A Comparison between an A-V and V Formulation in Transcranial Magnetic Stimulation

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Abstract: The prediction of the exact location and intensity of the electric field induced in the human brain during Transcranial magnetic stimulation (TMS) is a nontrivial computational task. Numerical simulations of TMS can be used to acquire first approximations in a safe and controlled environment. In order to make this approach more applicable, it is necessary to reduce computation time of simulations as much as possible while maintaining a satisfactory level of accuracy. This proves to be very important, especially in cases when it is required to run a large number of simulations with different parameters. The focus of this paper is to compare **A**-V (magnetic vector potential, electric scalar potential) and V (electric scalar potential) formulations in terms of accuracy, memory requirements and computation time.

Keywords: Electromagnetic induction, Finite element method, Transcranial magnetic stimulation

1. Introduction

Transcranial Magnetic Stimulation (TMS) is method to stimulate cortical nerves non-invasively by inducing eddy currents using strong and rapidly changing magnetic fields. In consequence, neuronal structures are excited by hyperpolarization or depolarization. The procedure was first introduced by Barker *et al.* in 1985 at the Royal Hallamshire Hospital and the University of Sheffield [1]. TMS has achieved a more widespread use than its electrical equivalent, Transcranial Electrical Stimulation (TES) [2]. TES requires direct contact of electrodes to the skull and therefore could cause pain and discomfort to the patient unlike to TMS.

Today, TMS has become a tool widely used in an increasing range of applications. It can be used as a research tool in neurosciences, as well as a diagnostic and treatment tool in neurology and psychiatry [3, 4, 5]. Because TMS is a relatively new treatment method, it is still to be determined which treatment technique works best and whether it has any long-term side effects.

Numerical simulations are essential to gain a better understanding and to identify possible unwanted effects TMS may cause. In order to warrant practicability of the computer simulations, it is required to build economic and time efficient models. Different methods can be used to potentially reduce the required computation time. In this paper two different formulations of the given electromagnetic field problem are presented and compared. First a combined magnetic vector potential – electric scalar potential **A**-V formulation is presented and compared to a reduced electric scalar potential V formulation in the framework of TMS.

2. Methodology

2.1 Geometrical modelling

To ensure a given level of accuracy it is necessary to construct a detailed model of the human head. For this purpose, different brain tissues were segmented from T1 and T2-weighted magnetic resonance images (MRI) using SimNIBS, a free available software tool created at the Max-Planck Institute for Biological Cybernetics (Tuebingen, Germany) [7]. The output data generated by SimNIBS describes surface geometries of different brain tissues and given in *Stereolitography* (STL) file format. Using the .stl-files, the brain model was generated and meshed by the 3D finite element mesh generator Gmsh [8]. The mesh was exported in a *Nastran Bulk Data File* (bdf) which is supported by COMSOL Multiphysics. In a last step the mesh which is shown in Figure 1 was imported into COMSOL.

Same head models and meshes were used for both the **A**-V and V formulation in order to ensure a valid and fair comparison.
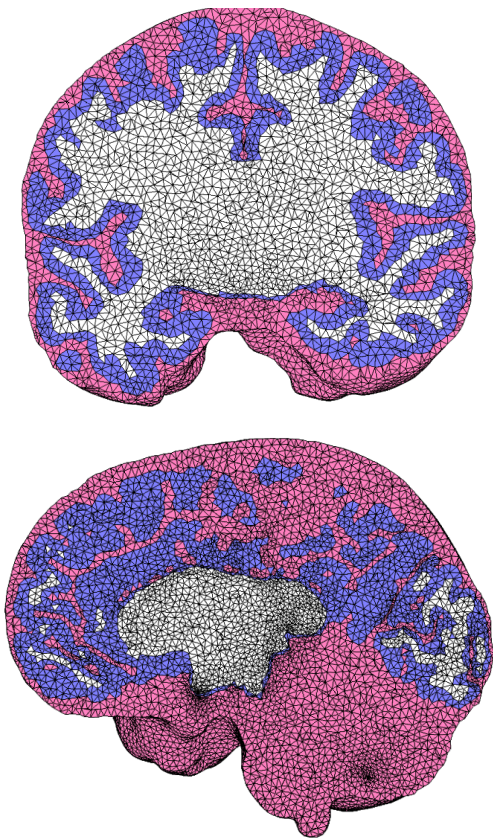
Figure 1. (top) Coronal and (bottom) sagittal view of the finite element mesh generated by means of MRI data and imported into COMSOL.
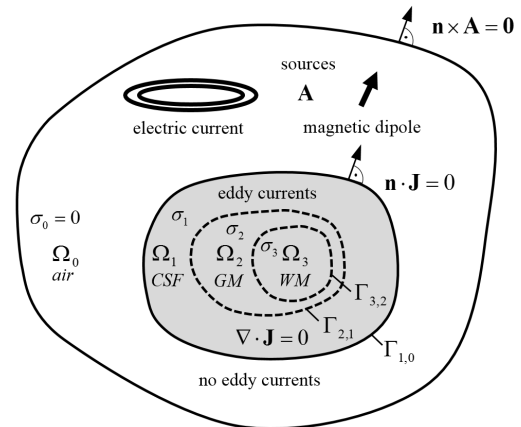

Figure 2. Physical interpretation of domains, subdomains, and boundaries included in the **A**-V formulation.

conductivity of the material. Figure 2 illustrates the physics of the **A**-V formulation in the framework of TMS.

The skull's low electrical conductivity, acts as an isolator surrounding inner brain tissues which prevents induced eddy currents to flow outside the inner skull area. Therefore, in order to calculate intensity and position of the induced electric field inside the human brain, it is only necessary to model the air region and brain tissues inside the skull (cerebrospinal fluid, gray matter and white matter), while the characteristics of the skull can be defined by appropriate boundary conditions.

2.2 A-V Formulation

The **A**-V formulation is a standard modeling approach, where the conducting domain (head) is surrounded by an air region and the calculation of the induced electric field is achieved numerically by solving the respective differential equations for the magnetic vector potential **A** and the electric scalar potential V in the frequency domain assuming harmonic excitation.

$$\left(j\omega\sigma - \omega^2\varepsilon_0\varepsilon_r \right)\mathbf{A} + \left(\mu_0\mu_r\right)^{-1}\nabla\times\nabla\times\mathbf{A} = \mathbf{J}_e \quad (1)$$

$$\nabla\cdot\left(-j\omega\mathbf{A} - \nabla V\right) = 0 \quad (2)$$

Whereby $\mathbf{J}_e$ and $\mathbf{J}$ denote the external and total current density respectively, ω representing the angular frequency, μ representing magnetic permeability of the material and σ the electrical

2.3 V Formulation

The magnetic field generated during TMS can be divided into two components: primary magnetic field $\mathbf{B}_p = \nabla\times\mathbf{A}_p$ generated by the external source (e.g. coil) and the secondary magnetic field $\mathbf{B}_s = \nabla\times\mathbf{A}_s$ generated by the induced currents inside the conducting region. Considering the estimation of the electrical conductivity ($\sigma \approx 1$ S/m) within the non-ferromagnetic ($\mu_r \approx 1$) human head and assuming a typical excitation frequency ($f \approx 3$ kHz) [10] and putting that values into the equation for the skin depth approximation,

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = 9.19m \quad (3)$$

it can be concluded that skin depth is approximately 100 times larger than the radius of the human head (R ≈ 0.09 m). In this case the primary magnetic field is dominant and the back reaction from the secondary magnetic field is negligible.

The induced electric field **E** can be calculated by,

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}_p}{\partial t} \qquad (4)$$

The electric scalar potential V arises from charges accumulated on the surface to preserve current conservation. Because there is no free charge inside the different tissue, one has to solve Laplace's equation for V inside the head region:

$$\Delta V = 0 \qquad (5)$$

Hence, in case of the V formulation the number of degrees of freedom inside the human head can be significantly reduced from four to one per node. Furthermore, the number of mesh elements decreases by avoiding the surrounding air region. When solving the Laplace equation for V, one has to ensure the uniqueness of the solution. Since we are only interested in the gradient of V, this can be accomplished by defining an arbitrary potential value (e.g. $V = 0$ V) at an arbitrary point inside the computational domain.

In order to include the excitation into the model, the primary magnetic field has to be calculated in advance.
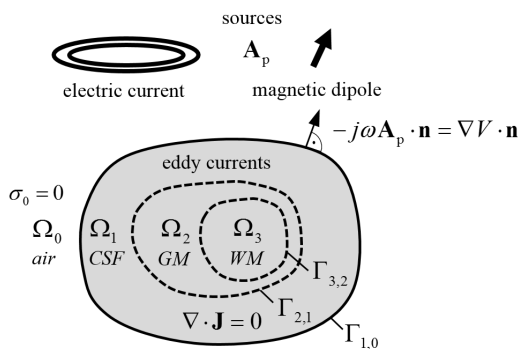
Figure 3. Physical interpretation of domains, subdomains, and boundaries included in the V formulation where the air region was omitted and the total magnetic field is approximated by the primary magnetic vector potential $\mathbf{A}_p$.

For the purpose of the comparison, it is not of interest to accurately model actual coil geometries used in TMS studies. Without limiting the generality, a magnetic dipole acts as the source of the magnetic field to induce eddy currents inside the human brain. Thielscher *et al.* showed that the coil area can be subdivided into small areas and approximated by magnetic dipoles [9]. In a second step, the total magnetic vector potential of the coil can be calculated by the principle of superposition.

Another approach is to discretize the coil windings into small segments and to calculate the magnetic vector potential analytically or numerically by means of Biot-Savarts law.

In case of the single magnetic dipole, the magnetic vector potential is given by:

$$\mathbf{A}_p(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \mathbf{r}}{r^3} \qquad (6)$$

Where **m** denotes the magnetic moment of the dipole and **r** the position at which the potential is evaluated in 3D space.

Taking into account continuity of the current density, the boundary between head and air must fulfill the following condition:

$$\mathbf{J} \cdot \mathbf{n} = 0 \qquad (7)$$

As a result, scalar potentials inside and outside the head can be observed separately thus making the calculation of the scalar potential outside the head irrelevant for the calculation of the intensity of the induced electric field inside. For that reason it is unnecessary to model the air region outside the head.

In order to calculate the induced electric field (4), one has to calculate the magnetic vector potential $\mathbf{A}_p$ (6) in order to solve the Laplace equation (5) for V with the boundary conditions (7). After acquiring values for the electric scalar potential, the induced field can be determined by combining $\mathbf{A}_p$ and V in equation (3).

3. Use of COMSOL Multiphysics

Both the **A**-V and the V formulation were set up and solved using COMSOL Multiphysics 4.3b. The modules *magnetic and electric fields (mef)* and a modified version of *electric currents (ec)* from the AC/DC module were used to set up the appropriate physics settings.

The material properties used in the numerical simulations are listed in Table 1.

In case of the **A**-*V* formulation, an appropriate point in the vicinity of the head was defined as a magnetic point dipole.

In case of the *V* formulation the primary magnetic vector potential from (6) and the induced electric field from (4) was defined as variables as shown in Figure 4. The parameters x_d, y_d and z_d indicate the position of the magnetic point dipole and m_x m_y m_z are parameters defining the magnetic dipole moment.

The equations for the *x*, *y* and *z* component of the electric field vector had to be modified in the *electric currents* module. The standard equations

for E_x, E_y and E_z were replaced by the expression given in (3), as shown in Figure 5.

4. Results

The distribution of electric field induced in gray matter calculated by the **A**-*V* and *V* formulation are shown in Figure 5 and 6. The plots show transverse and coronal views respectively. The intensity represents the electric field magnitude normalized by the maximum value obtained by the **A**-*V* formulation. No significant differences can be observed in terms of magnitude and distribution.

Both formulations were tested several times

Table 1: Electrical conductivity used for different brain tissues.

Type of tissue	Electrical conductivity [S/m]
Grey matter	0.1056
White matter	0.0650
Cerebrospinal fluid	2

Variables

Name	Expression	Unit
r	sqrt((x-xd)^2+(y-yd)^2+(z-zd)^2)	m
Ax	1e-7[H/m]/r^3*(my*(z-zd)-mz*(y-yd))	Wb/m
Ay	1e-7[H/m]/r^3*(mz*(x-xd)-mx*(z-zd))	Wb/m
Az	1e-7[H/m]/r^3*(mx*(y-yd)-my*(x-xd))	Wb/m
Ex	-j*omega*Ax-d(mod1.V,x)	V/m
Ey	-j*omega*Ay-d(mod1.V,y)	V/m
Ez	-j*omega*Az-d(mod1.V,z)	V/m

Figure 4. Variables of the induced electric field **E** and magnetic vector potential **A** used in the *V* formulation.
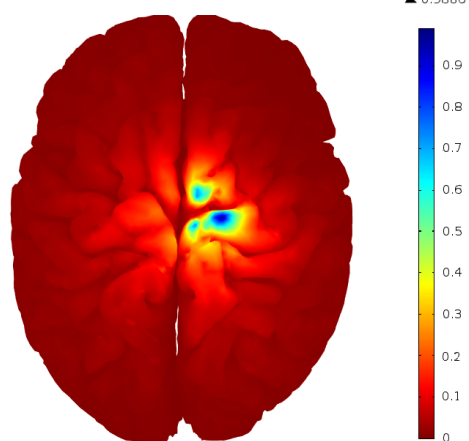
Variables

Name	Expression	Unit	Description
ec.dWe	ec.Weav	J/m³	Integrand for total e...
ec.Weav	0.25*epsilon0_const*(realdot(ec.e...	J/m³	Electric energy dens...
⚠ ec.Ex	Ex	V/m	Electric field, x com...
⚠ ec.Ey	Ey	V/m	Electric field, y com...
⚠ ec.Ez	Ez	V/m	Electric field, z com...
ec.tEx	-VTx	V/m	Tangential electric fi...
ec.tEy	-VTy	V/m	Tangential electric fi...
ec.tEz	-VTz	V/m	Tangential electric fi...
ec.normE	sqrt(realdot(ec.Ex,ec.Ex)+realdot(...	V/m	Electric field norm

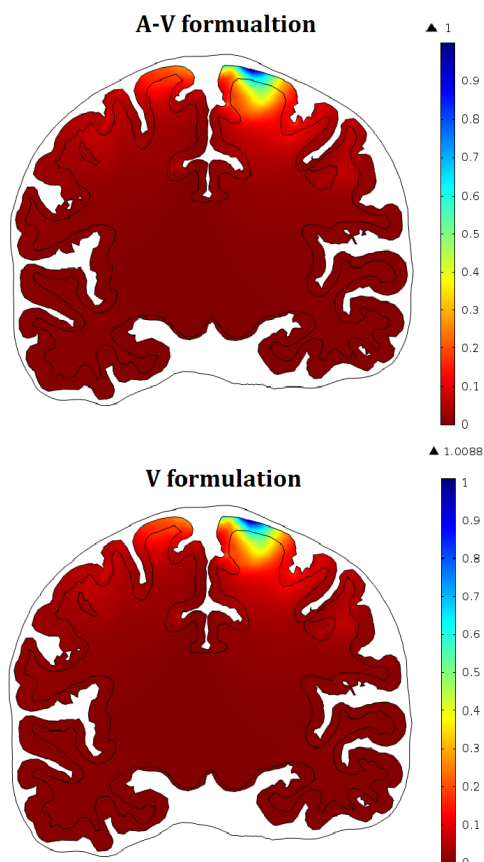**Figure 5**. Modified equations of the *electric currents (ec)* module.

A-V formualtion
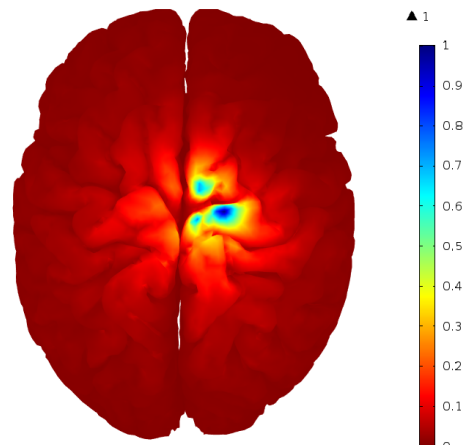
V formulation

Figure 6. Normalized induced electric field distributions in grey matter obtained by **A**-*V* and *V* formulations (transverse view).

A-V formualtion

V formulation

Figure 7. Normalized induced electric field distribution in grey and white matter obtained by **A**-*V* and *V* formulations (coronal view).

Table 2: Comparison of computational resources between an **A**-*V* and *V* formulation in TMS.

Category	**A**-*V* formulation	*V* formulation
Time	595 s	74 s
Peak memory allocation	28 GB	4 GB
Number of elements	1 106 239	889 478
Degrees of freedom	8 487 402	1 217 556

on the same computer (Intel i5, 16GB RAM) using the same set of parameters in order to acquire an appropriate set of simulation times. Data acquired during testing is shown in Table 2. The time values represent mean values from the different runs. Peak memory allocation represents the largest amount of memory allocated by COMSOL during the simulations.

5. Conclusion

In order to make a conclusion about the comparison between both formulations based on the data acquired from the test setup, it is necessary to determine if improvements of the *V* formulation in terms of computation time can be considered significant and if accuracy of the results have not been deteriorated compared to the **A**-*V* formulation. Considering the time necessary to run the simulations it can be noticed that the *V* formulation is approximately eight times faster while using fairly 14% of memory compared to the **A**-*V* formulation. The magnitude of the induced electric field shows differences lower than 2%. These relatively small differences in the electric field intensity can be considered negligible in most cases.

Data about the approximated location of induced electric field acquired by computer simulations coupled with experimental patient data could prove useful in precise mapping of brain regions.

It can be concluded that using a *V* formulation does reduce computation time significantly, while maintaining a satisfactory level of accuracy. A reduction of the computation time could prove to be very important in case of optimization studies or therapeutic applications. Precise coil positioning, and even automation, could be achieved for example by running a certain number of simulations on MRI derived head models.

6. References

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7. Acknowledgements