An Innovative Solution for Water Bottling Using PET

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Abstract: We study an innovative technology for water bottling using PET. The goals are the reduction of the thickness of the plastic bottles and consequently of the amount of plastic used for a single water bottle, the reduction of the packaging costs and more environmental sustainability. We notice that the required thickness of the bottle depends on its structural function: when carried, the bottles are piled one on top of the other and, consequently, they are subject to a mechanical loading. Our innovative solution introduces in the bottle a suitable amount of pressurized inert gas in the free space over the water. The pressure of this gas partially balances the external loads, and thereby the thickness of the bottle can be reduced. Here, we study the structural behaviour of a particular thin pressurized bottle, and then analyse the effectiveness and the criticality of this lay out.

Keywords: Fluid-structure interaction, shells, moving mesh, geometric instability, bottling

1. Introduction

The aim of this work is the study an innovative solution for reducing the amount of PET needed for the production of a single water bottle. This solution must satisfy certain conditions on the geometry of the bottle coming from the structural functions of this plastic packaging.

The first condition arises from the need of no residual strains. When carried, the bottles are piled one on the top of the other, so that the resulting stress must not exceed the elastic limit of the PET, otherwise residual strains arise, and then the bottles loose their shape. This is unacceptable: besides aesthetic matter, there will be practical problems like the impossibility of standing alone on the table.

A second condition is that the thickness of the wall of the bottles must be large enough to avoid geometric instabilities of the bottle under the standard working loads (i.e., the weight of the bottles piled on his top).

The proposed idea is to partially balance the external loads with a pressurized air introduced into the bottle, in order to satisfy the former two conditions and to reduce the thickness at the same time.

An important feature of this problem is that there are three phases in mutual interaction: a solid phase (the wall of the bottle), a gas phase (the pressurized air into the bottle) and a liquid phase (the water into the bottle). When loaded, the geometry of bottle changes and consequently the volume available for the pressurized air changes. Since the amount of air in the bottle is fixed, a change of volume implies a variation of the air pressure. Mathematically speaking, the geometry of the body and the pressure of the air are function one of the other. This relationship can be described by a thermodynamic process; by assuming that the deformations are sufficiently small, in this work the isentropic process is preferred [4]. For this study the water is considered incompressible; thus, because a change in the internal pressure of the bottle does not bring a change in the volume of the water, the liquid phase does not interact with the other phases.

2. Use of COMSOL Multiphysics

The wall of bottles has a thickness of few tens of millimeters, while the overall dimensions of the bottles measures tens of centimeters. This enables for applying the theory of thin shells for modeling the structural behavior of the solid phase [3]. In the STRUCTURAL MECHANICS module of COMSOL the SHELL interface is available and it can be applied only for modeling surfaces; therefore a mesh made only of two-dimensional elements is required.

It is important to consider in the material settings geometric nonlinearities in order to enable a correct modeling of the geometric instabilities.

Since the change of the air pressure is closely related to the available volume, a suitable variable for evaluating the volume is required. In particular, we define a probe as the integral of the unit over the whole internal volume.

Obviously, a mesh of the volume is needed, but it should be very coarse because its task is only to cover the volume (no structural functions).

Since during the resolution the geometry of the body changes, the mesh of the volume must follow the displacements of the wall. In particular, we employ the MOVING MESH module to update the mesh of the volume. The displacements of the solid elements of the mesh are set equal to the displacements u, v, w of the elements of the wall.

We describe the behavior of the air inside the bottle by a relationship set in the SHELL interface. In particular, the pressure of the air (that is a function of the volume) is set as a distributed load on the internal surface of the bottle.

We choose a segregated stationary solver; thus, the unknown variables are grouped in two steps: the first step solves the variables of the SHELL interface, while the second step updates the mesh by means of the MOVING MESH module.

3. A preliminary test case

The settings described above have been tested with respect to a problem whose solution is analytically known (see [2]). A thin walled linearly elastic sphere contains a pressurized gas and it is subjected to an external distributed constant load. The sphere has a radius r_0 of 1 m and a thickness s = 1 mm. The elastic modulus of the material is E=200 GPa and the Poisson ratio is ν=0,29. If the sphere is filled with a gas whose pressure is $p_{int,1}$=6 bar the radius becomes:

$$r_1 = r_0 + \frac{p_{int,1} * r_0^2}{2Es}(1 - \upsilon) = 1{,}001065\, m$$

and the internal volume becomes V_1=4,2022 m^3. Appling an external distributed load p_e and keeping the mass of the contained air constant, the evolution of the system obeys to the following two conditions:

$$r_i = r_0 + \frac{(p_{int,i} - p_e) * r_0^2}{2Es}(1 - \upsilon)$$

$$p_{int,i} = p_{int,1} * \left(\frac{V_1}{V_i}\right)^{\gamma}$$

with γ=1.4 and V_i the initial internal volume. Setting p_e=3 bar, after some iterations one find that the gas pressure p_{int} reach the value of 601366 Pa, with internal radius of 1,000534872 m and the internal volume of 4,1955 m^3.

The same problem has been solved with COMSOL by means of a mesh made of about 37.000 tetrahedra and 7.000 triangles.

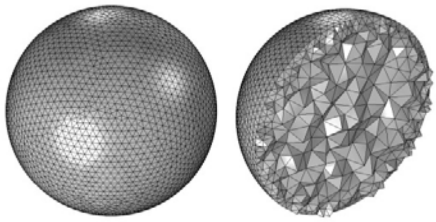

Figure 1. Triangles and tetrahedra of the mesh.

After four global iterations, the results are the following. The gas pressure is 601374 Pa; the displacements, i.e. the differences between the radius of the deformed sphere and the initial radius, range from $5{,}30 \cdot 10^{-4}$ m to $5{,}44 \cdot 10^{-4}$ m. The distribution of the displacements is shown in Figure 2.

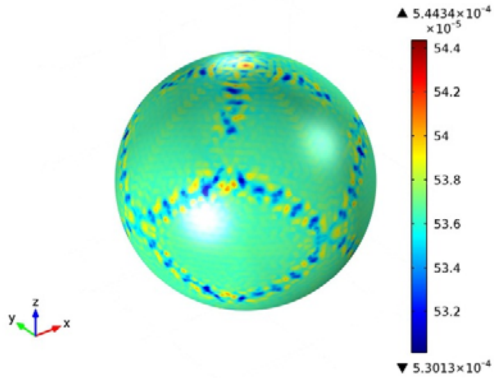

Figure 2. Total displacements [m] of the loaded sphere.

Considering that the mean value of the displacements is near $5{,}37*10^{-4}$ m, the results obtained from the numerical model show a very good agreement with the analytic solution.

4. Results

Our analysis is carried on an ordinary bottle containing 0,5 liters of plain water and having the shape shown in Figure 3 and a weight of 18,5 g. This bottle has a surface of 0,04 m^2: considering for PET a density of 1350 kg/m^3 [5], the wall has an average thickness of 0,34 mm.

The aim of the study is to verify the possibility to halve the mass of PET by introducing a suitable amount of pressurized inert gas in the packaging. Thus, we analyze the structural behavior of a bottle having the same shape, but thickness of the PET walls reduced to 0,17 mm. In particular, we consider a load of 9,4 daN applied on the top of the bottle which represents the weight of the bottles piled on the top. Since the deformations are small, we consider the PET as a linearly elastic material with elastic modulus E=2,8 GPa and Poisson ratio υ=0,33 [3].

The mesh shown in Figure 3 is made of 23.000 triangles and 76.000 tetrahedra. As explained before, the solid elements of the mesh are very coarse because their task is only to cover the internal volume of the bottle for the volume probe.

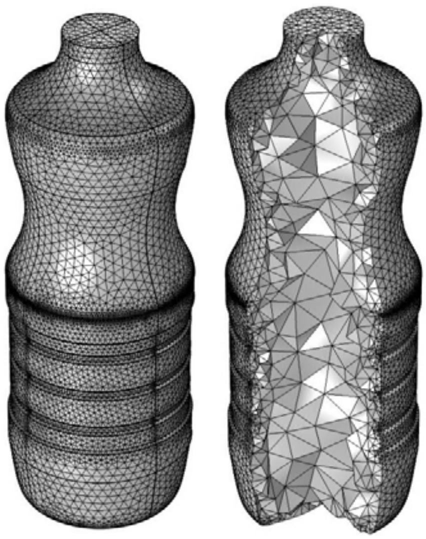

Figure 3. Triangles and tetrahedra of the mesh.

A first analysis is needed to evaluate the volume available for the gas. We perform this analysis without the external load and keeping the relative gas pressure constant and equal to $2 \cdot 10^4$ Pa. Figure 4 shows the von Mises stresses while Figure 5 shows the displacements along the axis of the bottle: the highest values of von Mises stresses are on the bottom of the bottle and are near the elastic limit of PET (55–75 MPa).

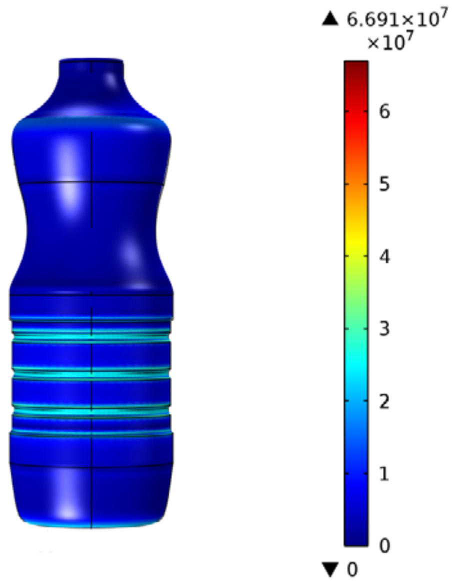

Figure 4. von Mises stresses [Pa]; analysis without the external load.

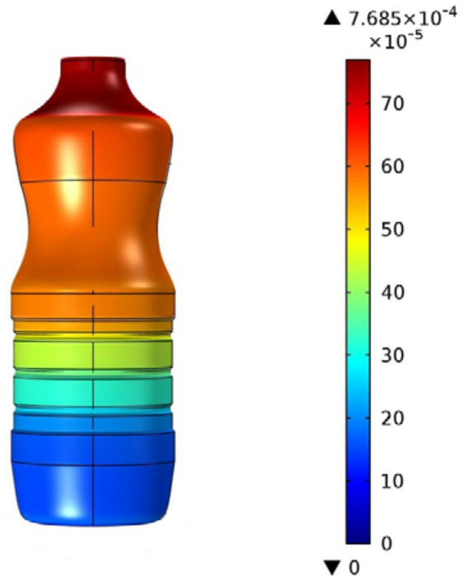

Figure 5. Displacements [m] along the axis; analysis without the external load.

The deformed body has a volume of 541,29 cm^3; thus, considering that the Italian laws require that the bottle must be filled with 520 cm^3 of water, the pressurized air takes a volume V_1=21,29 cm^3.

The second analysis is performed by considering a distributed load on the cap of 9,4 daN. Since the bottle contains a constant mass of air, the relationship between the volume V and the relative pressure is the following:

$$p = 1{,}2 * 10^5 * \left(\frac{2{,}129 * 10^{-5}}{V - 5{,}2 * 10^{-4}}\right)^{1{,}4} - 1 * 10^5$$

The solver has needed 6 global iterations to reach convergence. As shown in Figure 6, the variables of the SHELL interface exhubit errors greater than the variables of the MOVING MESH module.

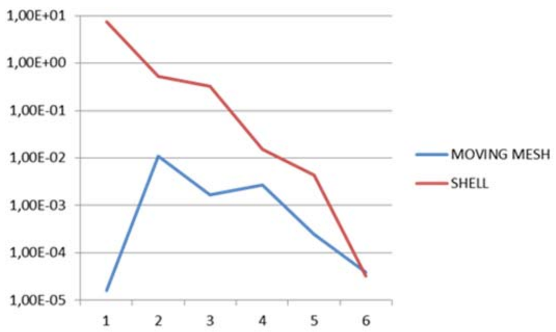

Figure 6. Histories of convergence.

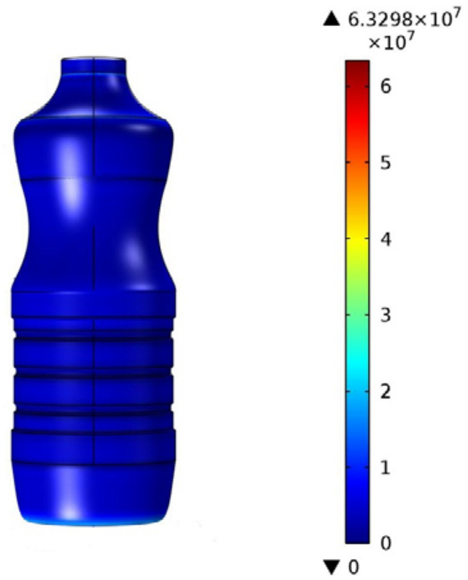

Figure 7. von Mises stresses [Pa]; analysis in presence of the external load.

The von Mises stresses and the displacements along the axis are shown in Figures 7 and 8, respectively.

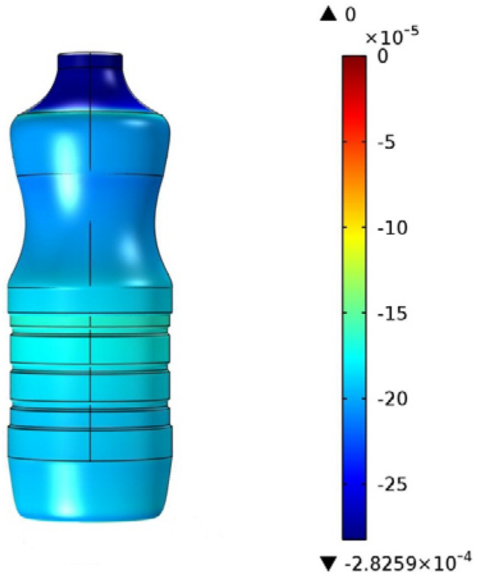

Figure 8. Displacements [m] along the axis; analysis in presence of the external load.

The highest von Mises stress (65 MPa), reached on the bottom of the bottle, is near to the elastic limit of PET (55–75 MPa); consequently, residual strains when the load is removed can be easily avoided by a more detailed design of the bottle. Obviously in this case the displacements are negative.

The last analysis aims at demonstrating the necessity of the pressurized gas in pressure inside the bottle when the thickness is reduced to 0,17 mm. In fact, if the relative pressure of the air into the bottle is set equal to zero, the bottle undergoes a form of geometric instability and consequently collapses under the external load (see Figure 9).

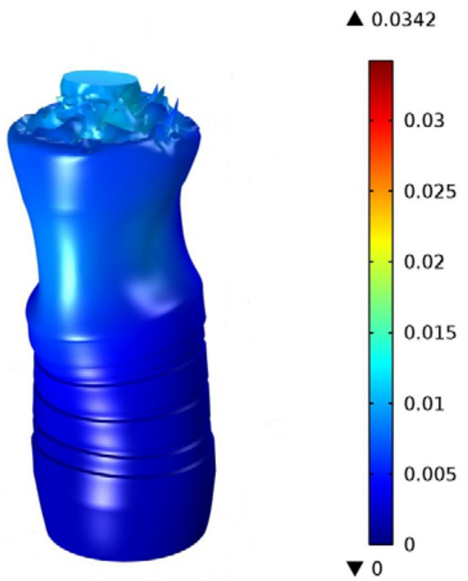

Figure 9. Displacements [m] along the axis for the collapsed bottle without pressurized gas.

5. Conclusions

An innovative technique for water bottling has been verified successfully by means of advanced multiphysics simulations for the particular case of a 0,5 l bottle. The solver combines two module of COMSOL: the SHELL interface and the MOVING MESH module. The introduction of pressurized inert gas in the packaging allows to reduce the amount of PET halving the thickness of the bottle without the occurrence of residual strains or of forms of geometric instability of the bottle. The pressurized inert gas plays an essential role: in absence of the pressurized gas the bottle becomes instable. Since the thickness and the relative pressure of the gas play the role of parameters, the proposed approach can be easily extended to a gradient-based optimization procedure in order to better improve the values of such parameters.

The stability analysis here developed may be further improved by modeling the body within the context of nonlinear elasticity theory and using some innovative procedures for studying stability issues like those developed in [7].

6. References

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