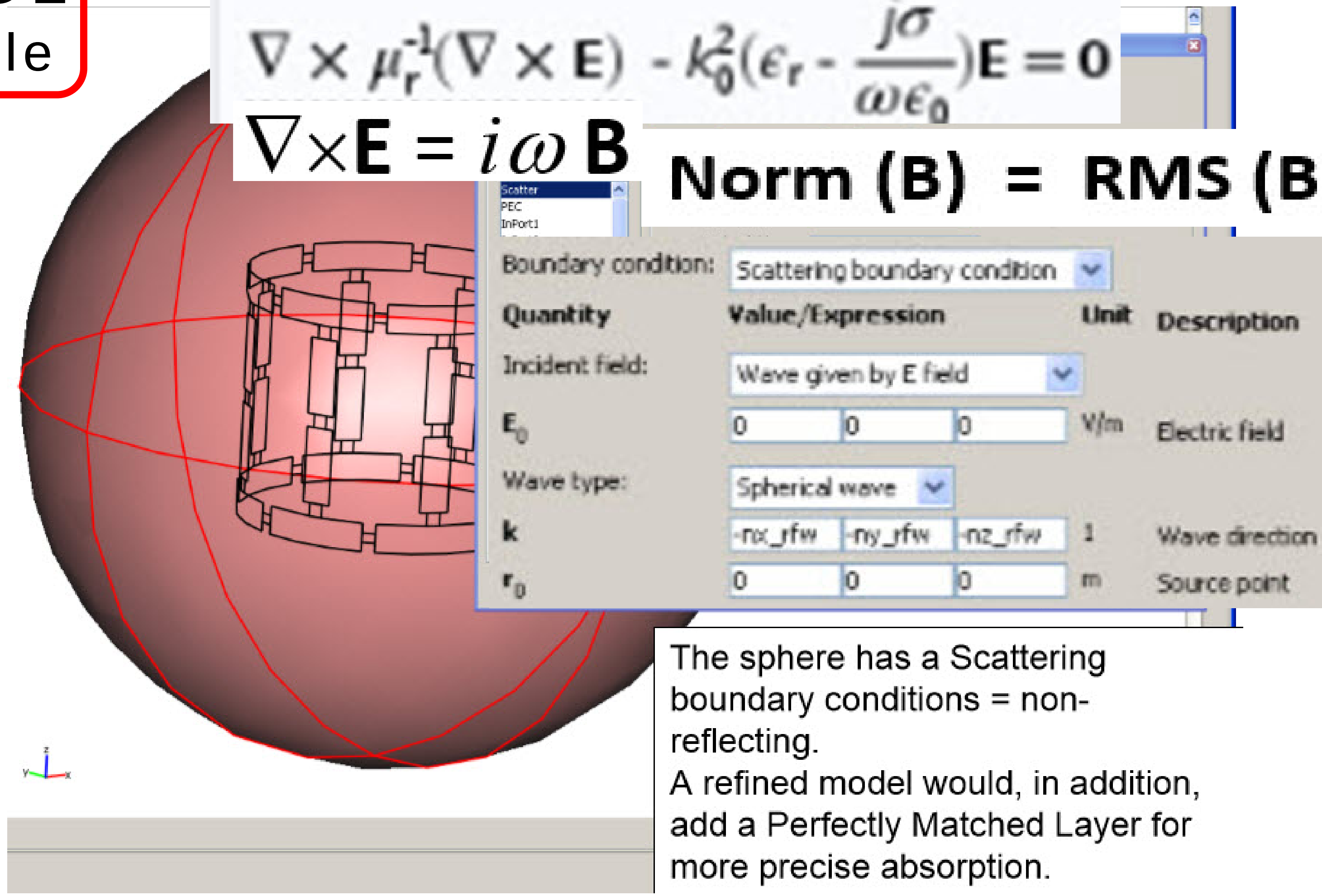
COMSOL
RF module

$$\nabla \times \mu_r^{-1}(\nabla \times \mathbf{E}) - k_0^2\left(\epsilon_r - \frac{j\sigma}{\omega\epsilon_0}\right)\mathbf{E} = 0$$

$$\nabla \times \mathbf{E} = i\omega \mathbf{B}$$

Norm (B) = RMS (B).



The sphere has a Scattering boundary conditions = non-reflecting.
A refined model would, in addition, add a Perfectly Matched Layer for more precise absorption.

Jeffrey T. Fong, NIST, Gaithersburg, MD
20899 USA. Tel: 1-301-975-8217
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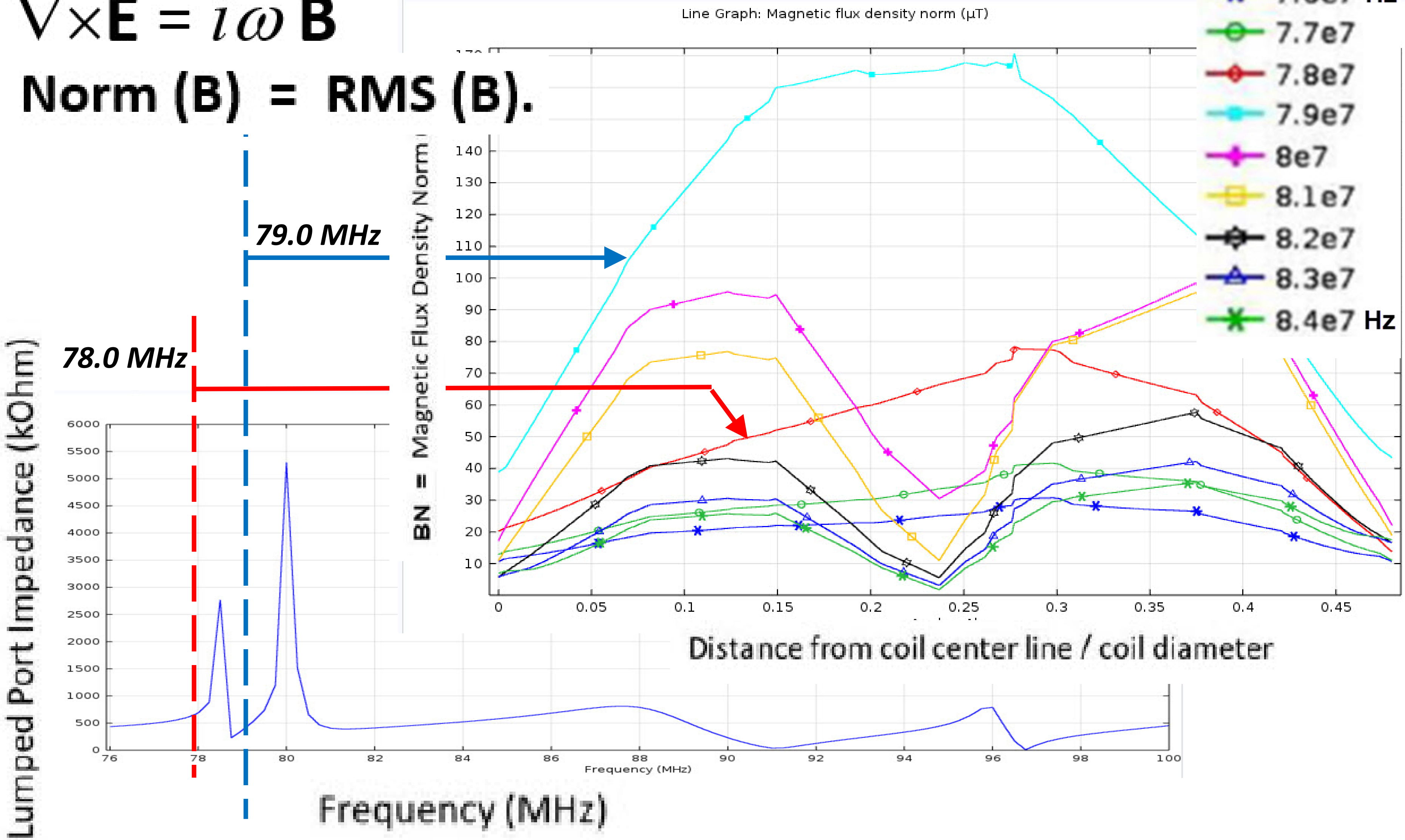
$$\nabla \times \mu_r^{-1}(\nabla \times \mathbf{E}) - k_0^2\left(\epsilon_r - \frac{j\sigma}{\omega\epsilon_0}\right)\mathbf{E} = 0$$

$$\nabla \times \mathbf{E} = i\omega\mathbf{B}$$

Norm (B) = RMS (B).



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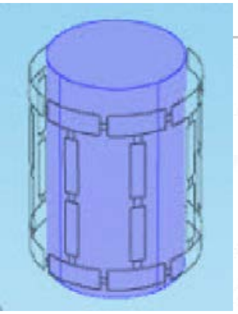
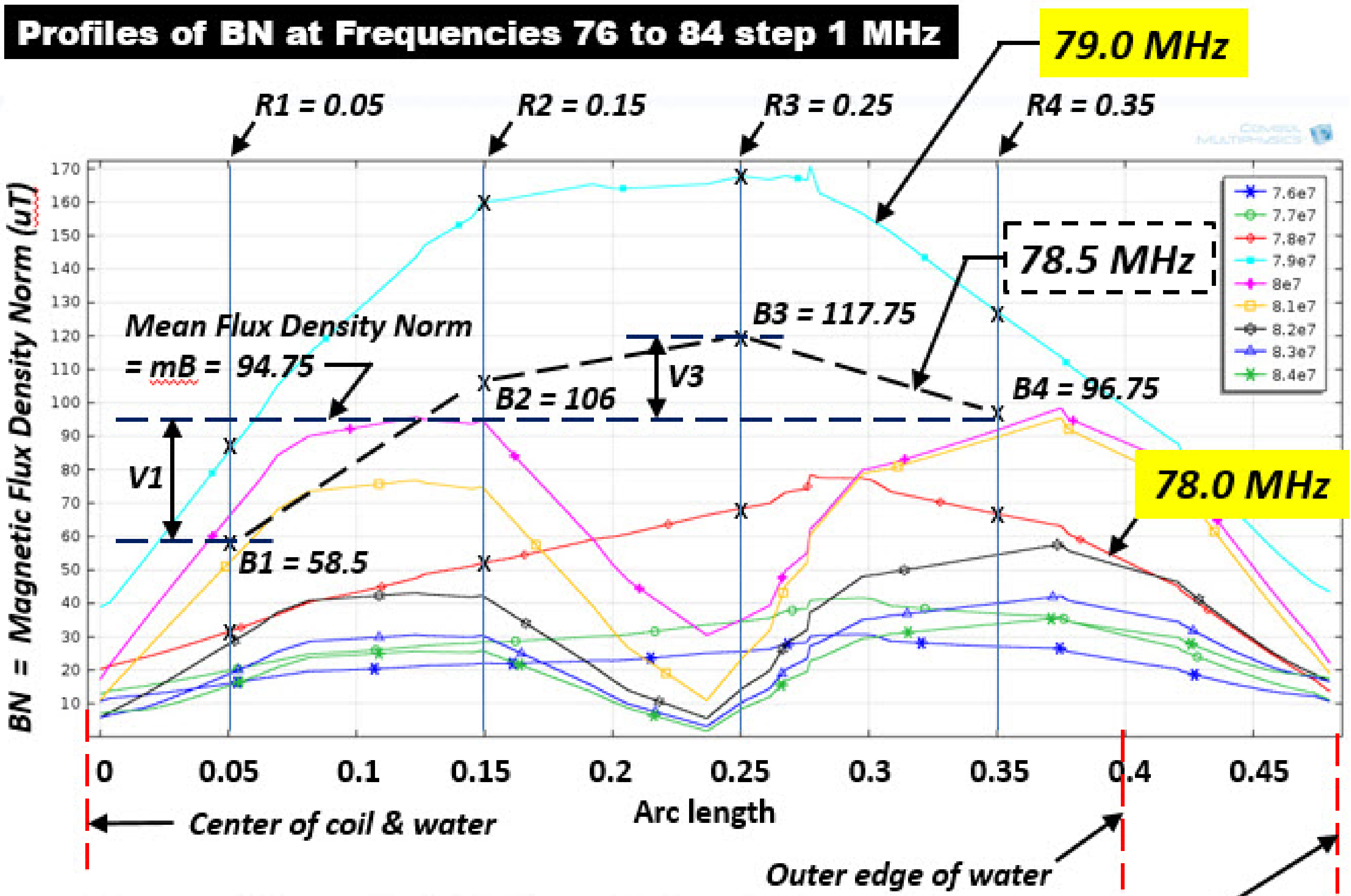
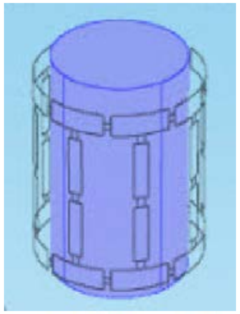




Distance from coil center line / coil diameter

Jeffrey T. Fong, NIST, Gaithersburg, MD
20899 USA. Tel: 1-301-975-8217
Email: fong@nist.gov



X1	X2	X3	X4	X5	X6	X7
-1	-1	-1	+1	+1	+1	-1
+1	-1	-1	-1	-1	+1	+1
-1	+1	-1	-1	+1	-1	+1
+1	+1	-1	+1	-1	-1	-1
-1	-1	+1	+1	-1	-1	+1
+1	-1	+1	-1	+1	-1	-1
-1	+1	+1	-1	-1	+1	-1
+1	+1	+1	+1	+1	+1	+1
+1	+1	+1	-1	-1	-1	+1
-1	+1	+1	+1	+1	-1	-1
+1	-1	+1	+1	-1	+1	-1
-1	-1	+1	-1	+1	+1	+1
+1	+1	-1	-1	+1	+1	-1
-1	+1	-1	+1	-1	+1	+1
+1	-1	-1	+1	+1	-1	+1
-1	-1	-1	-1	-1	-1	-1

	X1	X2	X3	X4	X5	X6	X7
	Sigma	Epsilonr	C	V0	w1	L3	bet2
Base Run (00)	0.0001	80	177	500	80	35	5
Unit	*S/m*	*1*	*pF*	*volt*	*mm*	*mm*	*degree*
+ / - variation	10 %	5 %	2 %	2 %	5 %	10 %	10 %
Run No. (01)	0.00009	76	173.46	510	84	38.5	4.5
Run No. (02)	0.00011	76	173.46	490	76	38.5	5.5
Run No. (03)	0.00009	84	173.46	490	84	31.5	5.5
Run No. (04)	0.00011	84	173.46	510	76	31.5	4.5
Run No. (05)	0.00009	76	180.54	510	76	31.5	5.5
Run No. (06)	0.00011	76	180.54	490	84	31.5	4.5
Run No. (07)	0.00009	84	180.54	490	76	38.5	4.5
Run No. (08)	0.00011	84	180.54	510	84	38.5	5.5
Run No. (09)	0.00011	84	180.54	490	76	31.5	5.5
Run No. (10)	0.00009	84	180.54	510	84	31.5	4.5
Run No. (11)	0.00011	76	180.54	510	76	38.5	4.5
Run No. (12)	0.00009	76	180.54	490	84	38.5	5.5
Run No. (13)	0.00011	84	173.46	490	84	38.5	4.5
Run No. (14)	0.00009	84	173.46	510	76	38.5	5.5
Run No. (15)	0.00011	76	173.46	510	84	31.5	5.5
Run No. (16)	0.00009	76	173.46	490	76	31.5	4.5

Jeffrey T. Fong, NIST, Gaithersburg, MD
20899 USA. Tel: 1-301-975-8217
Email: fong@nist.gov

760file1.txt - Notepad

File Edit Format View Help

```
 Filename:  760file1.txt   Date:  June 25,
 Num of Factors
7
factors
  X1  X2  X3  X4  X5  X6  X7
 key to factors
  X    factor name     symbol (on a newline
   Elec. Conductivity of Water
   sigma
   Relative Permitivity of Water
   epsilon
   Capacitance
   C
   Voltage
   V0
   Ring Width
   w1
   Strip Gap Length
   L3
   Ring Gap Angle
   bet2
 center Point values
0.0001  80  177  500   80   35   5
 variability (%)
10.0  5.0  2.0  2.0  5.0  10.0  10.0
```

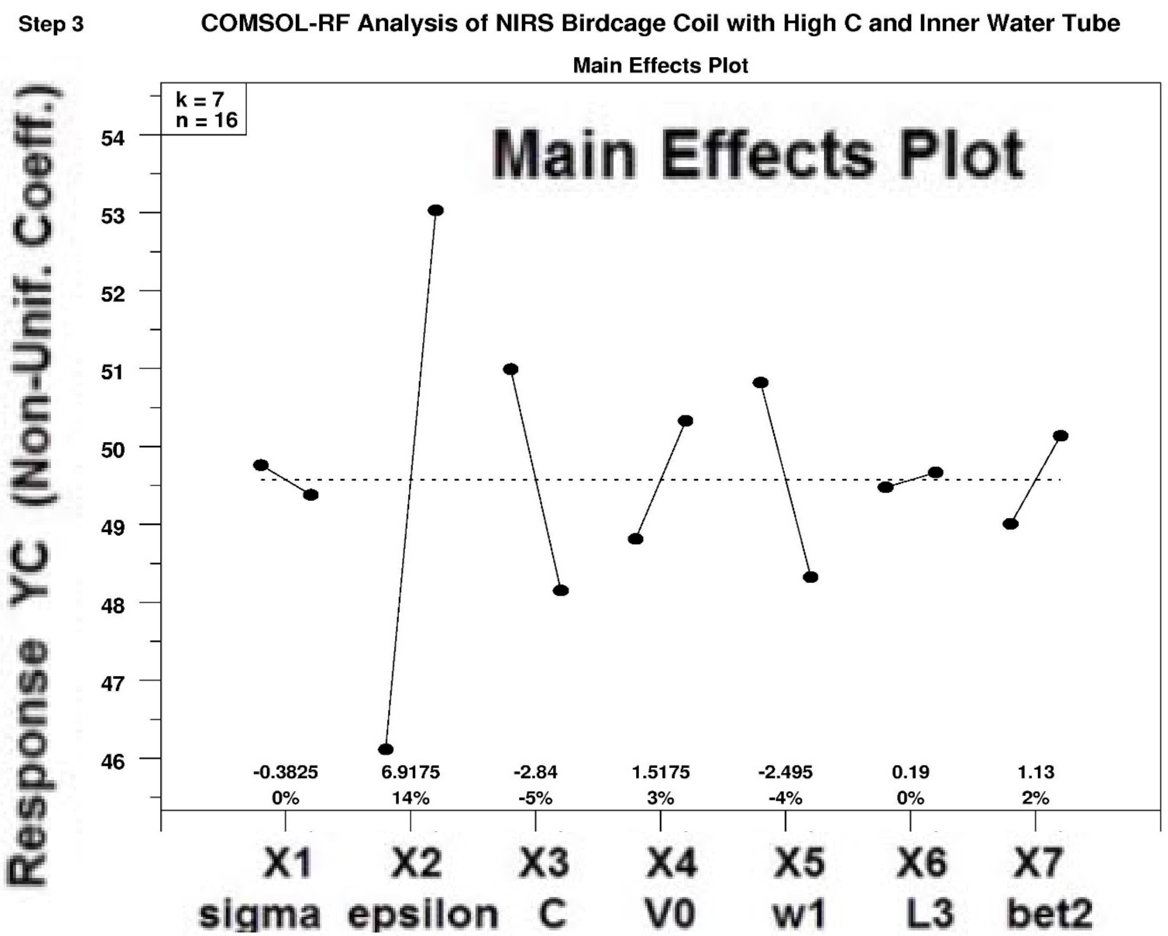
760file2.txt - Notepad

File Edit Format View Help

```
 Filename:  760file2.txt   Date:  Aug. 10, 2014
 Num of Factors
7
num of runs
  8  16
 num of runs chosen for DOE
16
 runs number
 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
 key to results
  YC     result name     symbol (next line)
   Non-Uniformity Coeff.
 YC
 results for runs
46.89
42.92  49.28  52.17  57.99  43.01  44.73  50.58  54.25
48.95  51.86  45.52  46.34  47.85  60.62  46.50  50.63
```


Jeffrey T. Fong, NIST, Gaithersburg, MD
20899 USA. Tel: 1-301-975-8217
Email: fong@nist.gov

DATAPLOT



Jeffrey T. Fong, NIST, Gaithersburg, MD
20899 USA. Tel: 1-301-975-8217
Email: fong@nist.gov

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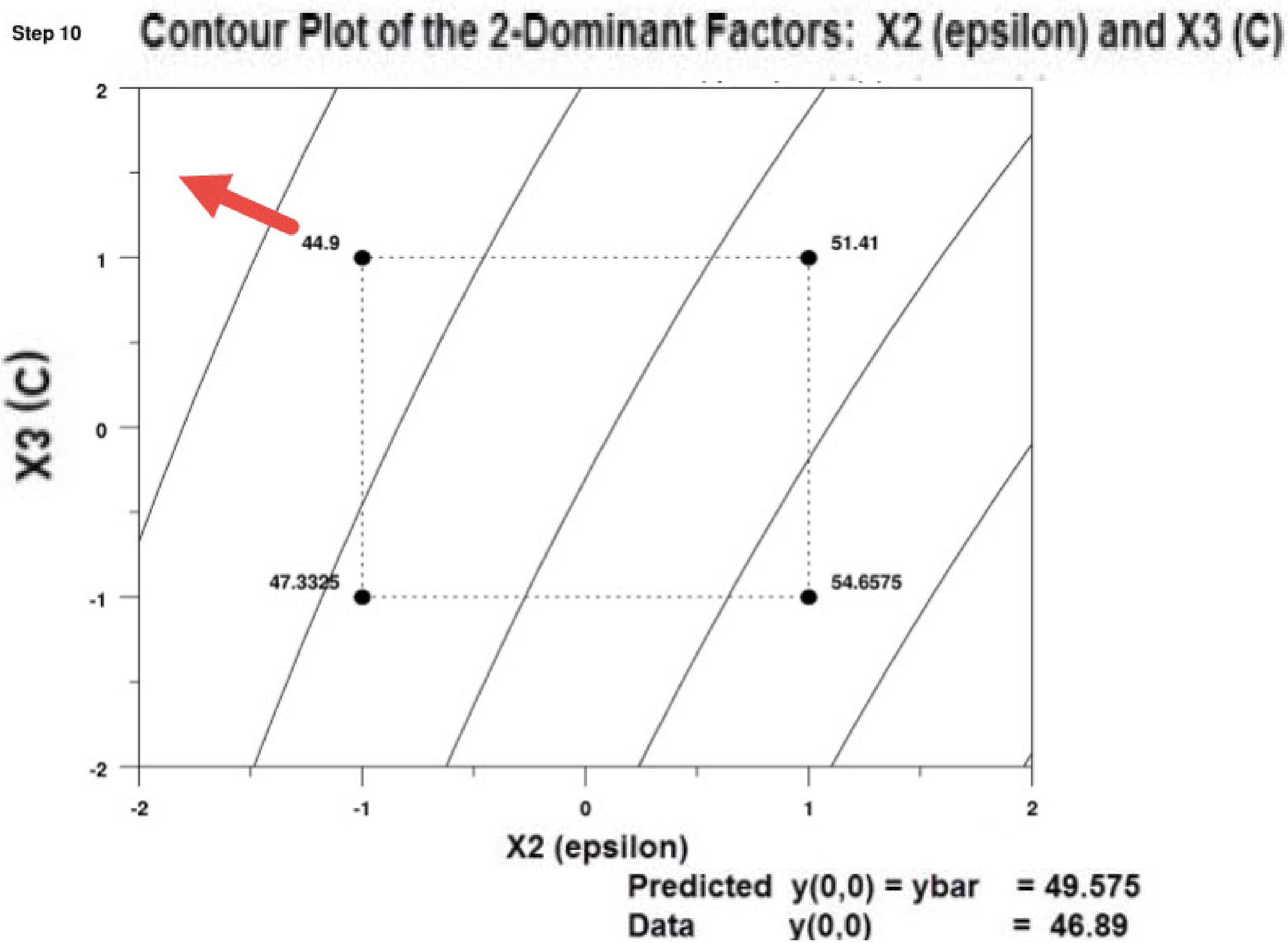
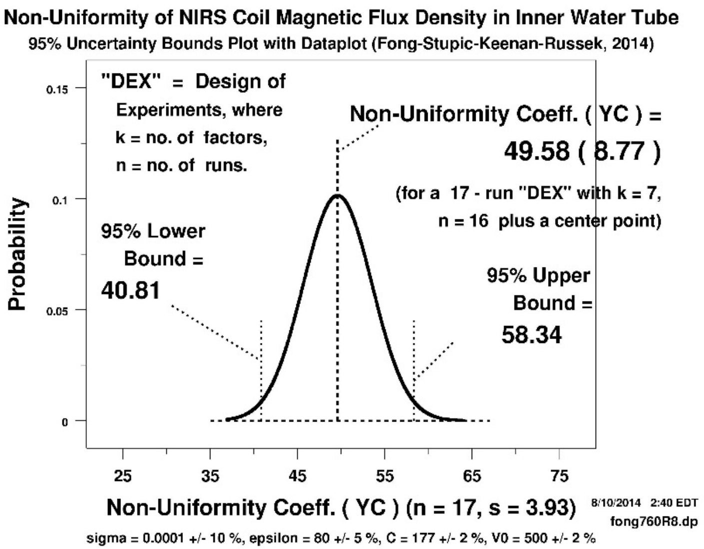
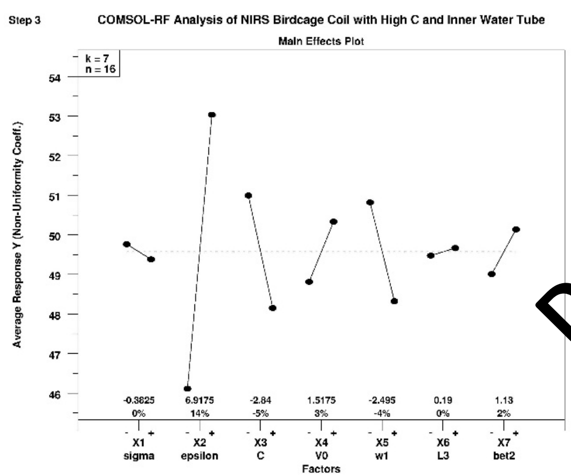


Jeffrey T. Fong, NIST, Gaithersburg, MD
20899 USA. Tel: 1-301-975-8217
Email: fong@nist.gov





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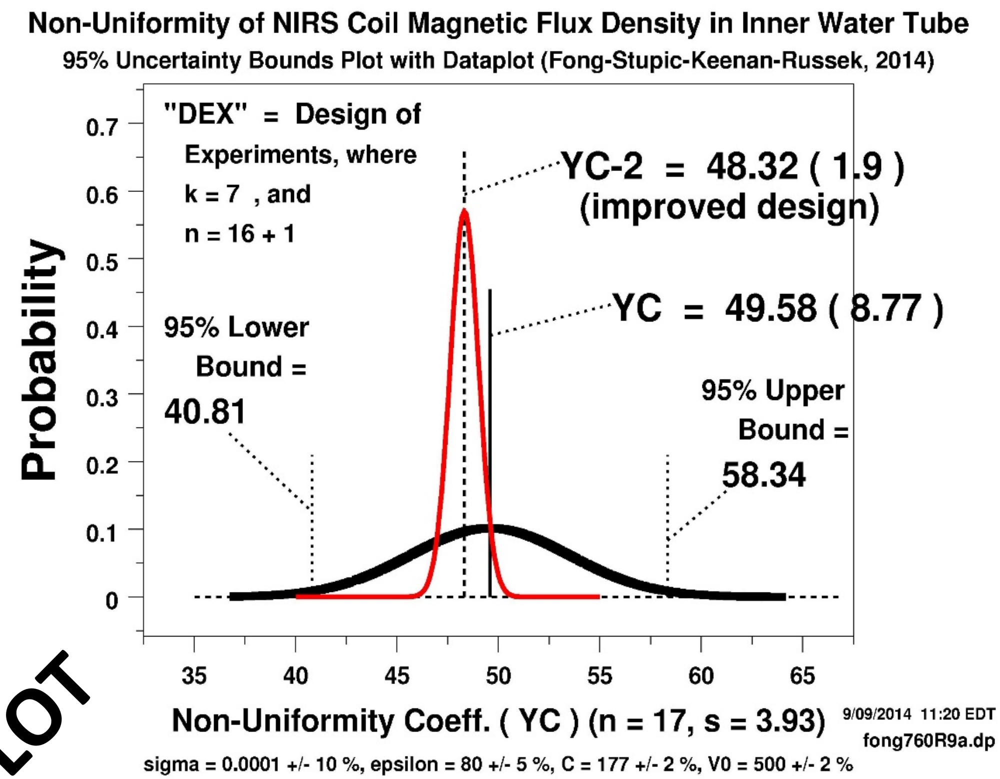


Before:

NUC = 49.58 (8.77)

After:

NUC = 48.32 (1.90)

Jeffrey T. Fong, NIST, Gaithersburg, MD
20899 USA. Tel: 1-301-975-8217
Email: fong@nist.gov

Future Work

1. *A new model with geometry and physics comparable to an actual coil.*

2. *Application of the DOE approach to the new model.*

3. *Experimental verification of the newly-optimized model.*

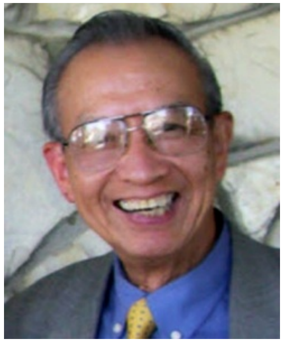
Jeffrey T. Fong, NIST, Gaithersburg, MD
20899 USA. Tel: 1-301-975-8217
Email: fong@nist.gov

Disclaimer

Certain commercial equipment, instruments, materials, or computer software are identified in this talk in order to specify the experimental or computational procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards & Technology, nor is it intended to imply that the materials, equipment, or software identified are necessarily the best available for the purpose.

Dr. Jeffrey T. Fong, P. E., Physicist
NIST, Gaithersburg, MD 20899-8910 USA
Email: fong@nist.gov

A Biographical Sketch of the Speaker



Dr. Jeffrey T. Fong has been Physicist and Project Manager at the Applied and Computational Mathematics Division, Information Technology Laboratory, National Institute of Standards and Technology (NIST), **Gaithersburg, MD, since 1966.**

He was educated at the University of Hong Kong (B.Sc., Engineering, first class honors, 1955), Columbia University (M.S., Engineering Mechanics, 1961), and Stanford (Ph.D., Applied Mechanics and Mathematics, 1966). Prior to 1966, he worked as a design engineer (1955-63) on numerous power plants (hydro, fossil-fuel, nuclear) at Ebasco Services, Inc., in New York City, and as teaching & research assistant (1963-66) on engineering mechanics at Stanford University.

During his 40+ years at NIST**, he has conducted research, provided consulting services, and taught numerous short courses on mathematical and computational modeling with uncertainty estimation for fatigue, fracture, high-temperature creep, nondestructive evaluation, electromagnetic behavior, and failure analysis of a broad range of materials ranging from paper, ceramics, glass, to polymers, composites, metals, semiconductors, and biological tissues.**

A licensed professional engineer (P.E.) in the State of New York since 1962 and a chartered civil engineer in the United Kingdom and British Commonwealth (A.M.I.C.E.) since 1968, he has authored or co-authored more than 100 technical papers, and edited or co-edited 17 national or international conference proceedings. He was elected Fellow of ASTM in 1982 and Fellow of ASME in 1984. In 1993, he was awarded the prestigious ASME *Pressure Vessels and Piping Medal.* Most recently, he was honored at the 2014 International Conference on Computational & Experimental Engineering & Sciences (ICCES) with a *Lifetime Achievement Medal.*

Since 2006, he has been Adjunct Professor of Mechanical Engineering and Mechanics at Drexel University **and taught a graduate-level 3-credit course on "Finite Element Method Uncertainty Analysis." Since Jan. 2010, he has given every 6 months an on-line 3-hour short course at** Stanford University **on "Reliability and Uncertainty Estimation of FEM Models of Composite Structures." In 2012, he was appointed Adjunct Professor of Nuclear and Risk Engineering at the** City University of Hong Kong**, and Distinguished Guest Professor at the** East China University of Science & Technology**, Shanghai, China, to teach annually a 1-credit 16-hour short course on "Engineering Reliability and Risk Analysis."**

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20899 USA. Tel: 1-301-975-8217
Email: fong@nist.gov