Self-consistent Modeling of Thin Conducting Wires and their Interaction with the Surrounding Electromagnetic Field

Göran Eriksson[*1]

[1]ABB AB, Corporate Research

*Corresponding author: SE-721 78, Västerås, Sweden, goran.z.eriksson@se.abb.com

Abstract: It is demonstrated how the RF module can be used to approximately model thin conducting wires or cables and how they interact with a surrounding electromagnetic field. Despite being non-stringent the method can reasonably well predict currents induced by an applied electromagnetic field in wires, and networks of wires, as well as fields radiated from current-carrying wires (antennas).

Keywords: Thin wires, Induced currents, Radiated fields, Cables.

1. Introduction

In Finite Difference Time Domain (FDTD) methods, where a regular structured mesh is used, it is in fact possible to include the effect of conducting wires having a radius much smaller than the mesh size into a global 3D model [1]. A Telegrapher's equation is employed to solve for the wire current, thus allowing for wave propagation and resonances. The trick is to take into account the electromagnetic energy very close to the wire surface by analytically including it into the expressions for the wire inductance and capacitance that appear in the Telegrapher´s equations. Also the parallel component of the global electric field is used to drive the wire current. Thanks to the regular FDTD mesh structure it is possible to describe the two-way interaction with the surrounding electromagnetic field in a fully self-consistent way. The FDTD scheme has therefore become very popular for modeling of all kinds of situations where thin wires or cables are involved, such as wire antennas and field coupling to and from cables and cable networks. Note that this method allows computation of both the field radiated from a current-carrying wire and the current induced in the cable due to an external time-varying electromagnetic field.

Despite many attempts, see for example [2], no similar technique being of practical use has been presented for finite element methods (FEM). Basically, the reason is the unstructured mesh into which the wire is embedded and the mathematical formulation of FEM. The proposed methods have either required elements almost as small as the wire radius and/or resulted in very complex implementations. A consequence of this is that FEM based solvers are not being considered particularly efficient for solving problems related to ElectroMagnetic Compatibility (EMC) issues and wire antennas. EMC deals with problems occurring due to cross-talk and interference to or from conducting wires or cables; a typical situation is illustrated by Fig. 1.

Here, we present an approximate method which is very easy to implement and that can be used to include wires having a radius smaller than the typical element size into a 3D model. The technique works for both low and high frequencies. In version 4.3 of COMSOL Multiphysics an implementation of the Telegrapher's equation is introduced in the RF module. Starting from this and adding the appropriate couplings to the external fields we can build up an approximately self-consistent model of the wire and its surrounding environment. The methodology is explained and illustrated with some examples.

Figure 1. An example where electromagnetic interference (EMI) between cables is very difficult to model using conventional FEM techniques.

2. The Telegrapher´s equations

In the Transmission Line Equation physics node in COMSOL Multiphysics the frequency domain equation for the voltage V along a two-wire transmission line is given by

$$\frac{\partial}{\partial x}\left(\frac{1}{R+i\omega L}\frac{\partial V}{\partial x}\right)-\left(G+i\omega C\right)V=0 \quad (1)$$

Here, $V(x)$ is the voltage between the two parallel wires and x is the distance along the line. R, L, G, and C are the resistance, inductance, conductance, and capacitance per unit length of the line. The line current $I(x)$ is related to $V(x)$ via

$$I=\frac{-1}{R+i\omega L}\frac{\partial V}{\partial x} \quad (2)$$

It is important to note here that for the transmission line it is assumed that the two wires are so close to each other that all of the corresponding electromagnetic energy is confined to the region between the wires and that no interaction takes place with the surroundings.

For the case of wires having radius a and axis separation distance d, the field problem can be solved analytically, resulting in the line inductance and capacitance given by

$$L=\frac{\mu}{\pi}\ln\left(\frac{d}{a}\right) \quad (3)$$

$$C=\frac{\pi\varepsilon}{\ln(d/a)} \quad (4)$$

3. Subcell model for a single thin wire

For a single wire far from other conductors it is not possible to define and calculate line parameters such as L and C describing all of the electromagnetic energy associated with the wire current and charge. It turns out, however, that one can still use the Telegrapher´s equation (1), but now with the line parameters describing only that part of the energy which is localized in the immediate vicinity of the wire surface, out to a distance of the order of the mesh element size Δ. In the FDTD formulation, with its regular

elements, one can derive the following expressions for these parameters:

$$L=L_w=\frac{\mu}{2\pi}\ln\left(\frac{\Delta}{2a}\right) \quad (5)$$

$$C=C_w=\frac{2\pi\varepsilon}{\ln(\Delta/2a)} \quad (6)$$

In order to include the driving force from external sources and current in distant parts of the wire, one adds the parallel component E_x of the global electric field to the local field $-\partial V/\partial x$ in (1):

$$\frac{\partial}{\partial x}\left(\frac{1}{R+i\omega L_w}\left(\frac{\partial V}{\partial x}-E_x\right)\right)- \quad (7)$$
$$\left(G+i\omega C_w\right)V=0$$

The current is now given by

$$I=\frac{1}{R+i\omega L}\left(E_x-\frac{\partial V}{\partial x}\right) \quad (8)$$

In the finite element method, where the elements are non-structured, it is not expected to have the same degree of accuracy as in FDTD but, as we will demonstrate, the agreement is acceptable for many purposes. If one has the same element size everywhere along the wire, this value is a good approximation of Δ. Moreover we use a suitable average value for the parallel electric field E_x. For instance it is possible to employ the diskavg or circavg operators.

Finally it should be noted that equation (7) retains the form of equation (1), implying that current conservation and continuity remain natural boundary conditions.

4. Validation case I: A square-shaped loop in an oscillating applied magnetic field

The first validation case consists of a planar square-shaped wire loop immersed in an external magnetic field. The method described above is implemented using the electromagnetic wave equation and transmission line physics nodes in the RF module. Figure 2 shows the magnetic field lines and the magnetic field amplitude in the plane
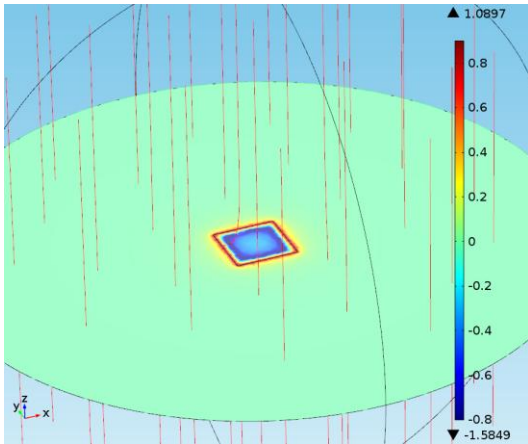
Figure 2. A conducting loop placed in a vertical magnetic field. The amplitude of B is shown in the plane of the loop (logarithmic color scale).

of the loop. It is clearly seen that the induced current in the loop reduces the field amplitude inside the loop, in accordance with Lenz´law.

Figure 3 illustrates the variation of V (green curve), $E_{parallel}$ (blue curve), and I (red curve) along the loop. As expected, the voltage, i.e. the wire charge density, varies but the current is approximately constant around the loop. However, the fluctuations in current indicate that the convergence is relatively poor due to the non-structured mesh and the associated difficulty to define a proper $E_{parallel}$,
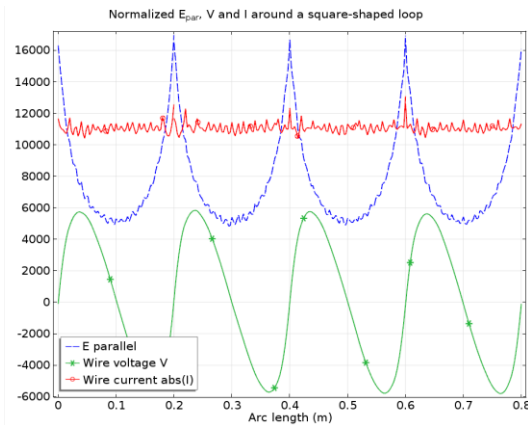

Figure 3. Parallel electric field, wire voltage, and current along the closed loop.
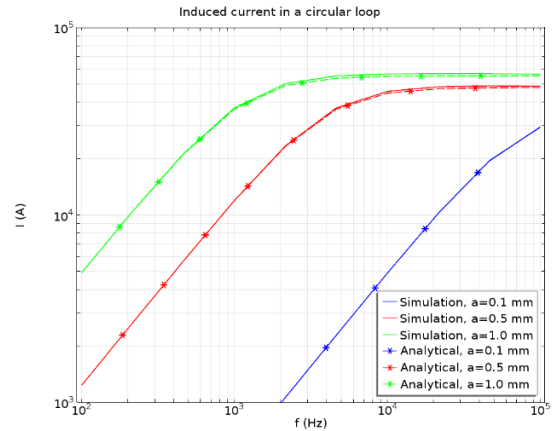

Figure 4. The induced loop current as function of frequency and for three different values of the wire radius a. Comparison between simulated and analytical results

Since L_w and C_w depend on the wire radius a, a series of simulations were performed, where the loop current was calculated as function of frequency for three different values of a. These results were then compared to an analytical expression derived using an exact formula for the full loop inductance. As can be observed in Figure 4, the agreement is good. For low frequencies the resistive part R of the loop impedance dominates, resulting in a linear relation between I and f. As the frequency increases the inductive part $i\omega L$ becomes dominant and the current is saturated. Note that L consists of two parts: L_w and the part represented by the extra term E_x in (7). If the latter is omitted in (7), the current curves would still saturate but on a much higher level. We have thus shown that our model indeed gives a realistic description.

5. Validation case II: A half wave dipole antenna

In the former validation case we have demonstrated that our model is realistic for describing the induction of wire currents due to external fields. We now also want to verify that we can model the electromagnetic field radiated from a current-carrying wire.

To do this we look at one of the simplest possible cases viz. a half wave dipole. This vertical antenna consists of two rods, each 1 meter long, connected via a short feed gap across which a voltage is applied. Two models are compared:
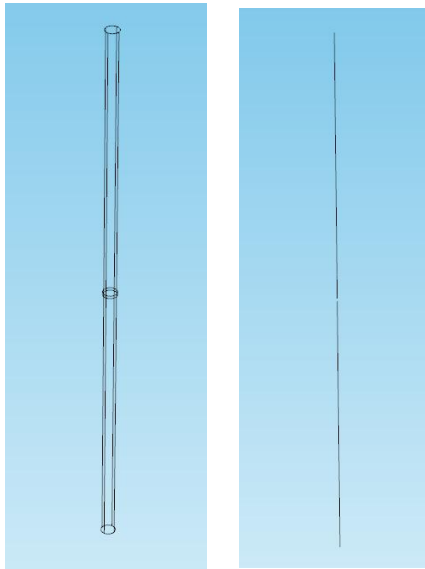
Figure 5. The two antenna models. Cylinder model (left) and wire model (right).

one where the rods are resolved and modeled as two perfectly conducting cylinders and one where the antenna is modeled using two thin wires. The two geometries are seen in Figure 5.

The current distribution in the wire model is shown in Figure 6. Despite some fluctuations due to convergence problems, the average current profile is as expected and the radiation pattern is almost identical to that of the resolved cylinder model. This is confirmed in Figures 7 and 8. Also the input impedance was calculated. It was found to be $106 + 43i\ \Omega$ for the cylinder model and $101-4i\ \Omega$ for the wire model.
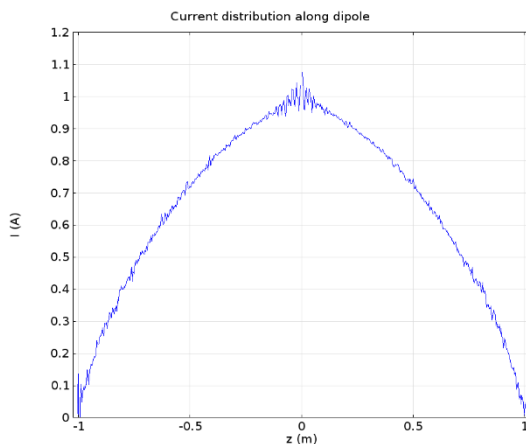

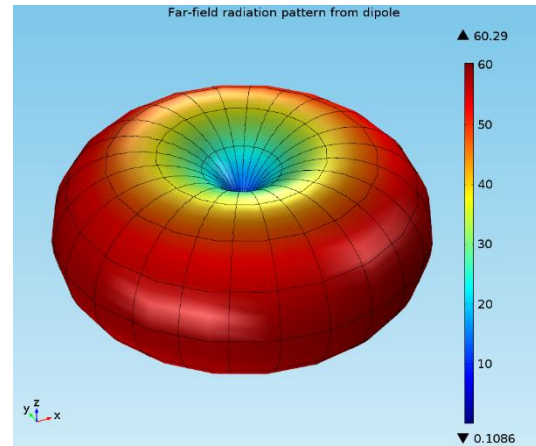**Figure 6**. Current distribution along the wire dipole.

Figure 7. 3D radiation pattern from the wire dipole.

The real parts, describing the radiated power, obviously agree quite well. That the imaginary parts, describing the inductive near-field energy, differ significantly but this is not surprising since the feed region is modeled very differently in the two models.
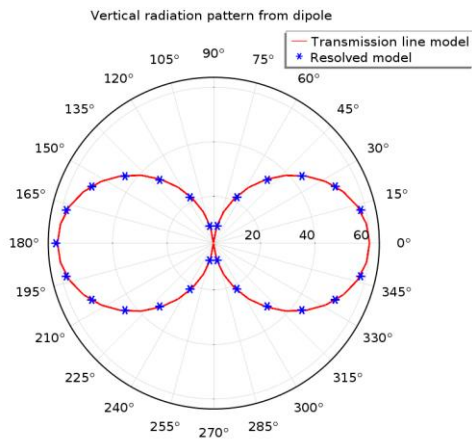

Figure 8. Radiation pattern in a vertical plane. Comparison between resolved cylinder and transmission line wire antenna models.

6. Conclusions

We have discussed and examined the possibility to employ a technique borrowed from the FDTD simulation method to model thin conducting wires and how the interact with the global electromagnetic field. Although the FEM method cannot exactly model such thin wires, the

examples shown here demonstrate that our method can be useful, provided the required accuracy is not too high. In areas such as EMC, where interference levels are measured in dB:s, it may provide a means to calculate first estimates.

7. References

1. Taflove, A. and Hagness, S.C., Computational Electrodynamics: The Finite-Difference Time-Domain Method, 3rd ed., Artech House, 2005.
2. Edelvik, F., Ledfelt, G., Lötstedt, P., and Riley, D.J., "An unconditionally stable subcell model for arbitrarily oriented thin wires in the FETD method", IEEE Trans. Antennas and Propagation, Vol. 51, No. 8, August 2003, pp. 1797-1805.