

Application of Hot Wire Technique and Comsol Multiphysics in the Heat Transfer Monitoring in Solids.

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Abstract: We applied the Hot Wire (HW) technique and Comsol Multiphysics software to study the heat transfer of a homogeneous and isotropic solid material. The HW technique is based on the application of a linear power density modulated by a rectangular pulse of heat, in a specific time period. The power density is applied over the symmetry axis of the sample to produce a radial heat flux and then, to investigate the thermal response of the sample. In this work, the solutions of the transient heat transport equation by means of the finite element technique with appropriate boundary conditions are reported and compared with the conventional theoretical model.

Keywords: Heat Transfer, Hot Wire, Conventional theoretical model.

1. Introduction

Heat transfer is the area which describes the energy transport between material bodies due to a difference in temperature, and its development and applications is of fundamental importance in many branches of engineering since provides economical and efficient solutions for critical problems encountered in many engineering items of equipment. Among the parameters that determine the thermal behavior of a material, the thermal conductivity is especially important because it represents the ability of a material to transfer heat, and it is one of the physical quantities whose measurement is very difficult and it requires high precision in the determination of the parameters involved in its calculations [1, 2].

The hot wire technique is an absolute, non-steady state and direct method which is considered an effective and accurate procedure to determining the thermal conductivity of a variety of materials, including ceramics, fluids, food and polymers [3-6]. However, this technique is based in a conventional

mathematical model which is an approximation of the physical reality in the experimental setup because the complexity of the mathematical problem has been an obstacle to obtain a more realistic theoretical model [7, 8]. Fortunately, nowadays the development of the advanced numerical methods and computing systems allow the application of high level software for obtain an approximate solution to a complex mathematical problem with a boundary conditions congruent with the physical reality. In particular, Comsol Multiphysics is a powerful Finite Element (FEM) Partial Differential Equation (PDE) solution engine [9] useful to obtain a numerical solution in complex problems.

In this work, the Comsol Multiphysics software is use to determinate the numerical solution of a transient temperature distribution in a sample measured by the hot wire technique configuration

2. Hot Wire Method

Typically a probe for this measurement consists of a needle with a heater and temperature sensor inside. A rectangular linear power density is passed through the heater and the temperature of the probe is monitored over a specific period of time. An analysis of the transient temperature response is used to investigate the thermal response of the sample.

In the conventional mathematical model the hot wire is assumed to be an ideal infinitely thin and long heat source which is immersed in an infinite surrounding material. By means of the solution of the heat conduction equation, in cylindrical coordinates, can be obtained the temperature distribution divided in two stages [7]:

a) The heating stage: $t \in (0, t_h]$

$$T_{hs}(t, r) = -\frac{Q_0}{4\pi k} Ei\left(-\frac{r^2}{4\alpha t}\right)$$

b) The cooling stage: $t \in (t_h, \infty)$

$$T_{cs}(t, r) = -\frac{Q_0}{4\pi k} \left[-Ei\left(-\frac{r^2}{4\alpha t}\right) + Ei\left(-\frac{r^2}{4\alpha(t-t_h)}\right) \right]$$

Here, r is the radial distance to the lineal source, k is the thermal conductivity, α is the thermal diffusivity, Q_0 is the linear heat source, t_h is the heating time, and Ei is the exponential integral [10]. By means of the fitting the expression of T_{hs} (heating stage) or T_{cs} (cooling stage) to the experimental data, we can obtain de thermal parameters of the sample. The black curve in Figure 1 shows the graph of T_{hs} , and, T_{cs} of the theoretical model, in function of t for $r = a$ (fixed). The radius of the needle probe is $a = 0.6$ mm

3. Comsol Multiphysics Solution

It was apply Comsol Multiphysics software with the purpose to solve the heat transport equations for the configuration of hot wire technique with boundary conditions corresponding to the physical reality.

Geometry

In the software Graphic user interface (GUI), it was built a 2D hollow cylinder of radius a and b , which represent the sample-probe arrangement. The hollow cylinder gives certain advantages in the numerical calculation process, one of these advantages is to reduce complexity in the model and the other is to simplify the boundary regions, see Figure 1.

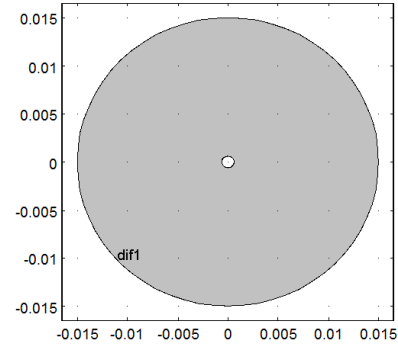


Figure 1. 2D Geometry.

Physics

Once established the geometrical aspects, the physical process that take place inside the sample need to be programmed. It was considered the heat transfer module of the Comsol software to reproduce the temperature behavior. The heat transfer module use the heat diffusion equation (HDE), given by:

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p u_{trans} \nabla T = \nabla \cdot (k \nabla T) + Q + W_p$$

In the simulation setup, only the Laplacian term and the time dependent contribution are needed, therefore the HDE is reduced to:

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T)$$

The boundary conditions and initial values proposed are given as follow:

For $r = a$, the heat flux is represented by a rectangular, or boxcar function, Figure 2:

$$Q(t) = \begin{cases} 1 & ; 0 \leq t \leq 30s \\ 0 & ; \text{elsewhere} \end{cases}$$

Or, in logical form as:

$$Q(t) = Q(0 \leq t) - Q(t \geq 30)$$

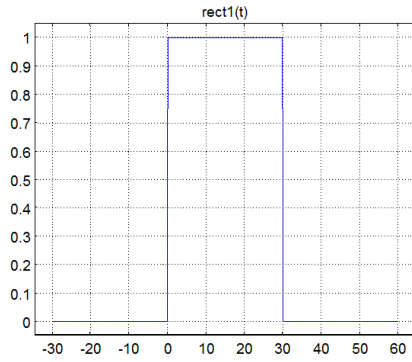


Figure 2. Rectangular function.

For the boundary $r = b$,

$$T(r = b, t) = T_{amb}$$

Where $T_{amb} = 294.15K$

And with the next initial conditions:

$$T_{ini} = 293.15K,$$

$$t = 0$$

Many simulations were realised with different boundary conditions, materials and initial values, but for lack of space, only the glycerine's (CAS 56-81-5) temperature behaviour with the described boundary and initial values are reported. Table 1 described the thermophysical properties of glycerine.

Table 1. Thermophysical properties of glycerin

Property	Magnitude	Unit
ρ	1261	Kg/m ³
k	0.285	W/mK
c	2470	J/KgK

Meshing

In order to obtain the most reliable solution for the problem, the mesh used in the simulation process was the free triangular elements with the element size parameters described in Table 2, and a free distribution of the element in all the geometry space, showed in Figure 3.

Table 2. Element size parameters.

Description	Magnitude	Unit
Maximum element size	0.0016	m
Minimum element size	$9 E - 6$	m
Maximum element growth rate	1.3	----
Resolution of curvature	0.3	----
Resolution of narrow regions	1	----

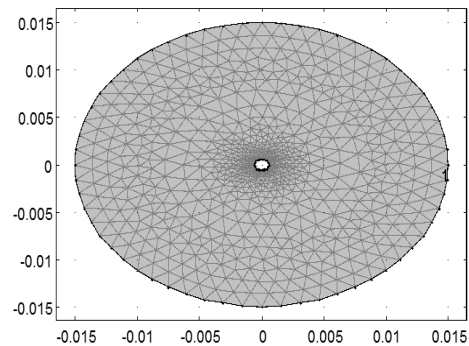


Figure 3. Free Triangular Meshing.

Solver

A direct solver with a constant damping method and a jacobian updating once per time step was used in order to obtain the solution of the hot wire simulation.

The time vector it was introduced for the transient response, in the close interval [0,60s], with increments of 0.1 seconds.

The resultant surface, with a rainbow representation, gives the radial heat distribution in the sample, Figures 4 and 5.

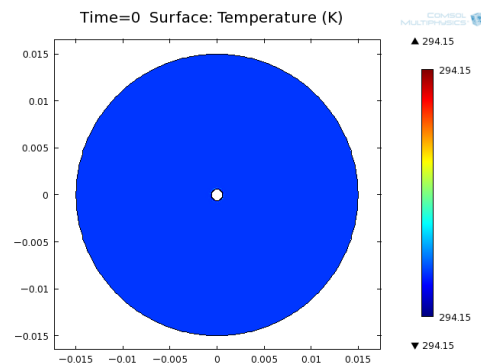


Figure 4. Temperature Field evolution for time.

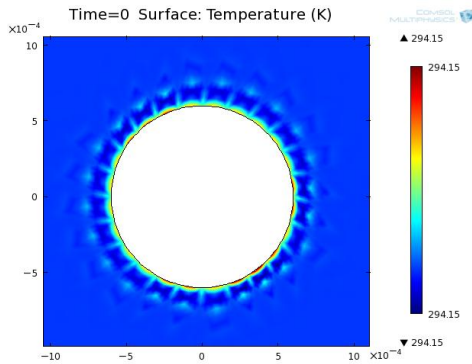


Figure 5. Temperature Field evolution for time, Neumann Boundary condition.

4. Results and Discussion

Figure 6 show the theoretical (red curve) and the simulated (black curve) results for the temperature difference versus time. It observed a significant difference between these results, which is better appreciated in Figure 7, where is showed the difference between the two curves of Figure 6.

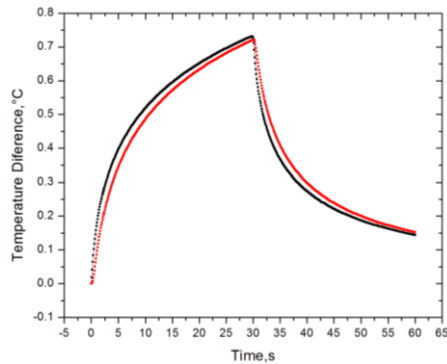


Figure 6. Comparison of conventional method Vs Comsol Simulation.

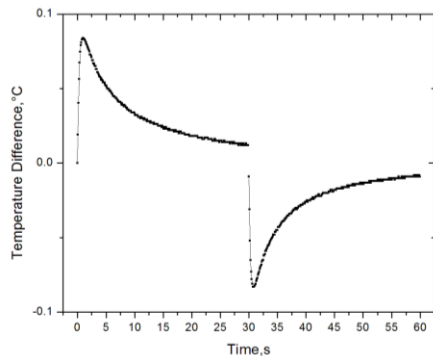


Figure 7. Differential Temperature between the theoretical and simulated results.

These results show that the Comsol simulation is an important tool for investigate the heat transfer in solids that can be used like complement in the application of the hot wire technique.

5. Conclusions

It was obtained that the finite element simulation has a remarkable reproducibility with respect to the conventional theoretical model. Besides this type of analysis provides a substantial advantage, being able to vary the different parameters of the experiment, such as the radial distance, the heat flux, the initial temperature, among others, and thereby optimize the results.

6. References

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8. Nomenclature and Physical Data

Table 1: Global Parameters and Functions.

Name	Expression	Description
r	6.0e-4 [m]	Radius of the needle
R	0.015 [m]	Radius of the material sample
Iog	$(0.8[\text{W/m}] * L) / A$ [W/m ²]	Heat Flow
L	0.028 [m]	Needle Length
A	1.056e-4[m ²]	Area of the Needle
Rect1	Lower Limit = 0 Upper Limit = 30	Pulse Definition
Pw1	Argument: t Extrapolation: Constant Smoothing : Continuous function Size of transition: 0.01	Other pulse definition manner