Sensitivity of wrinkling to mesh imperfections
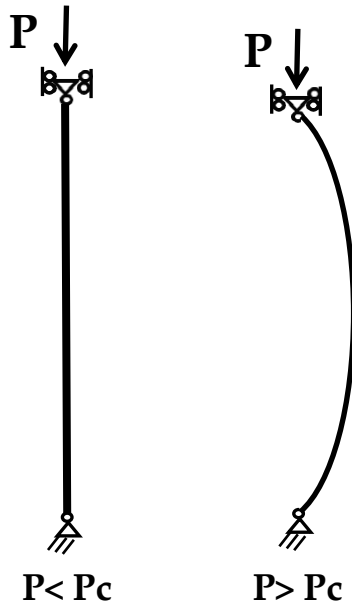
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Buckling instability generates wrinkles
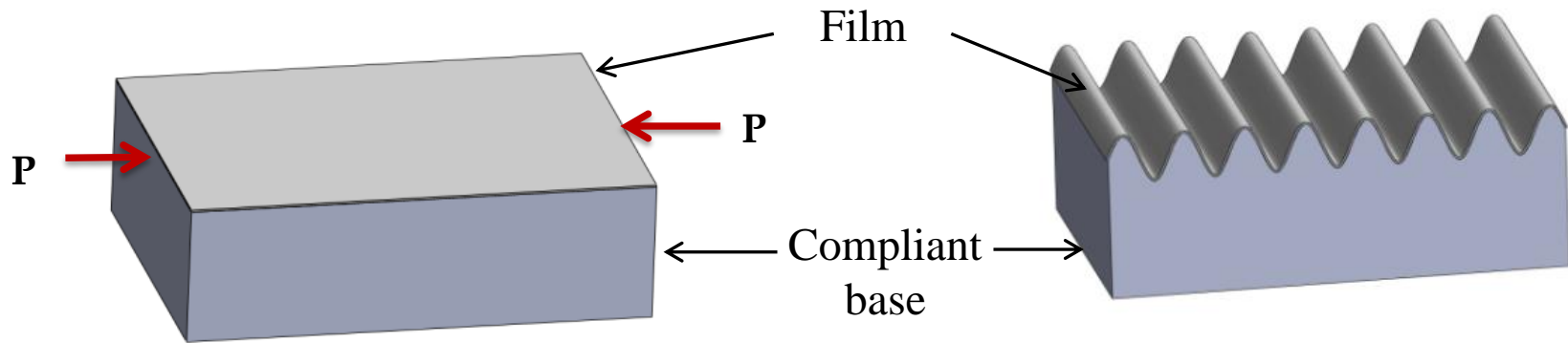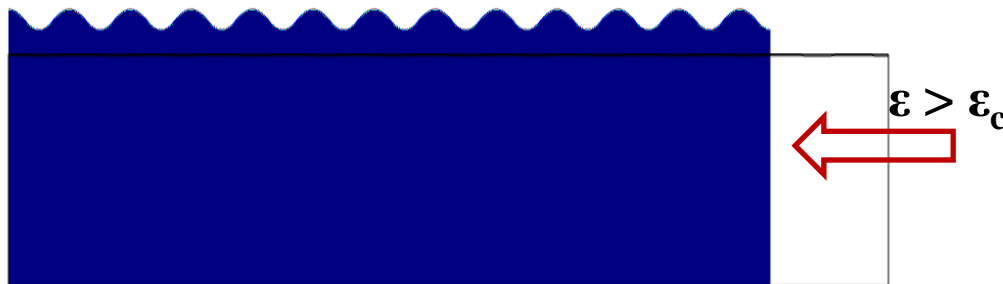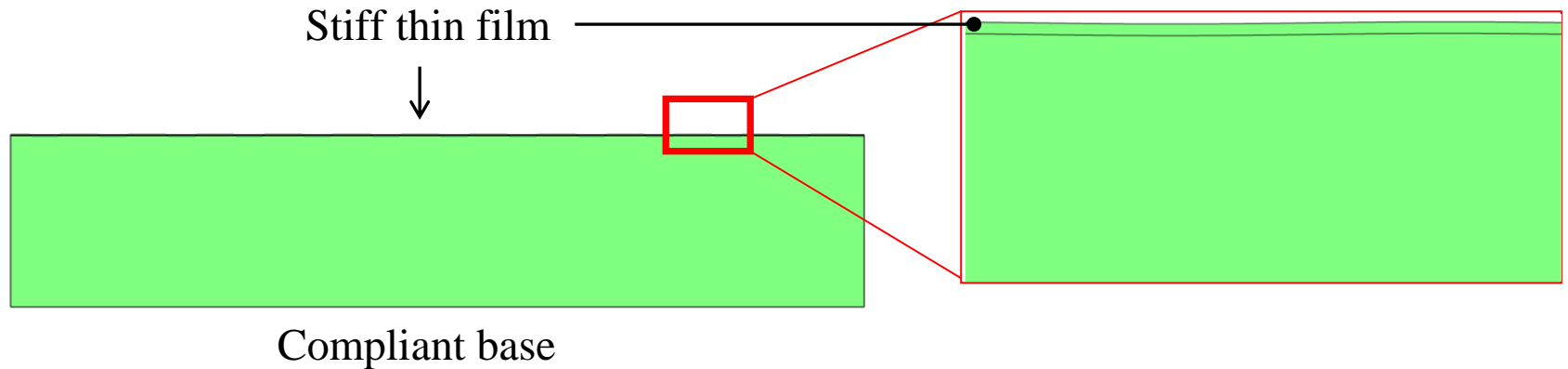
P < Pc   P > Pc

Beam buckling

Similar to Euler beam buckling

Mode shape determined by balance of bending & stretching energies

$$E_b \propto \frac{1}{\lambda^2} \qquad E_s \propto \lambda$$

Film

P

Compliant base

P

Imperfection is necessary for bifurcation

Stiff thin film

Compliant base

$\varepsilon < \varepsilon_c$

Flat state

$\varepsilon > \varepsilon_c$

What is the right amount of imperfection?

Wrinkled state

Two step modeling process

Modal analysis → Mesh imperfection → Post buckling analysis

Free surface

Plate

$u=0$ →

y

0 x

← $u=-\varepsilon L$

$v=0$

Foundation

$x=L$
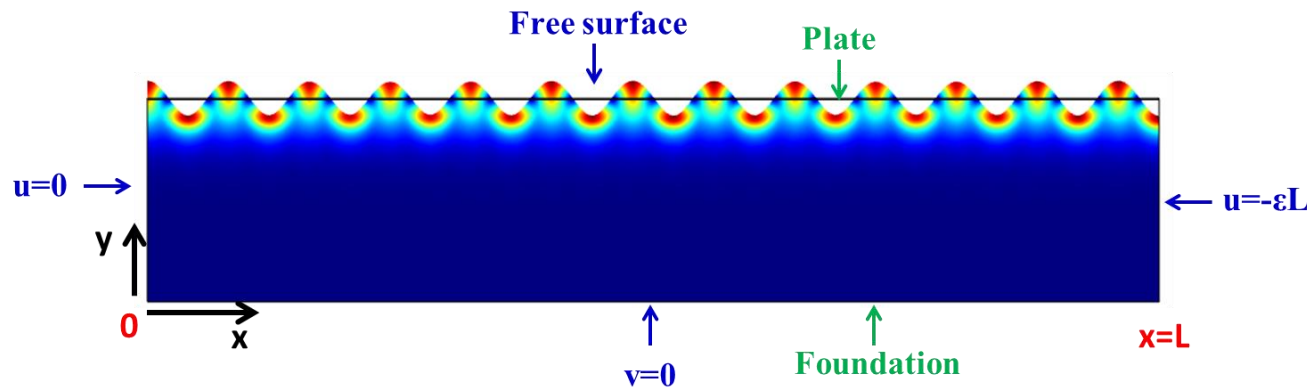
Wrinkling mode shape: 1st mode

LiveLink for MATLAB

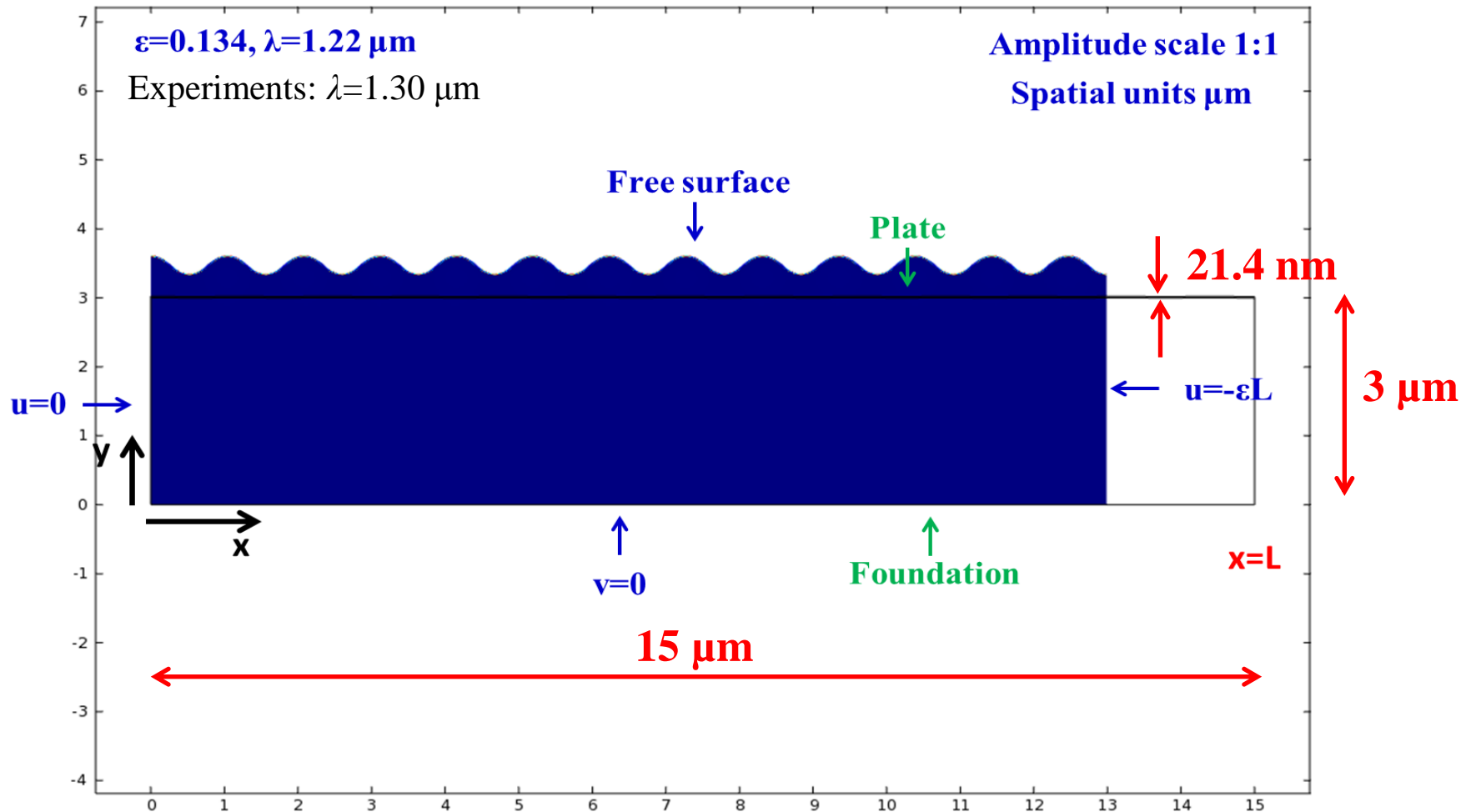Modal analysis

Extract mode shape & mesh
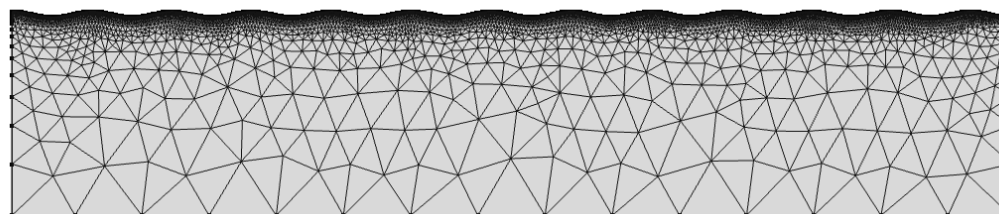
Generate mesh with imperfections

Mesh with 1st mode as geometric imperfection (8X)
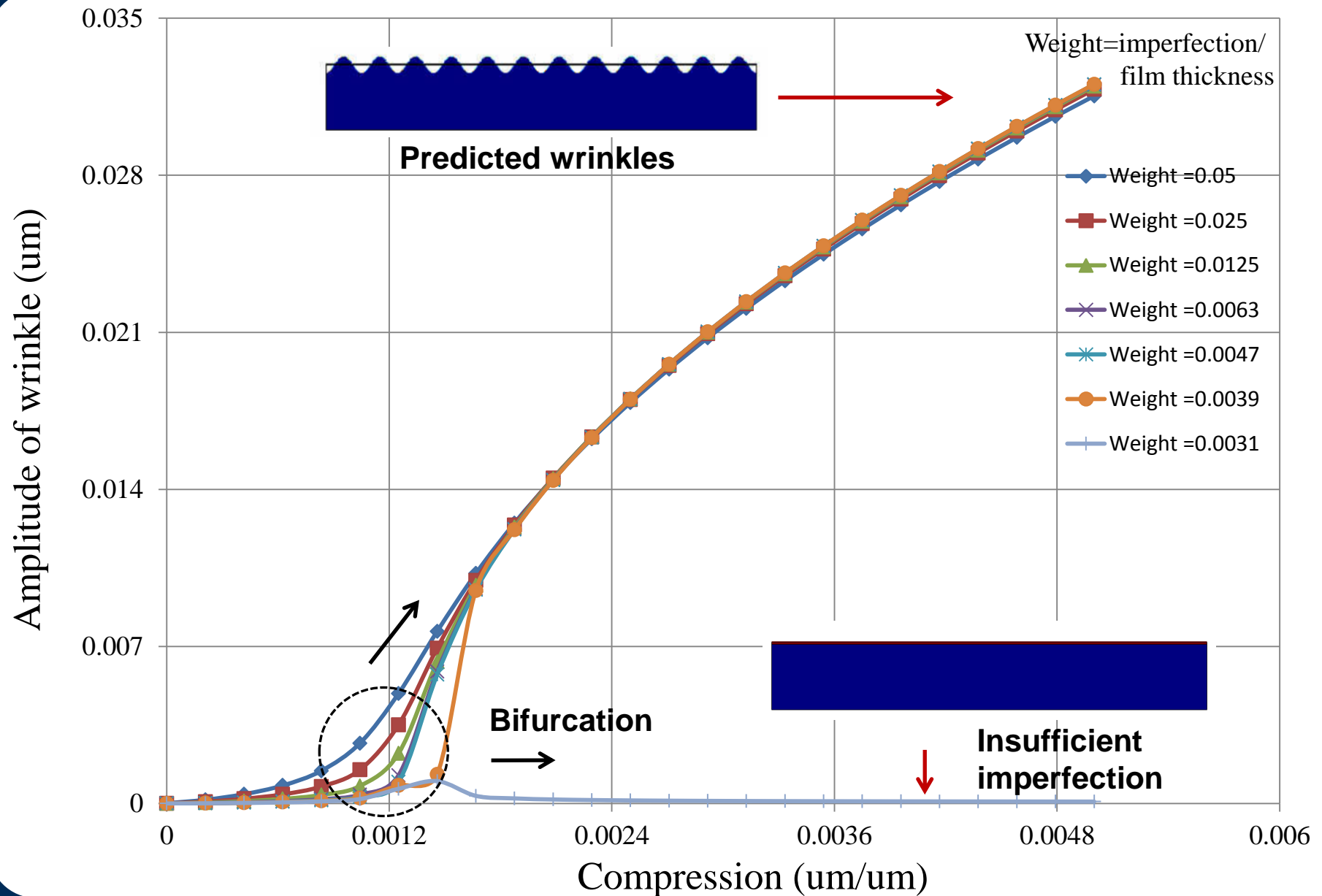
Two step modeling process

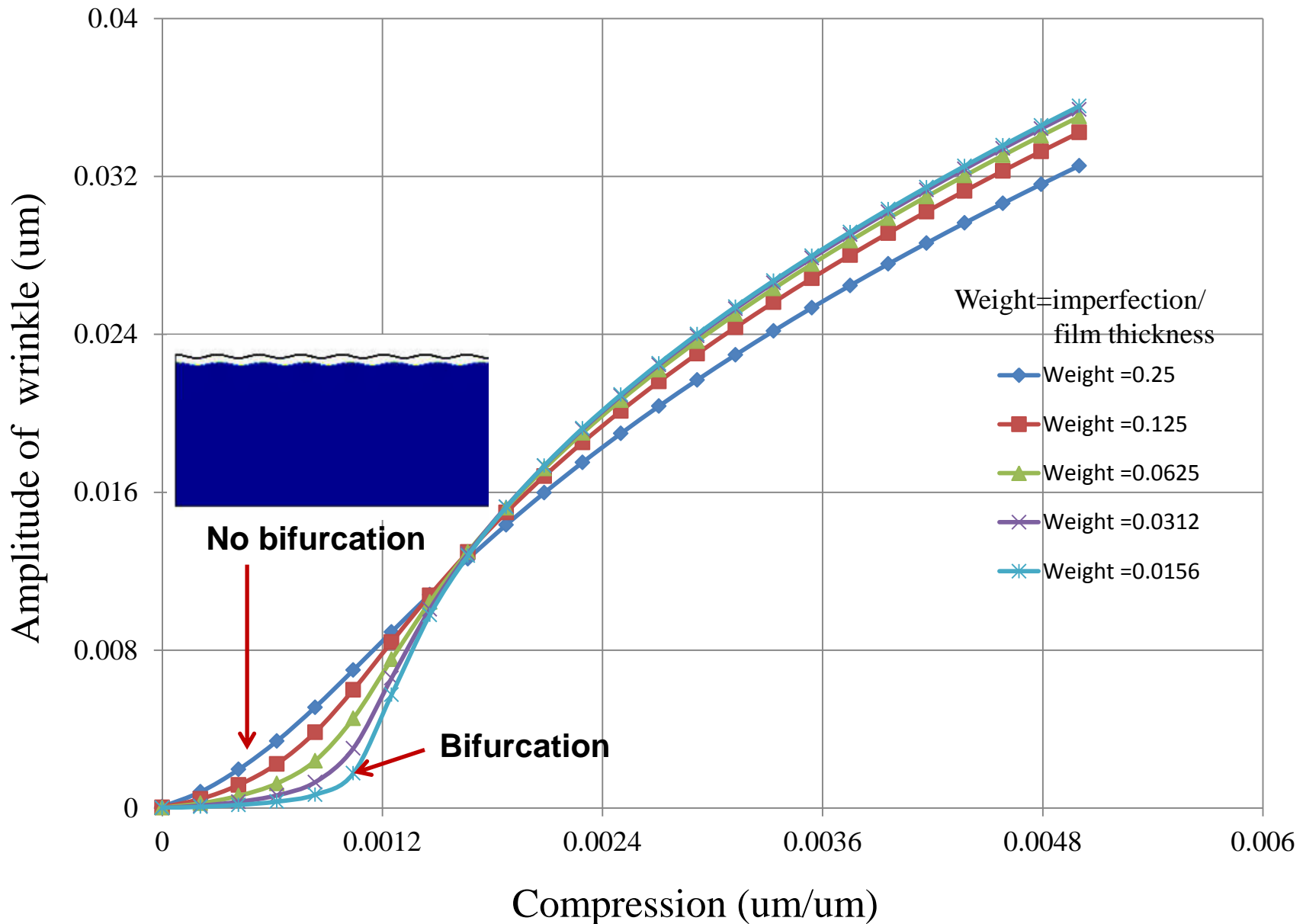Analysis performed on mesh with imperfections

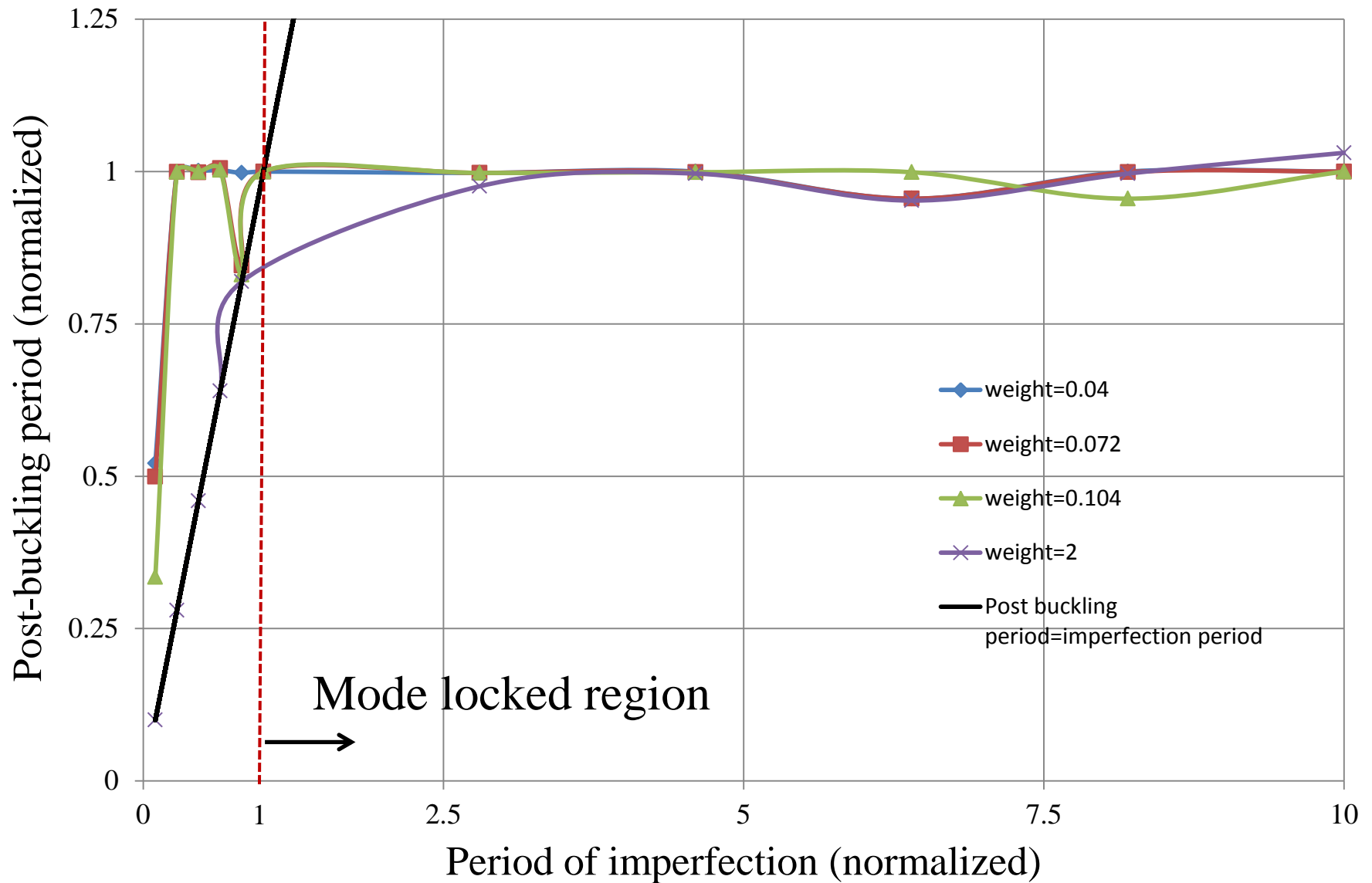**Non-linear post-buckling analysis for deformations**

No bifurcation with small imperfections

Amplitude of wrinkle (um) vs Compression (um/um)

Weight=imperfection/ film thickness

Predicted wrinkles

- Weight =0.05
- Weight =0.025
- Weight =0.0125
- Weight =0.0063
- Weight =0.0047
- Weight =0.0039
- Weight =0.0031

Bifurcation

Insufficient imperfection

No bifurcation with large imperfections

Mode lock-in at "natural" period

Conclusions

Imperfections necessary for wrinkling

Sensitive to height of imperfection

- ❑ Accurate when imperfection ~ 0.3% - 12.5% of film thickness

Insensitive to period of imperfection

- ❑ Very forgiving of large periods