Numerical Experiments on Deconvolution Applied to LES in The Modeling of Turbulent Flow

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Abstract: The Large Eddy Simulation is an important method to simulate turbulent flow. It does not produce a closed system of equations, to achieve this it is necessary to model the not-closed terms.The deconvolution can be used for this purpose.In this study some numerical experiments on this topic are performed with COMSOL Multiphysics®. The main objectives are to find an efficient way to implement deconvolution and to evaluate its numerical behavior, with particular attention to the boundary conditions, or rather to their LES-deconvolution modeling.

Keywords: LES, deconvolution, turbolence

Introduction

The main purpose of LES is to find a good estimate of filters of the fluid dynamic variables. Indeed, if a suitable filter is applied to the the fluid dynamic variables in a typical turbulent flow, then the filtered-variables will be much more smooth, in both time and space, than original ones. That justifies the hope that the equations governing the filtered-variables are much simple to solve than those governing the no filtered-variables. In general, it is not said that these equations exist, and this, unfortunately is the case of fluid dynamics. So the goal becomes to build equations that provide an adequate approximation of the filters of the fluid dynamic variables. A way to do is applying the deconvolution, i.e. the original variables are estimated as function of the filtered ones. Applying this estimate to the original equations the definitive equations, where the unknowns are the filtered-variables, are obtained. This is the method considered in our research work, whose main purpose is to build a methodology to produce turbulent models which require a reduced computational cost and are deducted only from the original equations. As a first step, the method is tested on its capacity to compute an estimate of the filters of original variables: the results from the unchanged equations and the LES-deconvolution modified ones are compared. A test of this type is reported in the present article.

Governing Equations

The starting equations are those of Navier-Stokes (with homogeneous and steady density and viscosity). Given a domain Ω and a sphere $S_\delta(\mathbf{x})$ of radius δ and center $\mathbf{x}$, defines the following operator:

$$G\langle u\rangle := \int_{\Omega_\delta} \Theta(\xi - \mathbf{x})\, u(\mathbf{x})\, d\Omega \tag{1}$$

where

$$\Omega_\delta := \Omega \cap S_\delta(\mathbf{x}) \tag{2}$$

Then the filter is defined by the following operator:

$$F\langle u\rangle := \frac{G\langle u\rangle}{G\langle 1\rangle} \tag{3}$$

The function $\Theta(.)$ and $u(.)$, are assumed sufficiently regular basis so that the operators above and the following mathematical processing are well defined.Once the filter, the deconvolution must be defined. As a first approach, a Taylor's series expansion is used, applying it to the function u in the filter operator:

$$u(\xi) = u(\mathbf{x}) + \partial_i u(\mathbf{x})(\xi - \mathbf{x}) + O(\delta^2) \tag{4}$$

Defining $\hat{u} := F\langle u\rangle$ and not showing the dependence on $\mathbf{x}$, is obtained:

$$\hat{u} = u + \partial_i u\, F_{\Delta_i} + O(\delta^2) \tag{5}$$

Where:

$$F_{\Delta_i} = \int_{\Omega_\delta} \Theta(\xi - \mathbf{x})(\xi_i - x_i)\, d\Omega \tag{6}$$

Then, deriving 5:

$$\partial_j \hat{u} = \partial_j u + \partial_i u\, \partial_j F_{\Delta_i} + O(\delta) \tag{7}$$

The equations above named 7 are a system that can be solved respect to $\partial_i u$ and definig $\tilde{\partial}_i u$ as its solution it is possible to write:

$$\partial_i u = \tilde{\partial}_i u + O(\delta) \tag{8}$$

Then, substituting in 5 is obtained:

$$\hat{u} = u + \tilde{\partial}_i u\, F_{\Delta_i} + O(\delta^2) \tag{9}$$

So, the deconvolution is:

$$u \approx \hat{u} - \tilde{\partial}_i u\, F_{\Delta_i} \tag{10}$$

Replacing u in the original equations by 10 we obtain the modified equations with the unknown factor $\hat{u}$, that is the sought after estimate of the filtered-variable.

Methods

Considering the complexity of the Navier-Stokes equations and the uncertainties in this type of research, you need to start with simple models and safe. For this reason it was chosen a 2D model consists of a circular ring. The inner circumference is stationary, the outer wheels around its center. In fact, this model has a very simple geometry and boundary conditions (no slip) very robust, so that the existence of the solution is ensured (for the unchanged Navier-Stokes equations). In addition, the system admits an know analytical solution for the stationary case. Always for simplicity, the deconvolution is applied only to velocity. It is important to note that the deconvolution is applied to all equations level (domain, boundary, ..). Time-dependent simulations are performed both for Navier-Stokes that for LES-Decon-Navier-Stokes equations. The aim is to evalate the numerical behavior of the modified equations and the accuracy filtered-variables estimate.

Numerical Model

The inner radius is 1.0 `m` and the outer 1.1 `m`, the angular velocity is 0.01 `rad/s`. It is applied with a linear ramp up to 200 `s` for a total simulation time of 500 `s`.The standard Laminar-Flow module of COMSOL Multiphysics® with the P1+P1 discretization was used.

Experimental Results

The velocity magnitude is shown in fig. 1, F_{Δ_1} in fig. 2 and F_{Δ_2} in fig. 3. In the graph of fig. 4 the filtered variable `FvTheo` and its estimate `v` are compared. These refers to the y component of velocity along a radial segment with $y = 0$.

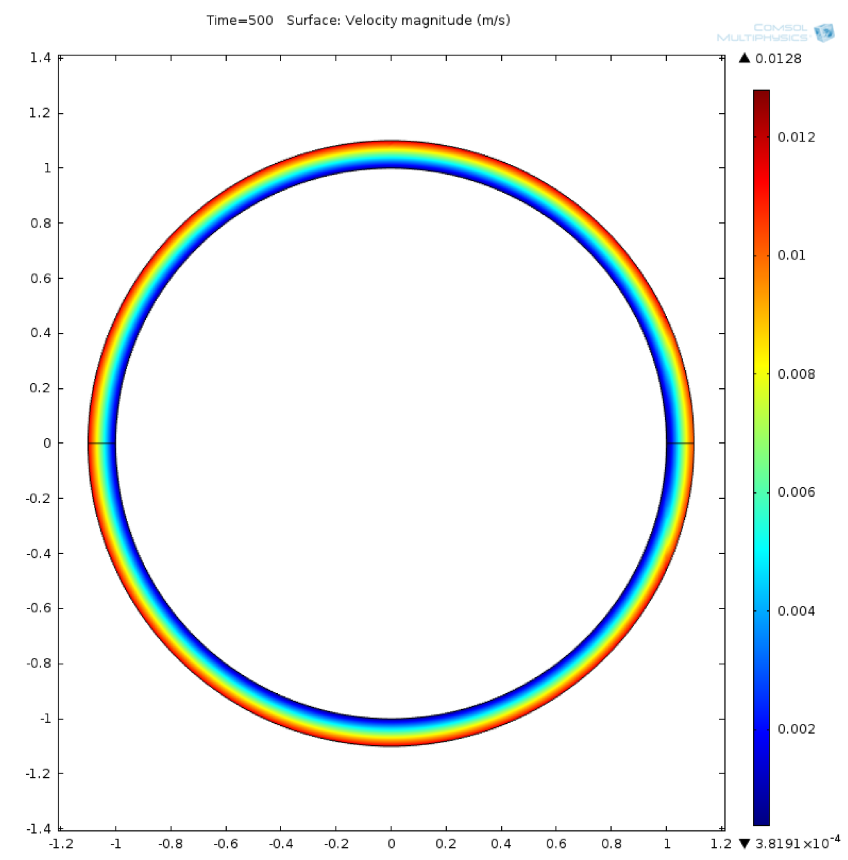

Figure 1: Velocity Magnitude

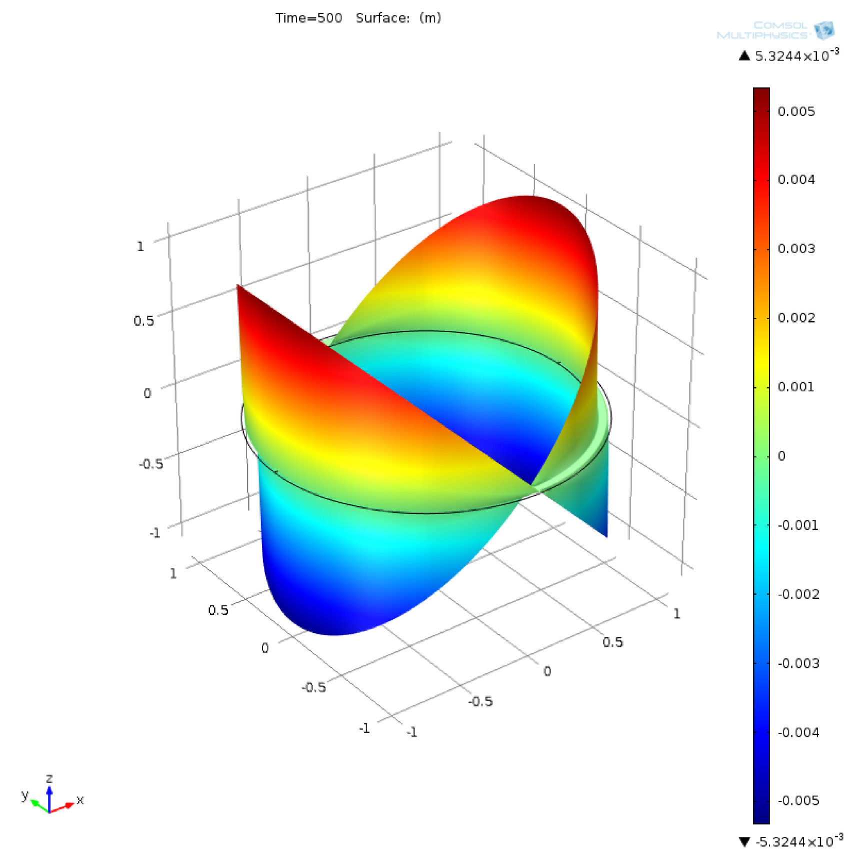

Figure 2: F_{Δ_1}

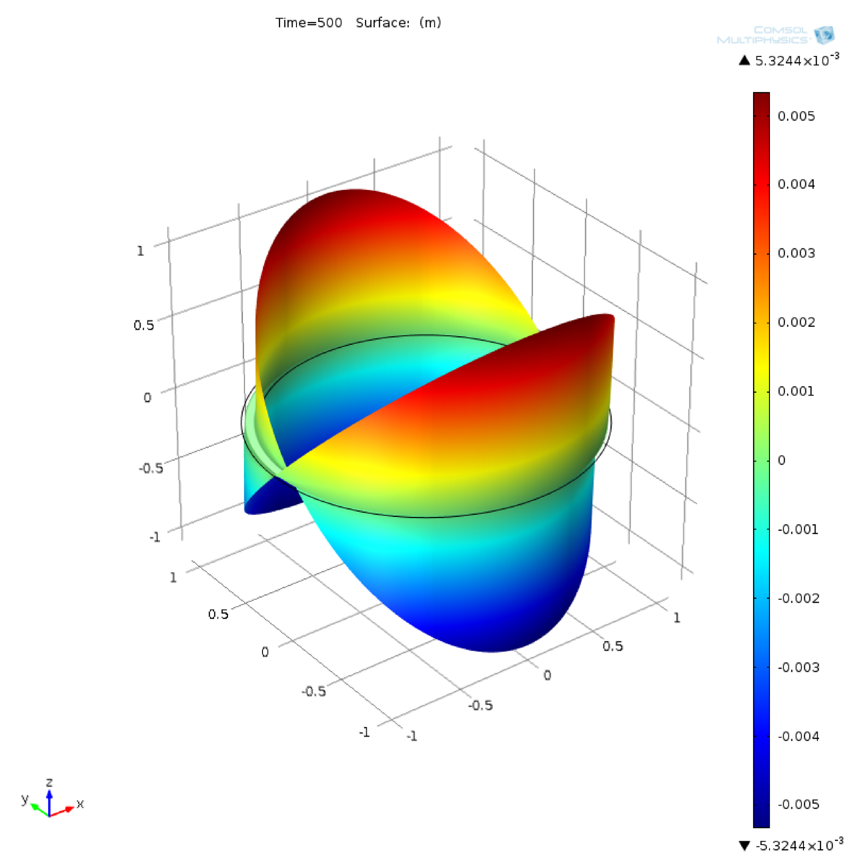

Figure 3: F_{Δ_2}

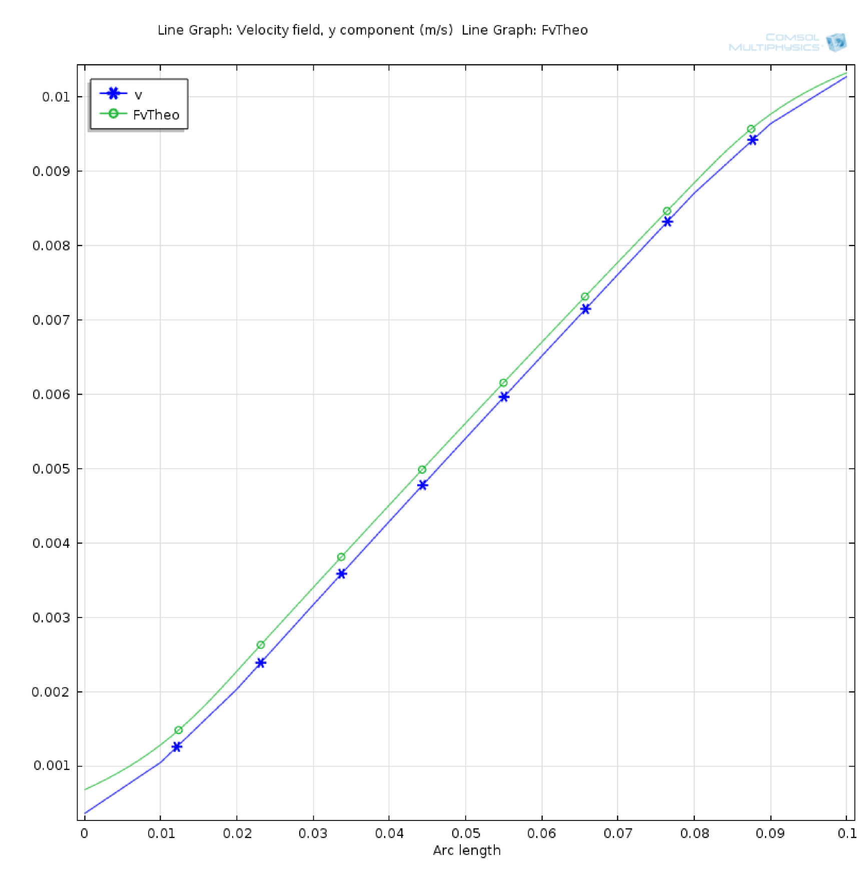

Figure 4:

Discussion

The numerical behavior of the LeDeNS equations is good and very similar to that of NS, for example the computation-times are very close ≈ 15 `min`. The fig. 4 shows that the estimate of the filtered-variable is globally good but it has a not negligible relative error near the inner radius.

Conclusions

This research work show that the LES-deconvolution approach to Navier-Stokes equations is feasible and (could be) coerent, i.e. the solved variables estimate sufficiently well the filtered-veocity. However, this is a preliminary result that needs to be consolidated. In particular, be necessary to seek the causes of the non-homogenous relative error along the radius.