Irrotational Motion of an Incompressible Fluid Past a Wing Section in an Unbounded Region

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Abstract

Developers of numerical models who address the title problem (Figure 1) face several hurdles, such as: (1) the need to formulate boundary conditions applicable in an unbounded region; (2) The need to specify conditions suitable to ensure a unique solution in a doubly connected region; and (3) The need to allow the interior boundary to have a sharp edge, such as a cusp. The aim of the work reported herein is to build a COMSOL model tree that addresses these challenges with as few additional complications as possible and to compare the numerical results with the corresponding results of an analytical solution, specifically one associated with the names of Kutta and Zhukovski, as described, for example, in Chapter 10 of the book by Lighthill,1986. Idealizations suited to this aim are: (1) Restriction to two-dimensions; (2) Exclusion of time dependencies; and (3) Exclusion of the effects of boundary-layers and wakes associated with enforcement of the no-slip boundary condition of a viscous fluid. The present work employs equation-based modeling in COMSOL Multiphysics. COMSOL's Weak Form PDE physics interface proves to be convenient and adequate for the present purpose. From the stated assumptions one may show that the cartesian velocity components (u,v) each satisfy Laplace's equation (cf. Section 33 of Lamb 1932). In the analytical theory of Zhukovski airfoils one employs a change of complex position coordinate (the Zhukovski transformation) that maps points in the unbounded exterior of a circle to points in the unbounded exterior of an airfoil. If one applies this transformation in the opposite direction one obtains two roots of a quadratic equation, one of which takes points in the unbounded exterior of the airfoil to points in the unbounded exterior of the circle and the other of which takes points in the unbounded exterior of the airfoil to points in a bounded region interior to a smaller circle. The present work employs the latter root to map the whole exterior of the airfoil maps to a bounded region suitable for meshing. By appeal to a theorem of complex variables one may show that the velocity components (u,v) satisfy Laplace's equation with respect to both the old and the new independent variables. The root model described herein solves the weak-form equations simultaneously in two adjoining sub-regions of the transformed geometry. Model-coupling operators of Identity Mapping type prove convenient as a means of exchanging data at the interface between these sub-regions. A third geometry, needed for plotting, represents the bounded neighborhood of the airfoil in the original variables. The results include those in Figure 1 as well as plots of the non-dimensional slip velocity, q/U, (Figure 2) over the upper and lower surfaces both from the COMSOL model and from the corresponding analytical solution. COMSOL Multiphysics does not support infinite elements in all of its modules. The model tree employed herein will be of use to intermediate-level users who encounter the need to build one.

Reference

1. James Lighthill, An Informal Introduction to Theoretical Fluid Mechanics. Oxford University Press, 1986
2. Sir Horace Lamb, Hydrodynamics, Cambridge University Press, Sixth edition, 1932. See also Dover reprint 1945.

Figures used in the abstract

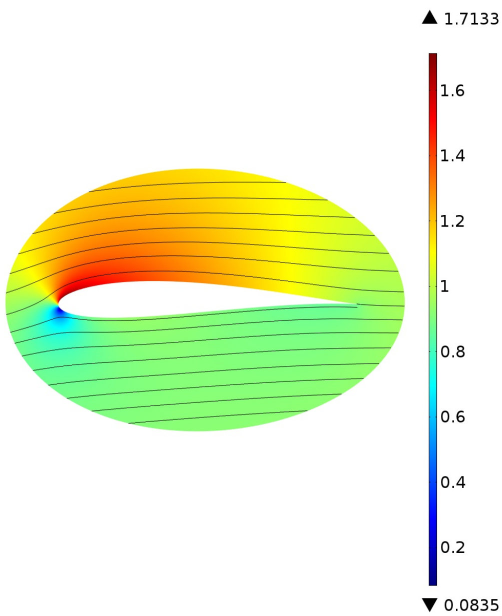

Figure 1: Distribution of the ratio of fluid speed, q, to free-stream speed, U, and the corresponding streamlines in the flow past a cambered Zhukovski airfoil at five degrees angle of attack.

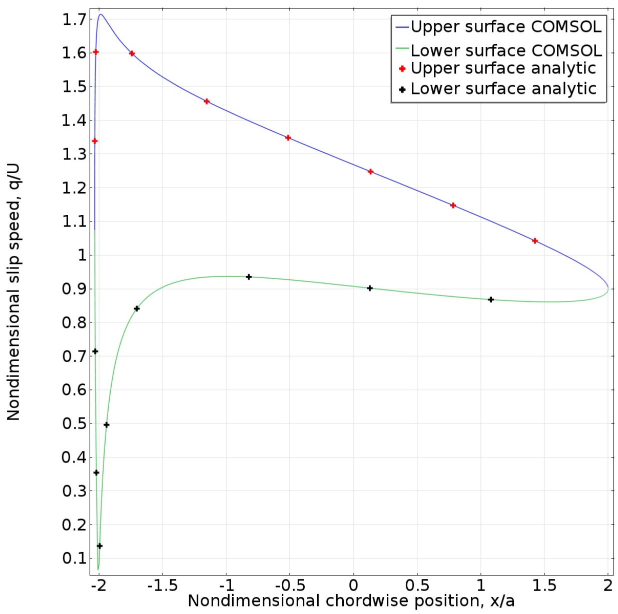

Figure 2: Distribution of the boundary value of q/U over the upper and lower surfaces at five degrees angle of attack.