Coupled PDEs with Initial Solution from Data in COMSOL 4

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Outlines

- Model Description
- Two Approaches
- Initial Data Input As Data File
- Results

Problem Statement

We consider an example from mathematical biology, the FitzHugh-Nagumo equations:

$$C_t - \nabla \cdot (D_{\text{eff}} \nabla C) = C(C-\alpha)(1-C) - \beta v,$$
$$v_t = \epsilon(C - \gamma v),$$

It is a reduced system of coupled time-dependent reaction-diffusion equations with:

- Square domain $\Omega = (0,150) \times (0,150) \subset \mathbb{R}^2$ in units of micrometers (μm).
- No-flow boundary conditions $\mathbf{n} \cdot (D_{\text{eff}} \nabla C) = 0$ for all $(x,y) \in \partial\Omega$.
- The initial conditions are given by data in `txt` files that specify the values of $C(x,y,0) = C_0(x,y)$ and $v(x,y,0) = v_0(x,y)$ on a 50×50 mesh of $\overline{\Omega}$.
- The physiological parameters of the problem are $D_{\text{eff}} = 1$, $\alpha = \beta = 0.1$, $\gamma = 0.2$, and $\epsilon = 0.07$.

Two Approaches for Coupled PDEs

The idea of the first approach is to model each PDE equation in the system separately and then couple them together.

- Each equation in the coupled system of PDEs is represented by one `Physics`, each need to set up source term, boundary condition, and initial condition.
- This method is convenient with smaller systems of PDE, or PDEs with different structure.

The idea of the second approach is to use only One `Physics` by using the matrix form of coefficients.

- In this case, **`Diffusion Coefficient`** is a diagonal 2×2 matrix with 1 and 0 on diagonal. The source term is a 2×1 vector with $f_1 = C(C - \alpha)(1 - C) - \beta v$ and $f_2 = \epsilon(C - \gamma v)$.
- The initial condition is in the form of a 2×1 vector with $C_0(x, y)$ and $v_0(x, y)$.
- This method suits the situation that we have multiple PDEs with similar shape.

COMSOL calculated identical results with both approaches.

Setting up Initial Conditions from Data Files

- The initial condition profiles for the excitation variable, $C_0(x, y)$, and recovery variable, $v_0(x, y)$, are provided in two separate `txt` files.
- We use **`Global Definitions`** and then **`Interpolation Function`** to create functions from our data file.
- Enter the name of the appropriate interpolation function as the initial value. at tab **`Coefficient Form PDE`**
- The benefit of using this function is our data does not have to agree with the mesh, see Fig 1.

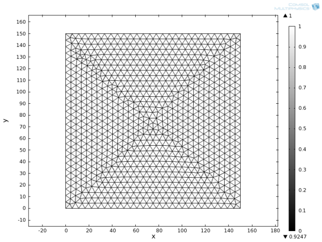
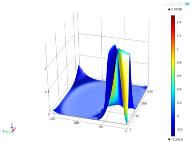
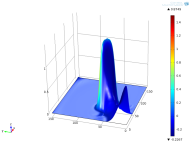


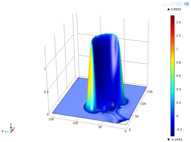
Figure 1: mesh

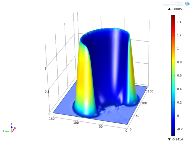
Three-dimensional view of the excitation variable C at different times

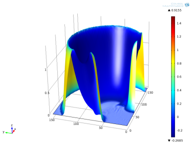
- The plots are `2D Plot Group` with `Height Expression`
- Figure shows how excitation is induced and then it proceeds into the resting part of the domain. This physiological process is characterized by the visual appearance of a double spiral wave.

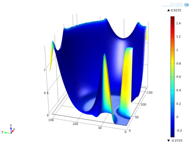

(a) $t = 0$ s

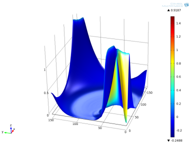

(b) $t = 50$ s

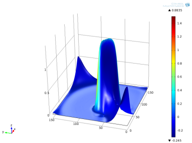

(c) $t = 100$ s


(d) $t = 150$ s


(e) $t = 200$ s

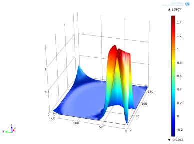

(f) $t = 250$ s


(g) $t = 300$ s

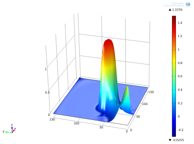

(h) $t = 350$ s

Three-dimensional view of the excitation variable v at different times

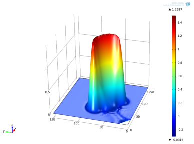
- Figure shows a recovery variable controls the local recovery of the excitation.



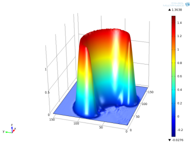
(a) $t = 0$ s



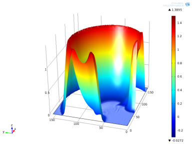
(b) $t = 50$ s



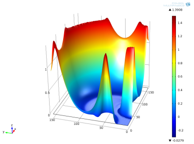
(c) $t = 100$ s



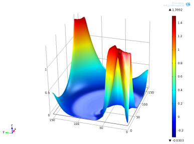
(d) $t = 150$ s



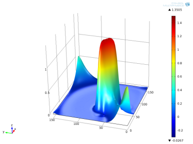
(e) $t = 200$ s



(f) $t = 250$ s



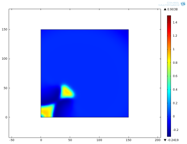
(g) $t = 300$ s

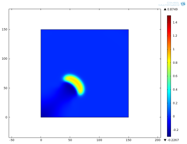


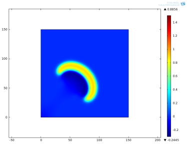
(h) $t = 350$ s

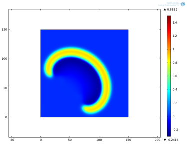
Two-dimensional view of the excitation variable C at different times

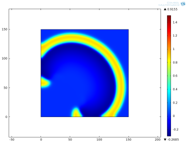
- Figure depicts the two-dimensional view of the excitation variable, where one can easily see the curl pattern.

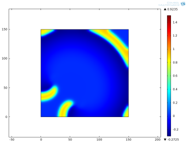

(a) $t = 0$ s

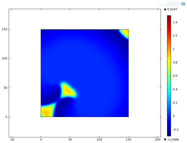

(b) $t = 50$ s

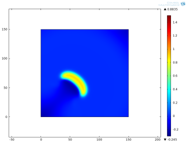

(c) $t = 100$ s


(d) $t = 150$ s


(e) $t = 200$ s


(f) $t = 250$ s


(g) $t = 300$ s


(h) $t = 350$ s

Conclusion

- We solved the coupled system of PDEs with two approaches, they gave the same results.

- COMSOL read initial conditions from Data files, then used interpolation to work with different mesh.