Coupled PDEs with Initial Solution from Data in COMSOL 4

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Problem: To illustrate how to set up coupled PDEs and how to load initial solutions from a data file, we consider an example from mathematical biology. The spread of the excitation variable $C(x,y,t)$ and the recovery variable $v(x,y,t)$ is modeled by the system of two coupled time-dependent PDEs

$$\begin{aligned} C_t - \nabla\cdot(D_{\text{eff}}\nabla C) &= C(C-\alpha)(1-C) - \beta v, \\ v_t &= \epsilon(C-\gamma v), \end{aligned}$$

where

$$D_{\text{eff}} = 1,\ \alpha = \beta = 1,\ \gamma = 0.2,\ \epsilon = 0.07$$

Cell is given as domain $\Omega = (0,150)\times(0,150)$

Method:
Two approaches used to couple the PDEs:

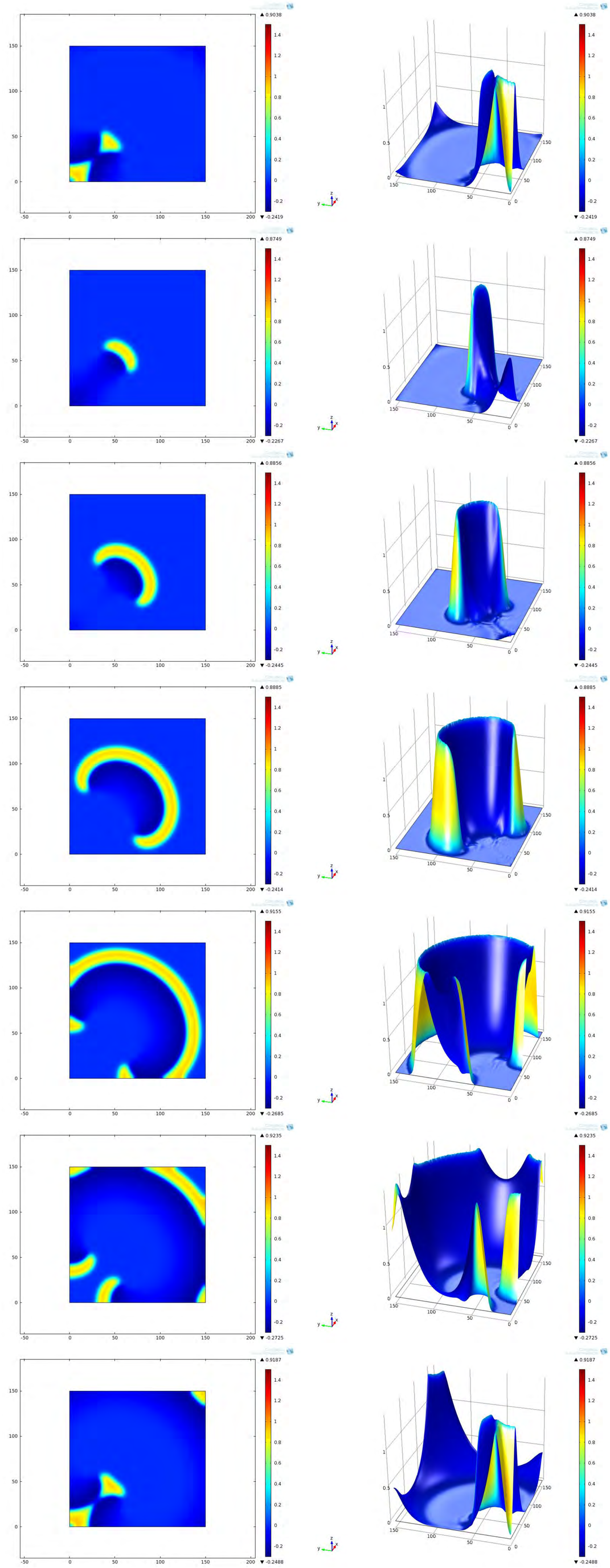
- Each PDE in its own `Physics` Model, then coupled together.
- The matrix form of coefficients couples the PDEs in one `Physics` Model.

Initial conditions from data files using COMSOL `Interpolation function`.

COMSOL calculated identical results with both approaches. The first approach is suitable for small systems, while the second one is convenient for larger systems of PDEs of the same type.

Results:

- In the initial frame of Figure 1, excitation is induced.
- Consequently, propagation of the excitation proceeds into the resting part of the domain.
- A recovery variable controls the local recovery of the excitation.
- This physiological process is characterized by a double spiral wave.
- Close resemblance of results at times t = 0 and t = 300 allows conclusion that the process has an approximate period of 300.



(a) **(b)**

Figure 1: (a) Two-dimensional and (b) three-dimensional view of *C* for *t* = 0, 50, 100, 150, 200, 250, 300

Reference: *Proceedings of the COMSOL Conference 2013*, Boston, MA.

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