Simulation of Pumping Induced Groundwater Flow in Unconfined Aquifer Using Arbitrary Lagrangian-Eulerian Method

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Abstract: A novel numerical method characterizing groundwater flow in an unconfined aquifer is demonstrated. In contrast to the conventional method (Dupuit approach considering horizontal flow only), hydraulic head is simulated both in horizontal and vertical directions. The new approach is introduced via developing a 2-D-axisymmetric model representing the vertical cross section of the aquifer. The model solves the groundwater flow equation derived from Darcy's law and the principle of mass conservation. Meanwhile, the dynamics of hydraulic head changes is modeled using arbitrary the Lagrangian-Eulerian (*ale*) method. In a verification test the numerical results agree well with an analytical solution (Thiem equation). An application is presented, evaluating a pumping test conducted at a test site in Germany. The limitations and advantages of the model approach are also discussed in the paper.

Keywords: Unconfined aquifer, pumping test, *ale*, Darcy's Law.

1. Introduction

Extensive research has been conducted to accurately characterize unconfined aquifers in the field of groundwater hydrology. The analysis of pressure response data (i.e. changes of hydraulic head) from pumping and recovery tests is commonly used to determine aquifer properties. In the confined situation the aquifer thickness is independent from hydraulic head and remains constant under the pumping regime. In the unconfined situation, on the other hand, hydraulic head appears in two roles, which describes the thickness of aquifer and its gradient determines velocity and flow according to Darcy's Law. The difficulties for the unconfined problem lie in the non-linearity of the mathematical analysis, while it is linear for confined situation.

In the case of steady flow, we usually assume that groundwater moves horizontally and the pressure is hydrostatic (followed by Dupuit assumption, 1863). Hence, in the conventional approach, by integrating the 3-D equation over the vertical, it is sufficient to consider flow in the 2-D horizontal plane. In the case of changing head in vertical direction, one may utilize the arithmetic average over depth.

Recent literature on analyzing complex pumping induced flow processes in unconfined aquifers has demonstrated the importance of considering vertical flow especially in the vicinity of operating wells (Bevan et al 2005, Bunn et al 2011). Also, Dagan et al (2009) proved that the Dupuit approach is inadequate in terms of characterizing groundwater flow in an unconfined aquifer. Aside from these inaccuracies, the classical approach does not suffice to describe several application problems. For example, the situation of pumping and injecting in a single borehole (Jin et al, 2011) at different depths cannot be treated with the classical method, if the aquifer is not separated by impermeable layers.

We developed an innovative numerical model that takes non-zero velocity components in vertical directions into account. The model is also flexible in terms of applying complex physics. The challenge of developing such model is to couple the moving boundary with the driving physics. Comparing numerical with analytical solutions tests the reliability of the model.

Within a project, funded by Deutsche Bundesstiftung Umwelt (DBU) within the framework of DSI-project, the described model development is accompanied by ongoing field experiments. Pumping tests, described below, were performed at the Plötzin test site in Brandenburg in Germany.

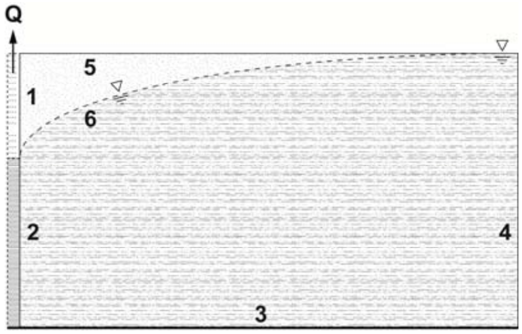

Figure 1. Sketch of pumping test concept. Q is the pumping rate and the dashed line indicates the dropped groundwater table during pumping. Labeled numbers illustrate the boundary set-up in the model.

2. Principle

The concept of classical pumping test is visualized in the sketch in Figure 1. Groundwater is pumped constantly from the fully penetrated borehole at the left site of the Figure. The groundwater table drops simultaneously until it reaches a steady state, if pumped constantly.

Figure 1 also describes the model domain with the boundary indication. The detailed boundary set-up will be illustrated in the following.

3. Equations

3.1 Stationary Solutions

In the model we solve the groundwater flow equations derived from Darcy's law and the principle of fluid mass conservation:

$$\nabla(\rho \cdot \mathbf{u}) = 0 \qquad \mathbf{u} = -\frac{k}{\mu}\nabla p \tag{1}$$

where ρ denotes fluid density [M/L^3], μ the dynamic viscosity [Pa·T], k the permeability [L^2] and p pressure [Pa]. From p as main dependent variable we calculate the velocity vector $\mathbf{u}$ and hydraulic head h.

The Thiem equation is the known analytical solution for steady-state flow towards a single perfect well in a homogeneous confined aquifer. The equation can be adapted to the unconfined situation (Holzbecher 2007). For homogenous isotropic and infinite unconfined aquifers, equation (2) delivers piezometric head $h(r)$ in the radial distance from a well:

$$h(r)^2 - h_0^2 = \frac{Q}{\pi K}\ln(\frac{r}{r_0}) \tag{2}$$

Q is the pumping rate [L^3/T], K is the aquifer hydraulic conductivity [L/T], r_0 is the distance from unchanged head point to pumped well [L] and h_0 is the unchanged head at r_0 [L]

3.2 Time Dependent Solution

Under unsteady conditions an additional physical property, storage, is used to characterize the capacity of an aquifer to release groundwater. The storage coefficient is interpreted as a compressibility of the aquifer material and the fluid in the pores. Equation (3) is solved respecting time and aquifer storage can be determined additionally,

$$\rho S(\frac{\partial p}{\partial t}) + \nabla(\rho \cdot \mathbf{u}) = Q_m \tag{3}$$

where S is storage coefficient [1/Pa], ρ is fluid density [M/L^3], p is pressure [Pa], t is time [T] Q_m is mass source term [M/T] and u the velocity vector from equation (1)

In our model, storage coefficient S is defined as linearized storage using the compressibility of fluid and solid in equation (4),

$$S = \varepsilon_p \chi_f + (1-\varepsilon_p)\chi_p \tag{4}$$

where χ_f is the compressibility of fluid [1/Pa], χ_p is the compressibility of aquifer material [1/Pa] and ε_p is the porosity [1].

4. Use of COMSOL Multiphysics

COMSOL Multiphysics 4.3 is used to set up the model. The main objectives of the modeling are 1) to test the novel modeling approach by comparing the numerical model results with analytical solution (steady-state) and 2) to implement the model to evaluate a pumping test conducted in field (unsteady-state).

Two modules, Darcy's Law (*dl*) and Moving Mesh (*ale*), are used in the model. Darcy's module uses pressure p as dependent variable and determines the hydraulic head in time and space. The location of the groundwater table is considered as free boundary in the model. The arbitrary Lagrangian-Eulerian (ALE) method is applied in order to compute the movement of the free boundary. These two modules are coupled by taking the deformed mesh as a part of the solution procedure in Darcy's Law. Within the ALE module, we use Laplace smoothing throughout:

$$\frac{\partial^2 R}{\partial r^2}+\frac{\partial^2 R}{\partial z^2}=0 \qquad \frac{\partial^2 Z}{\partial r^2}+\frac{\partial^2 Z}{\partial z^2}=0 \tag{5}$$

4.1 Model Region and Meshing

We present the novel approach through a 2D axisymmetric model, representing a vertical cross section of the aquifer. We consider an aquifer plane with 20m depth that extended 500m long from the pumping well. Note that the analytical solution assumes that aquifer is infinitively extended in the horizontal direction. Numerically, however, the model region has to be limited to a certain length, which should be significant enough to avoid any boundary influence. Here, we assume that groundwater table is not influenced by pumping at 500m distance away from the well. In the model, we simulate flow to a fully penetrated borehole in a homogenous, isotropic aquifer for steady state and unsteady state respectively.

Triangular (2D) fine meshes were used by default. Further meshing was made with a drastic refinement along the free boundary (top boundary with finer mesh) and especially in the vicinity of the borehole (see Figure 2). In the figure, the material frame (solid rectangle) represents the non-deformed initial condition. And the upper limit of the triangular mesh depicts the moved groundwater table at stationary state.

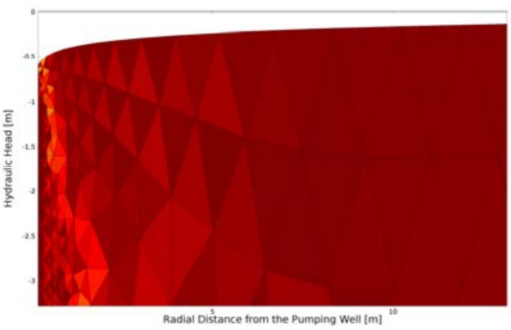

Figure 2. Finite element mesh of part of the model region with refined well at the left side of the vertical cross section. The deformation of the mesh with hydraulic head change is also presented

4.2 Boundary Condition

The boundaries as well as the model domain are shown in Figure 1. In the model, we only concern the saturated part of the aquifer, which has the groundwater table as upper (free) boundary. In the 2D-axisymmetric model, polar coordinates (r, z) are applied and z = 0 is set for the initial water table (boundary 5). Note here we do not consider recharge and/or discharge along the boundary, which means evaporation, precipitation as well as further influence of unsaturated zone are neglected.

The model region extends negatively that leads z = -D at the bottom (D is the aquifer depth [L]). The lower boundary is considered as the impermeable aquifer base, where no flow is prescribed accordingly (boundary 3).

The borehole itself is not considered in the model. Hence, we take the well radius ($r = r_{well}$) as horizontal basis at the left site (boundaries 1&2). Water abstraction is implemented via a mass flux (j) condition (see Equation 6) along the borehole with a constant pumping rate Q (positive for pumping). The negative sign indicates the direction of water flow at the boundary (outflow in the model).

$$j\left[{}^{M}\!\big/_{L^2\cdot T}\right]=-\frac{Q}{2\pi r_{well}\left|D-d\right|}\rho \tag{6}$$

where r_{well} is the well radius [L], D is the aquifer depth (which here equals with well screen length), and d (absolute value) is the distance of the deformed free boundary position relative to its initial position (also known as drawdown).

Zero hydraulic pressure condition across the model edges was specified at outer boundary for pressure constrains (boundary 4).

In ALE mode, the whole domain is free to deform. Since we assume that outer boundary is significantly far enough that the groundwater table is not influenced by pumping, we apply zero displacement accordingly (boundary 4). The same condition is enforced at the lower boundary for aquifer bottom. The move of the meshes are mainly described at the upper boundary (boundary 5), where the groundwater table moves simultaneously to force the total pressure to match the atmospheric pressure along the boundary. Also, meshes along the borehole and above the pump (boundary 1) are allowed to move respectively, but only in vertical direction.

4.3 Parameters

Table 1: Input parameter values for the model set-up

Parameters		
Name	*Value*	*Unit*
Hydraulic conductivity (K)	$1\cdot10^{-3}$	m/s
Pumping rate (Q)	60	m^3/h
Porosity (ε_p)	0.2	–
Well Radius (r_{well})	0.1	m
Aquifer Depth (D)	20	m
Aquifer Length (W)	500	m

Table 1 lists the reference parameters applied in the stationary model. The reference parameters are oriented respecting the field condition of Plötzin test site. The aquifer depth is determined through the direct push test. The stationary model aims to compare the model result with Thiem solution when the same parameters are applied. Since sand and gravels are identified as the dominating aquifer material in our test aquifer, typical values of hydraulic conductivity and porosity for sandy aquifer are used in the model.

5. Results and Discussion

5.1 Comparison with Analytical Solution

Figure 3 delivers the comparison of numerical result simulated in the model and the corresponding analytical solution (Thiem solution) by plotting hydraulic heads versus radial distance from the well. Although a large area (500m from the well) is considered in the model, we only present the enlarged results near the well.

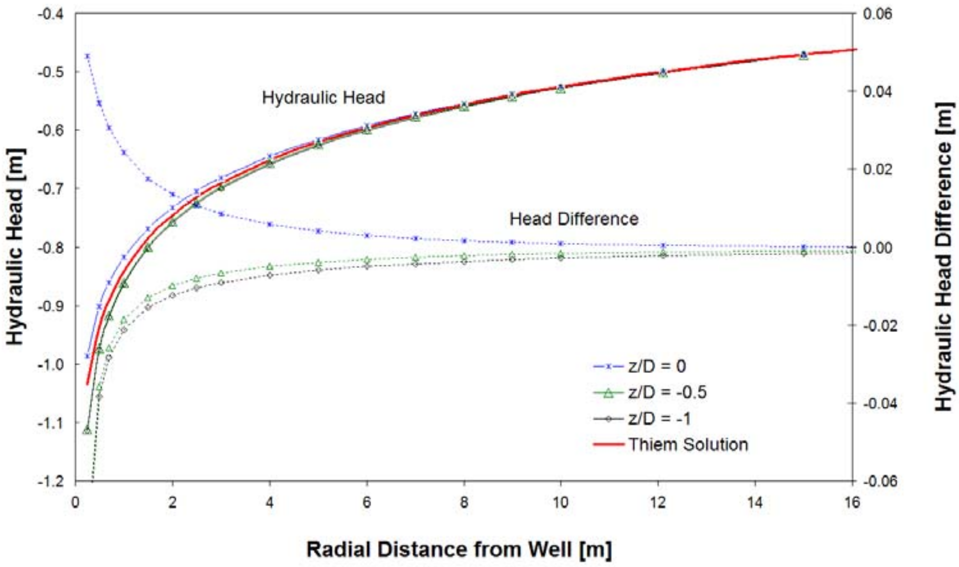

Figure 3. Comparison of numerical simulation and analytical solution. Numerical solution is obtained at three different depths (z/D =0 is at groundwater table, z/D = -0.5 and z/D = -1 represent the middle and bottom of the aquifer).

We present the typical hydraulic heads obtained at three different depths, which are at the groundwater table (z/D =0), at the middle and bottom of the aquifer (z/D = -0.5, z/D = -1 respectively). The hydraulic heads differences at various depths are also depicted in the Figure by subtracting the analytical results from the numerically simulated heads. The result shows lower estimation of hydraulic head at the groundwater table, while it is overestimated at the bottom of the aquifer. The mean hydraulic head over depth delivered by the Thiem solution matches best with the heads obtained at the middle depth of the aquifer. Moreover, it can be recognized in the graph, that the results from the numerical simulations coincide very well with the Thiem solution. The deviations are highest in the direct vicinity of the pumping well and decrease quickly in the far field.

As a result of numerical simulation, hydraulic head decreases exponentially from the groundwater table to the bottom of the aquifer in the vicinity of the well. The average of the heads at direct vicinity of the well (r = 0.1m) over depth is -1.163m, which matches excellent with the head delivered via Thiem solution at the corresponding distance from the borehole.

5.2 Field Test Evaluation

A 7 hours long pumping test was conducted at Plötzin. The lowering of the groundwater table was around 1.5m. Groundwater was pumped from the 20m deep borehole and the responses were measured in 18 observation wells in different distances and screening depths respectively. The borehole set up for the field test is depicted in Figure 4. At each location, drawdown was measured at two different depths, which are 6m and 8m deep from the ground.

The pumping test is evaluated using the demonstrated model set up in COMSOL Multiphysics. Aquifer parameters, such as hydraulic conductivity, porosity, storativity are calibrated via fitting the modeled data to the field observations.

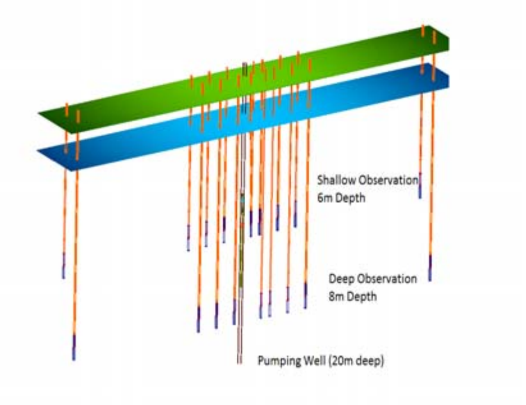

Figure 4. Borehole and observation well set up at Plötzin test site in Germany

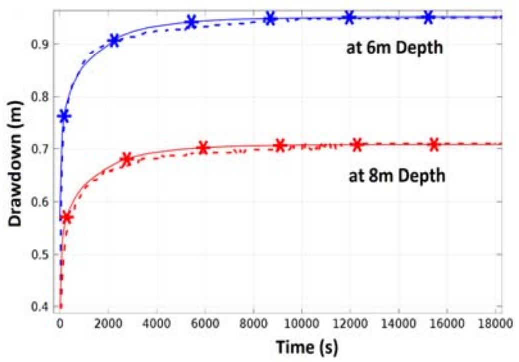

Figure 5. Comparison of observation data (dashed line) and simulation results (solid line). Observation is taken at 1m away from the pumping well and at 6m and 8m depths respectively.

The best fit is observed when the following aquifer parameter values are applied in the model: hydraulic conductivity of 1.735×10^{-3} m/s, porosity of 0.2, and storage coefficient of 0.8×10^{-8} 1/Pa. These values are within the expected range for sandy aquifer properties.

As one of the example results, we compare the simulated result with the field data recorded at closest (1m) distance from the pumping well (see Figure.5).

In the model, we usually set the initial head as zero and apply negative mass flux at the boundary for pumping condition. Hence, negative values of hydraulic heads are usually delivered as results. In groundwater hydrology, however, drawdown (positive) is normally used for describing the groundwater response when the borehole is pumped. Therefore, hydraulic heads subtracted from zero are plotted as finial result at the *y*-axis in the figure, which equals with drawdown.

The observed different pressure responses in depths at the same distance from the well prove that Dupuit approach is not suitable in terms of describing the groundwater flow especially in the vicinity of the pumping well. The vertical head variation decreases with the increasing distance and finally vanishes after 3m away from the well. Figure 5 also indicates that the vertical variation of the hydraulic heads can be evaluated through our model approach using COMSOL Multiphysics.

6. Conclusions

In the paper, we demonstrated a novel numerical method and studied the applicability of the concept through setting up models using COMSOL Multiphysics. The good agreement of the model results with the analytical solution shows that the moving mesh method works well simulating groundwater flow in an unconfined aquifer. The advantage of the method, with regard to the flexibility of coupling with other physical processes and of implementing complex boundary conditions, makes it a promising tool for application in groundwater technology. For example, complex problems, such as partial penetration of the well, heterogeneous and anisotropic aquifer formations, can be treated by the described modeling approach.

Using the described method, in previous work, we have already demonstrated the applicability of the method for a problem concerned with pumping and injecting within one borehole (for details see Hand 2012). The verification of the used numerical approach as shown here, gives indications for the model reliability in more complex situations, to be treated in future work.

The biggest limitation and difficulty of the model is the choice of the model region in order to avoid the influence of the outer boundary. The pressure constraint is required to set far enough from the well, which demands a large model region. On the other hand, larger model regions require bigger meshes, which may exceed the computational resources.

7. References

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