Coupled PDEs with Initial Solution From Data in COMSOL Multiphysics®

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Abstract

Many physical applications require the solution of a system of coupled partial differential equations (PDEs). In most cases, the analytic PDE solution does not exist for this system and we need to solve the problem numerically using the finite element method in COMSOL Multiphysics®. This paper presents information on techniques needed in COMSOL to enable computational studies of coupled systems of PDE for time-dependent non-linear problems. To illustrate the techniques, we consider a system of two time-dependent non-linear PDEs from mathematical biology. The first PDE for excitation variable is a time-dependent reaction-diffusion equation representing the excitable dynamic, and the second PDE for the recovery variable controls the local recovery of the excitation. The second PDE does not have any diffusivity term, and the coupling term is the only reason for spatial dependence of the recovery variable. The zero-flux boundary condition is applied for the first PDE, and each of the PDEs has an initial solution profile specified in a data file.

This paper extends the step-by-step instructions in [1] for solving one stationary linear PDE to a system of time-dependent non-linear PDEs. We present two different approaches to solve coupled systems of PDEs. In the first approach, each equation in the coupled system of PDEs is modeled independently in its own Physics model, and then they are coupled together. In the second approach, the matrix form of coefficients is used to specify all PDEs of the coupled system in one physics model. Furthermore, the technique to read initial solution from a data file into COMSOL is introduced.

Figures 1, 2, 3, and 4 depict the three-dimensional view of the excitation variable at times of 0, 100, 200, and 300 seconds, respectively. In the initial frame, excitation is induced (interior peak), and simultaneously the corner of the domain is excited analogous to a burn line to thwart a forest fire in a given direction. Consequently, propagation of the excitation proceeds into the resting part of the domain. The recovery is slow, but rapid enough for the corners of the domain adjacent to the burn line to begin to curl into the newly recovered region thus beginning the sustained and doubly spiraling excitation pattern.

The two different approaches for coupling the PDEs are used to solve the coupled system of PDE and yield the same results. For smaller systems of coupled PDEs (with two or three dependent variables) or for systems that couple different types of PDEs (multiphysics), the first method is convenient to apply. However, for larger systems of PDEs of the same type, it is

suggested to use the matrix form of coefficients to represent the whole system in one physics model.

Reference

[1] David W. Trott and Matthias K. Gobbert,
Conducting Finite Element Convergence Studies Using COMSOL 4.0,
In Yeswanth Rao, editor,
Proceedings of the COMSOL Conference 2010, Boston, MA, 2010.

Figures used in the abstract

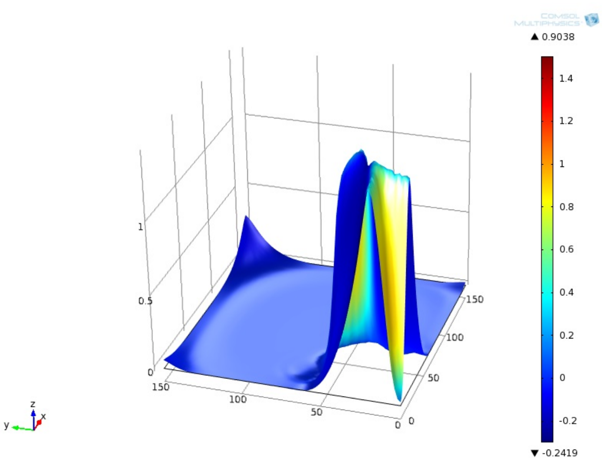

Figure 1: Excitation variable at 0 seconds.

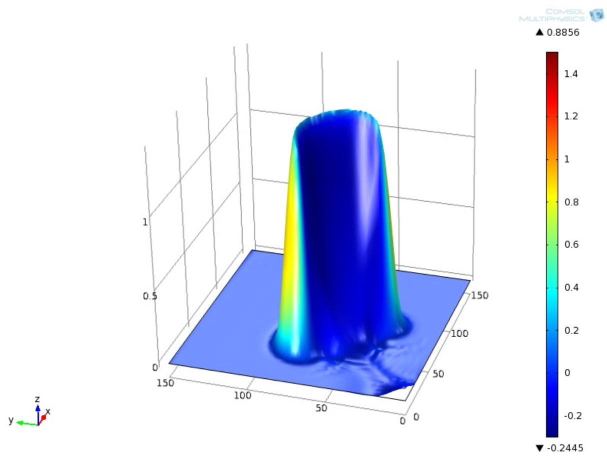

Figure 2: Excitation variable at 100 seconds.

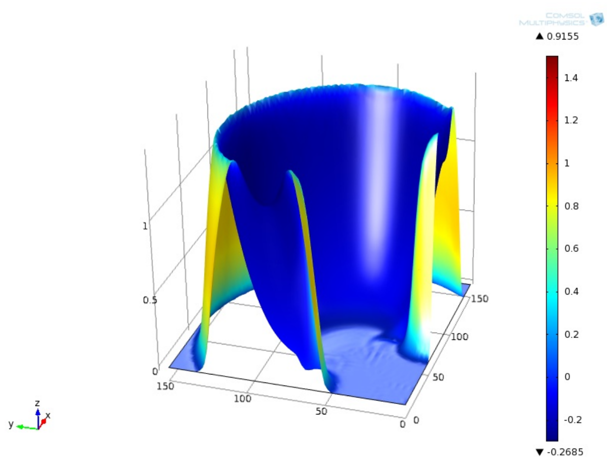

Figure 3: Excitation variable at 200 seconds.

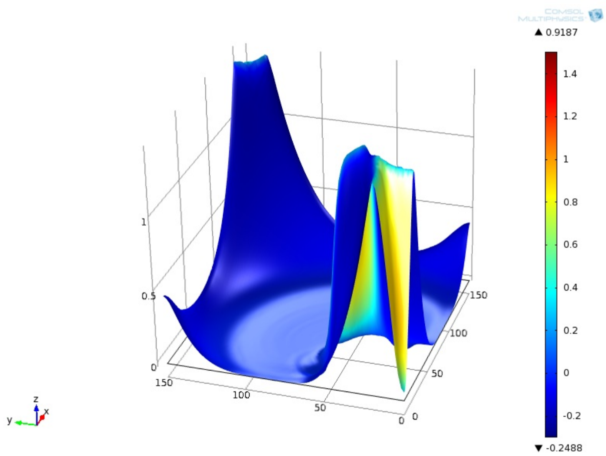

Figure 4: Excitation variable at 300 seconds.