Studying the Sensitivity of the Wrinkling Process to Mesh Imperfections Using COMSOL Multiphysics® and LiveLink™ for MATLAB®

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Abstract

Introduction:
Wrinkles are formed on a thin film as a result of buckling-based instabilities and the mechanism is similar to Euler buckling of beams under compressive loads (Figure 1). Wrinkling of thin films can be used as an inexpensive fabrication technique for generating micro and nano scale periodic patterns. In the absence of analytical models, finite element techniques are used for predictive design of complex wrinkling patterns. As wrinkles are formed via a bifurcation process, the accuracy of these models is dependent on the initial geometric imperfection to the system. Without this initial imperfection, the 'perfect' system does not bifurcate and wrinkled patterns are not observed; when excess imperfection is added, the system changes over into a different one. Thus, there exists a specific range of imperfection values over which 'proper' bifurcation occurs. Herein, we study the sensitivity of the wrinkling process to initial imperfection and identify this acceptable range of imperfections.

Use of COMSO Multiphysics®:
The wrinkling system consists of a thin film of a stiff material that lies on top of a soft base. We developed a 2-D finite element model of wrinkling by implementing buckling of wide plates in the plane strain condition. The top film is modeled as an elastic plate and the bottom layer as a Neo-Hookean foundation. Modeling was performed in two steps: (i) linear buckling analysis to predict the mode shapes, i.e., the period of the wrinkles and (ii) a non-linear post-buckling analysis to predict the shape and amplitude of the wrinkles after buckling bifurcation (Figure 2). In this two-step process, (i) displacements of the 1st mode shape obtained from linear buckling analysis were extracted and then (ii) added as the initial imperfection to the mesh for the second step. The displacements were weighted by a multiple of the top layer thickness. A set of nonlinear studies with varying amounts of mesh imperfection was generated by varying this weight multiplier. LiveLink™ for MATLAB® was used to (i) generate the initial weighted imperfections, (ii) set-up the nonlinear studies and (iii) post-process the nonlinear analysis for bifurcation.

Results:
The minimum imperfection was evaluated by identifying the smallest weight at which bifurcation occurs (Figure 3). It was observed that bifurcation does not occur when the amplitude of the

imperfection is below 0.1% - 0.5% of the top layer thickness. The maximum imperfection was identified by evaluating the weight at which the error in wrinkle amplitude crosses a threshold (Figure 4). It was observed that as the imperfection increases, the nonlinear bifurcation phenomenon changes into linear amplitude versus compression response.

Conclusion:
This imperfection sensitivity analysis eliminates the need for 'guesswork' in the process of setting up a wrinkling study by quantifying the acceptable range of mesh imperfection. Also, the two-step modeling process with a tunable imperfection enables one to rapidly perturb the geometry of a model while preserving the rest of the model definition. Therefore, this technique can be used to perform an automated search/exploration over a wide range of the design space for optimum patterns.

Figures used in the abstract

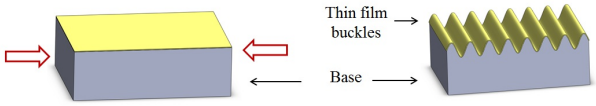

Figure 1: Schematic of the wrinkling process

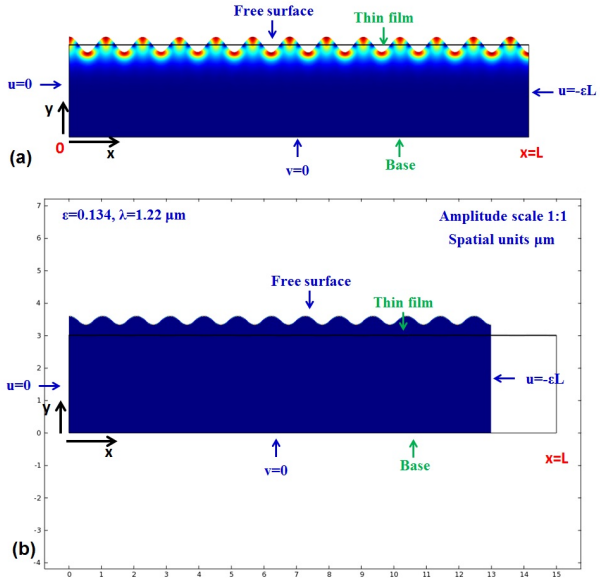

Figure 2: Steps of the finite element analysis process (a) Linear buckling analysis (b) Nonlinear post-buckling analysis

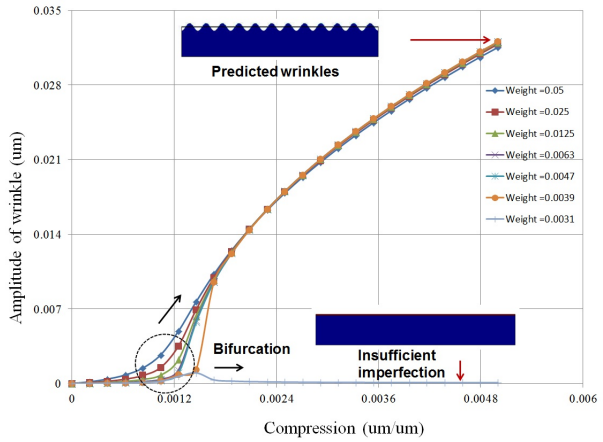

Figure 3: Bifurcation diagram for different imperfection values. The system does not bifurcate at low values of imperfection.

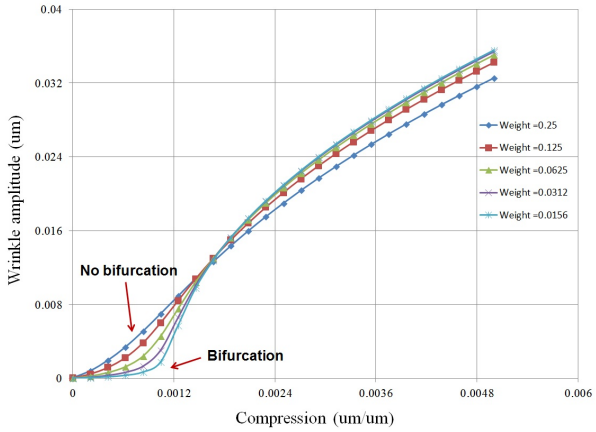

Figure 4: Bifurcation diagram for high imperfection values. At high imperfection values wrinkles are formed without bifurcation.