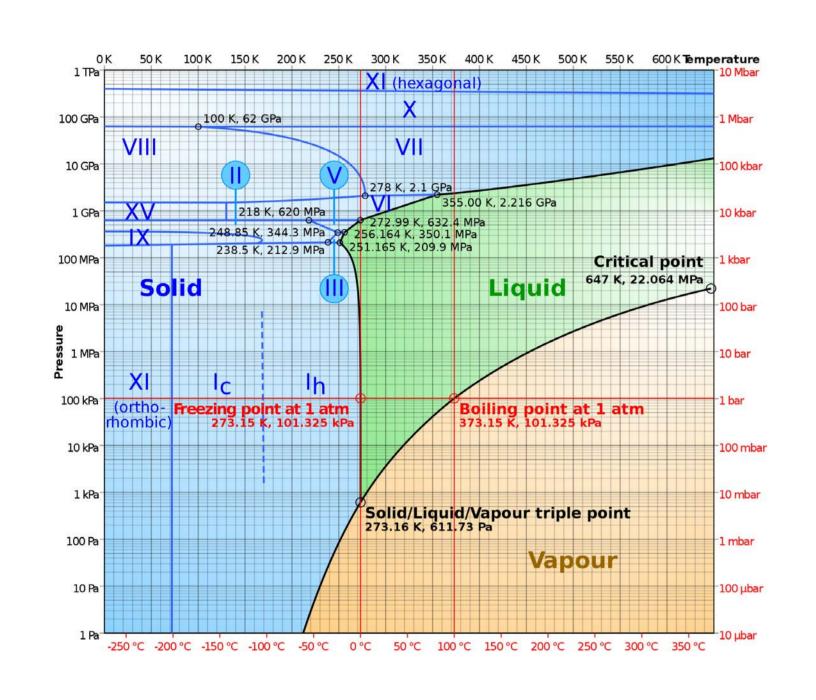
A Numerical Comparision of Dielectric based Measurement of Atmospheric Ice Using Comsol

Umair N. Mughal, Muhammad S. Virk & Mohamad Y. Mustafa Narvik University College, Department of Technology, 8505, Narvik, Norway

Introduction: Atmospheric ice is a very complex material with varying electrical properties due to different polymorphs of ice itself. Also if the medium is considered to be snow, then density becomes an additional parameter because it is a mixture of three dielectrics water, ice and air.



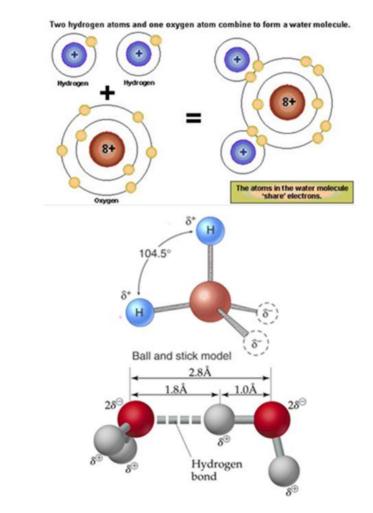


Figure 2. Water Molecule

Figure 1. Phase Diagram of Water

Mathematical Relations To measure dielectric properties of ice as a function of frequency we have used,

$$\varepsilon_{r_{ice}}^{'} = \varepsilon_{r\infty} + \frac{\varepsilon_{rs} + \varepsilon_{r\infty}}{1 + \left(\omega \tau_{h} e^{\frac{H}{kT}}\right)^{2}}$$

$$\varepsilon_{r_{ice}}^{''} = \frac{\lambda(\varepsilon_{rs} - \varepsilon_{r\infty})}{\left(\omega \tau_{h} e^{\frac{H}{kT}}\right)^{\lambda} + \left(\omega \tau_{h} e^{\frac{H}{kT}}\right)^{-\lambda}} + \frac{\sigma(0) + A\omega^{n}}{\omega \varepsilon_{0}}$$

And to measure dielectric properties of dry snow we have the mixing formula,

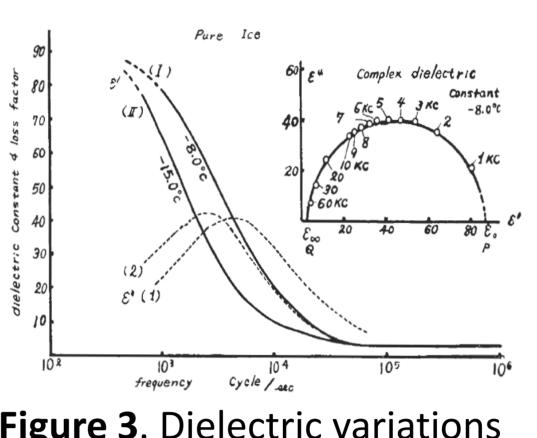
$$\frac{\varepsilon_m - 1}{\varepsilon_m + u} = (\rho) \frac{\varepsilon_1 - 1}{\varepsilon_1 + u} + (1 - \rho) \frac{\varepsilon_2 - 1}{\varepsilon_2 + u}$$

From which we have derived,

$$\varepsilon_{d}^{'} = \frac{\left(\rho u - u - \rho u \varepsilon_{ice}^{'} - \varepsilon_{ice}^{'}\right) \left(\rho \varepsilon_{ice}^{'} - \rho - u - \varepsilon_{ice}^{'}\right) - \left(\rho \varepsilon_{ice}^{''} - \varepsilon_{ice}^{''}\right) \left(\rho u \varepsilon_{ice}^{''} + \varepsilon_{ice}^{''}\right)}{\left(\rho \varepsilon_{ice}^{'} - \rho - u - \varepsilon_{ice}^{'}\right)^{2} + \left(\rho \varepsilon_{ice}^{''} + \varepsilon_{ice}^{''}\right)^{2}}$$

$$\varepsilon_{d}^{''} = \frac{-\left(\rho u - u - \rho u \varepsilon_{ice}^{'} - \varepsilon_{ice}^{'}\right) \left(\rho \varepsilon_{ice}^{''} - \varepsilon_{ice}^{''}\right) - \left(\rho \varepsilon_{ice}^{'} - \rho - u - \varepsilon_{ice}^{'}\right) \left(\rho u \varepsilon_{ice}^{''} + \varepsilon_{ice}^{''}\right)}{\left(\rho \varepsilon_{ice}^{'} - \rho - u - \varepsilon_{ice}^{'}\right)^{2} + \left(\rho \varepsilon_{ice}^{''} - \varepsilon_{ice}^{''}\right)^{2}}$$

Experimental Results: Kuroiwa [1], Cumming W. A. [2] and many other experimentally determined the dielectric properties of ice (Fig 1) and snow (Fig 2) which are in good agreement with each other.



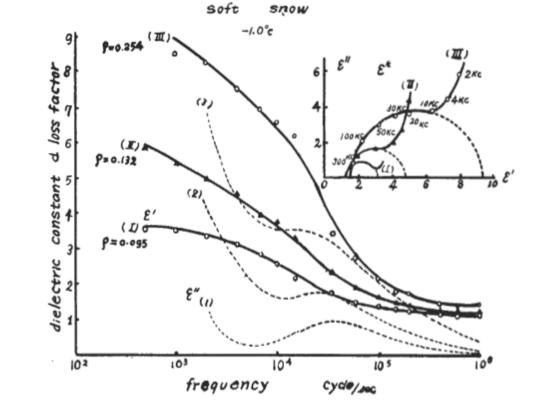


Figure 3. Dielectric variations in ice

Figure 4. Dielectric variations in dry snow

Simulation Results:

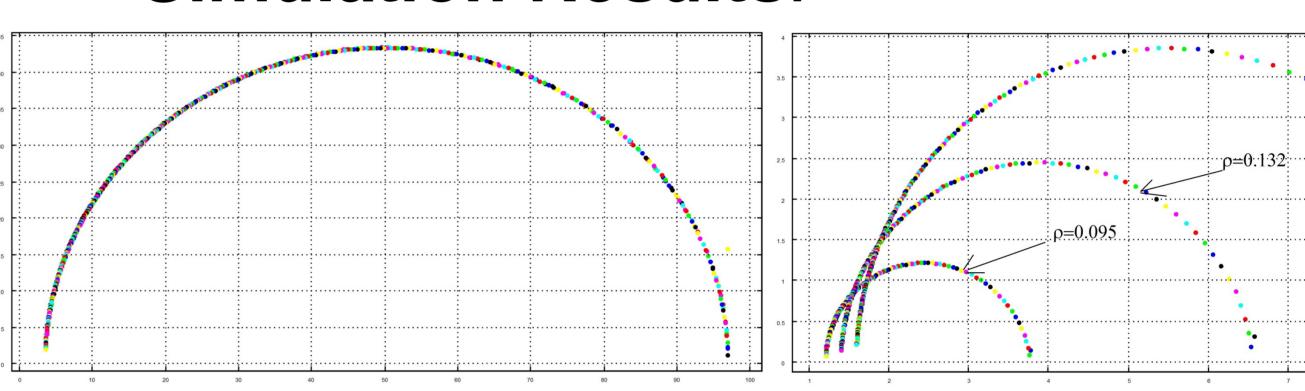
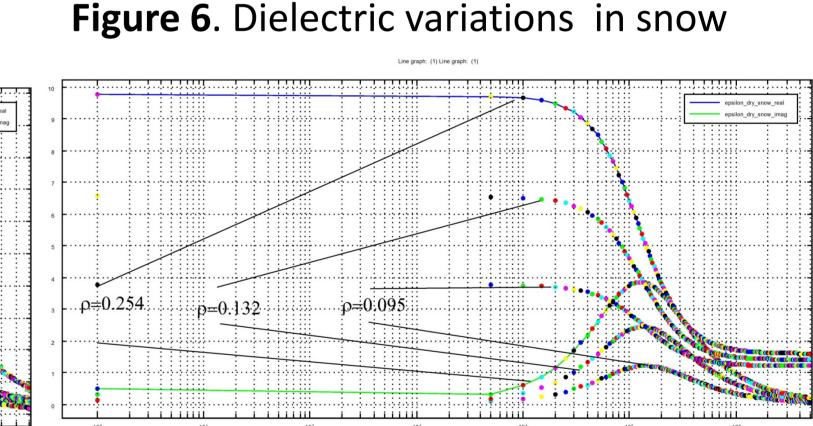
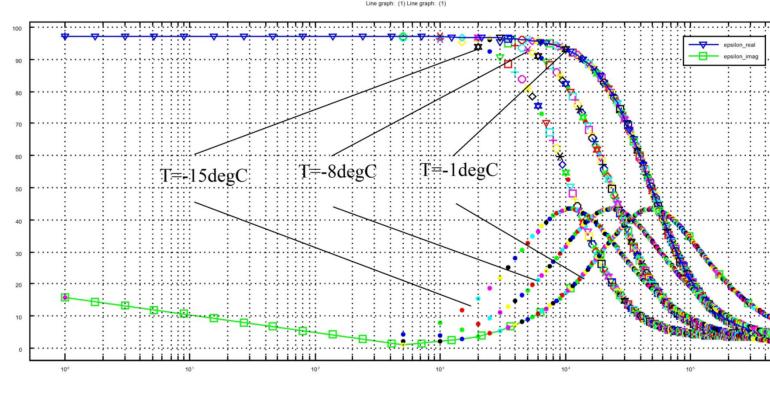


Figure 5. Dielectric variations in ice





Conclusions: As it can be seen that fig. 5 is quite similar with fig. 3 and fig. 6 with fig. 4. The small deviations between the two are because conductivity is not a function of temperature.

References:

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- 4. Kuroiwa, D., *The dielectric property of snow*1954: International Association of Scientific Hydrology.