Geometric Optimization of Micromixers

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Introduction

- **Micromixing is a key step in realizing fast analysis time in many bio-chemical, biological and detection applications of Lab-on-a-chip (LOC) devices.**
- **The conventional T-mixer design requires longer channel lengths and times to achieve complete mixing owing to its dependence on transverse diffusion.**
- **In this work, we investigate the effect of pattern/ groove shape on mixing performance. The geometric optimization is carried out for :**

(a) Heterogeneous charge bottom electro-kinetic micromixer and;

(b) Pressure driven flow based groove micromixer.

Mathematical Model

- **Navier Stokes equation and Continuity equation is solved for obtaining pressure and velocity field.**

$$\rho(u\cdot\nabla u) = -\nabla p + \mu\nabla^2 u$$

$$\nabla\cdot u = 0$$

- **Convection- Diffusion equation**

$$(u\cdot\nabla c) = D\nabla^2 c$$

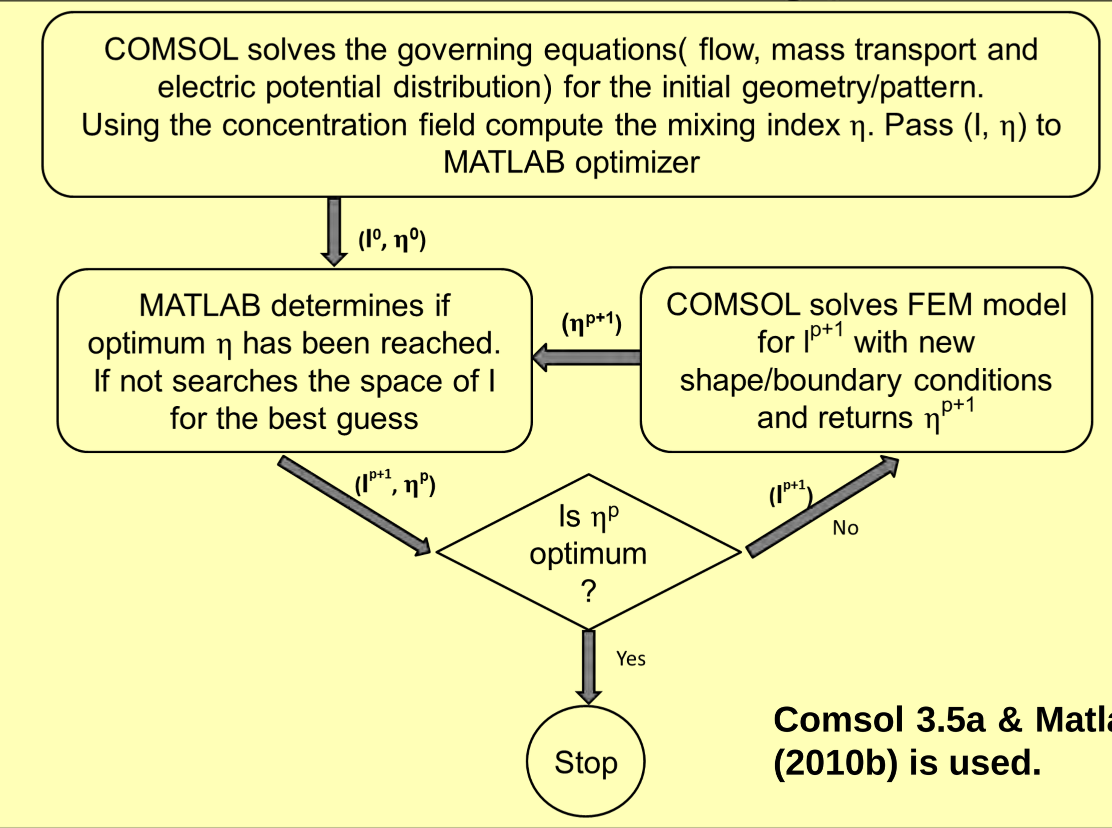
- **Electroosmotic flow (EOF) is modeled using the following additional equations; Laplace equation is solved for potential distribution.**

$$\nabla^2\psi = 0; \quad E = -\nabla\psi$$

- **Smoluchowski's slip velocity condition is used with an assumption of thin electric double layer (EDL).**

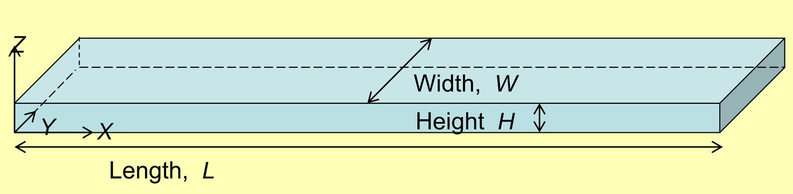
$$u_{slip} \sim -\frac{\varepsilon_m \varsigma}{\mu} E$$

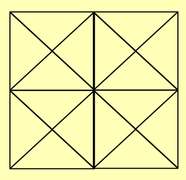
Methodology



Comsol 3.5a & Matlab (2010b) is used.

Design of Optimal Heterogeneous Charge Pattern using Binary Optimization

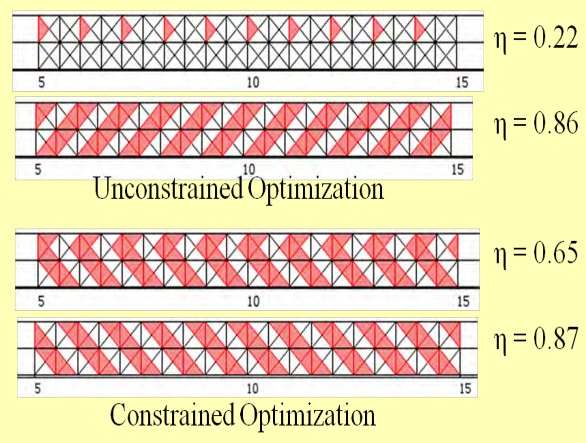




Heterogeneous charge pattern is represented by 10 unit blocks from x = 5-15. A unit block consists of 16 triangular elements; as shown on the left.

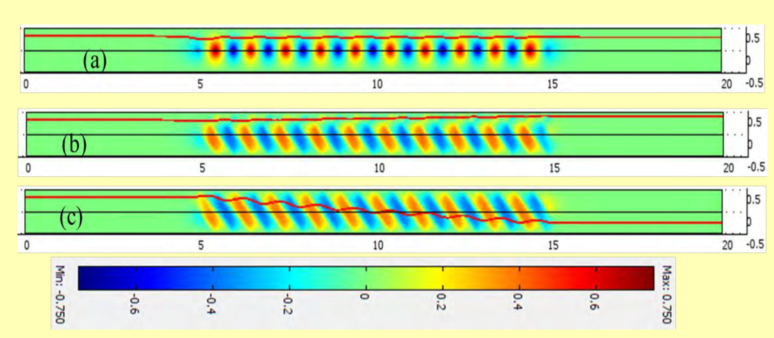
Surface charge on each triangular patch (S_i) in unit block is treated as a binary variable, i.e.

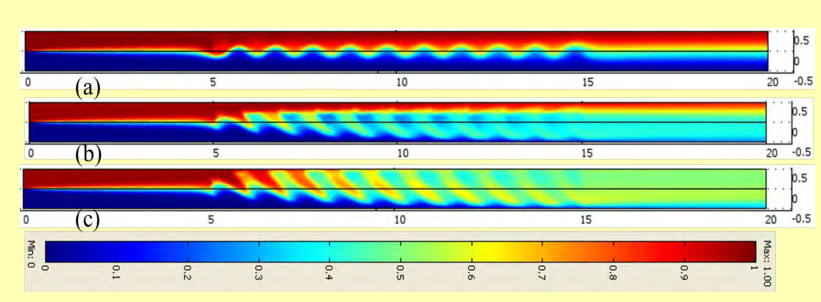
$$S_i = \begin{cases} 0 & \varsigma = \varsigma_f \\ 1 & \varsigma = \varsigma_h \end{cases} \quad i = 1 \text{ to } 16$$

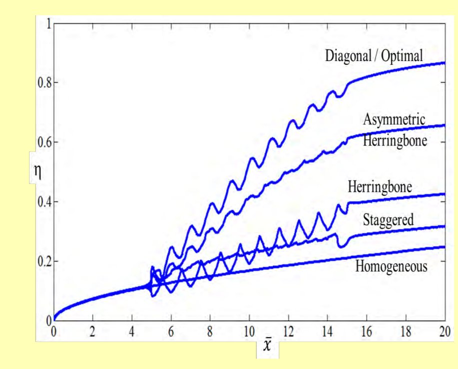


This discretization approach is capable of generating 2^{16} design patterns.

The diagonal heterogeneous charge pattern found to provide the best mixing performance for both unconstrained as well as constrained optimization case.







(a) Staggered block ; (b) Asymmetric herringbone and ; (c) Diagonal (optimal) pattern

The y-velocity component (scaled by average axial velocity) contour plots at the 'z' mid-plane are shown.

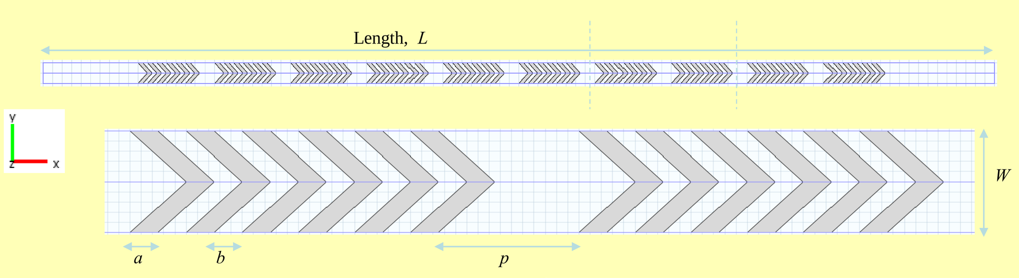
The diagonal patterns generate alternate bands of positive and negative y-velocity across the entire channel width which results in improved mixing performance.

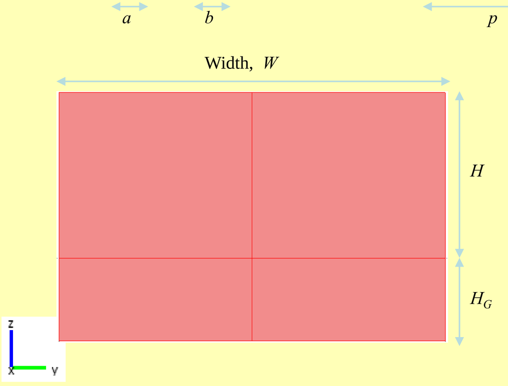
The concentration contour plots at the 'z' mid-plane are shown for various pattern designs.

The diagonal heterogeneous charge pattern generates the most favorable transverse flow structure to provide superior mixing performance with η = 0.87 at Re ~ 0.1 and Pe ~ 1075.

The axial mixing index plot for various designs is shown above. The optimal design provides more than three-fold improvement in mixing performance w.r.t. the homogeneous T-mixer.

Design of Optimal Groove Structure using Bézier Curve Approach

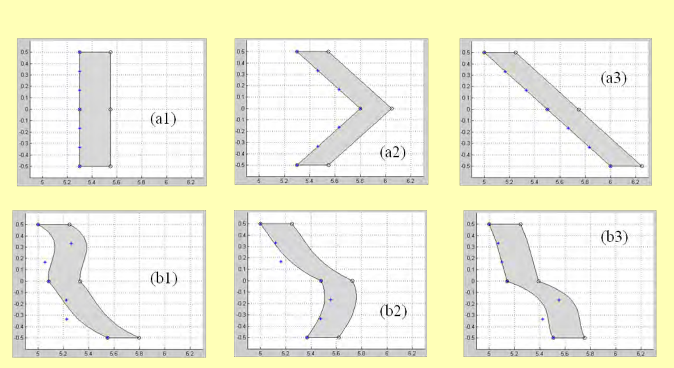




Geometric Parameter	Value
Channel Width (W)	200 µm
Channel Length (L)	1 cm
Channel Height (H)	80 µm
Groove Width (a)	50 µm
Groove spacing (b)	50 µm
Groove cycle spacing (p)	250 µm
Groove Height (H_G)	40 µm

A total of 60 grooves are axially distributed in form of 10 groove cycles containing 6 grooves per cycle.

The effect of groove shape on mixing performance is studied; a single groove shape is optimized and same shape is applied to all the grooves in the channel.

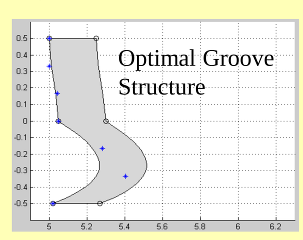


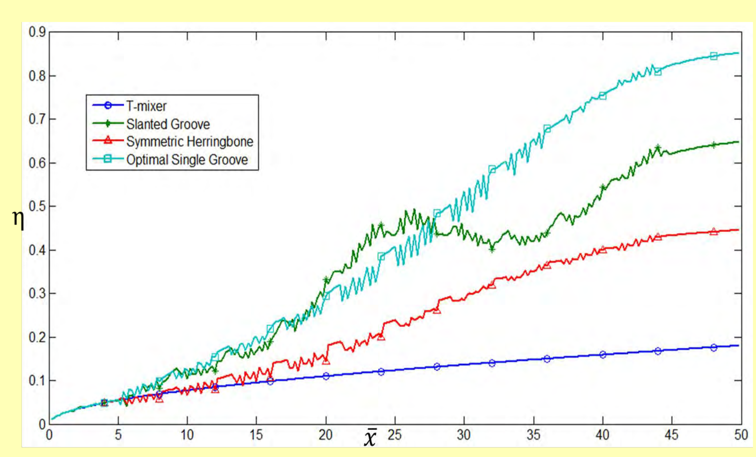
The groove shape is parametrically represented by the Bézier curve, $P(t)$.

$$P(t) = \sum_{i=0}^{n} B_i J_{n,i}(t) \qquad 0 \le t \le 1$$

$$J_{n,i}(t) = \frac{n!}{i!\,(n-i)!} t^i (1-t)^{n-i}$$

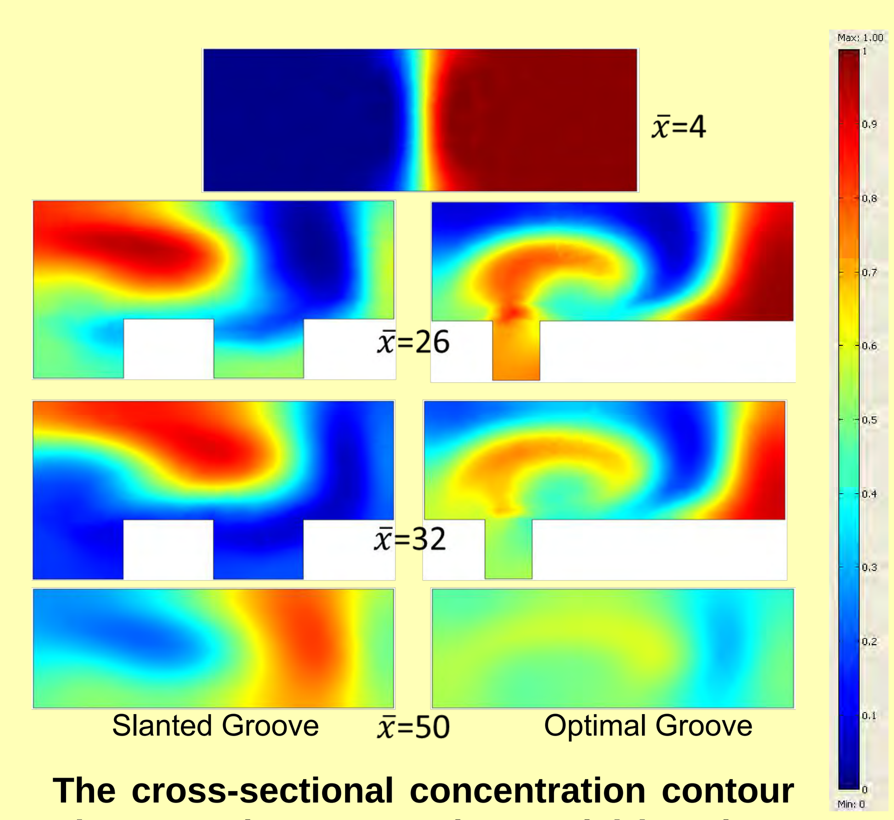
The optimal groove shape as identified by the optimization approach.





The axial mixing index plot for various groove designs is shown above.

The optimal groove structure generates the most favorable transverse flow structure to provide superior mixing performance with η = 0.85 at Pe ~ 4200.

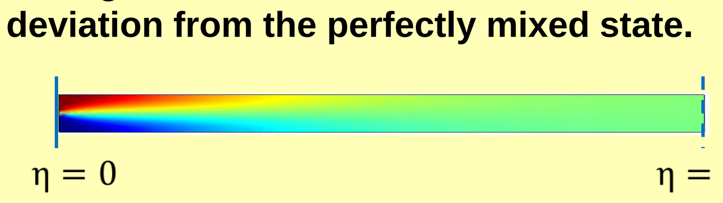


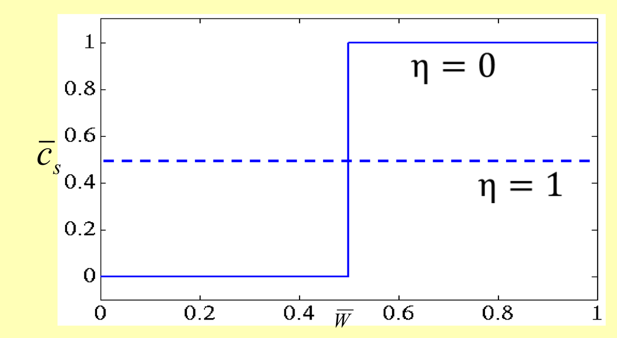
The cross-sectional concentration contour plots are shown at various axial locations for SGM and optimal groove designs.

Mixing Index

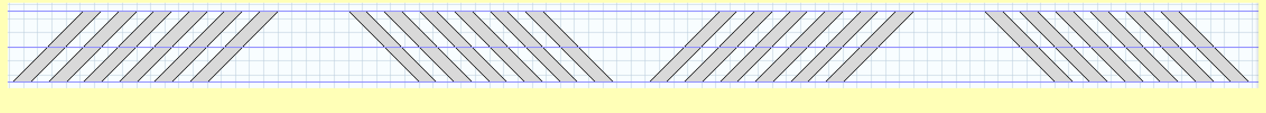
$$\eta = \left[1 - \frac{\sqrt{\frac{1}{N}\sum_{1}^{N}\left(\bar{c}_s - \bar{c}_s^{*}\right)^2}}{\sqrt{\frac{1}{N}\sum_{1}^{N}\left(\bar{c}_s^{0} - \bar{c}_s^{*}\right)^2}}\right]$$

Mixing index is based on the standard deviation from the perfectly mixed state.

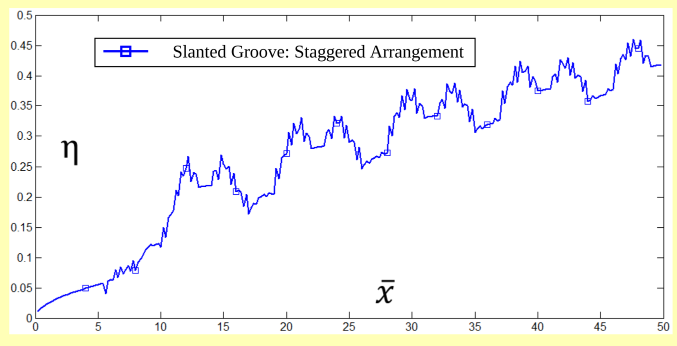


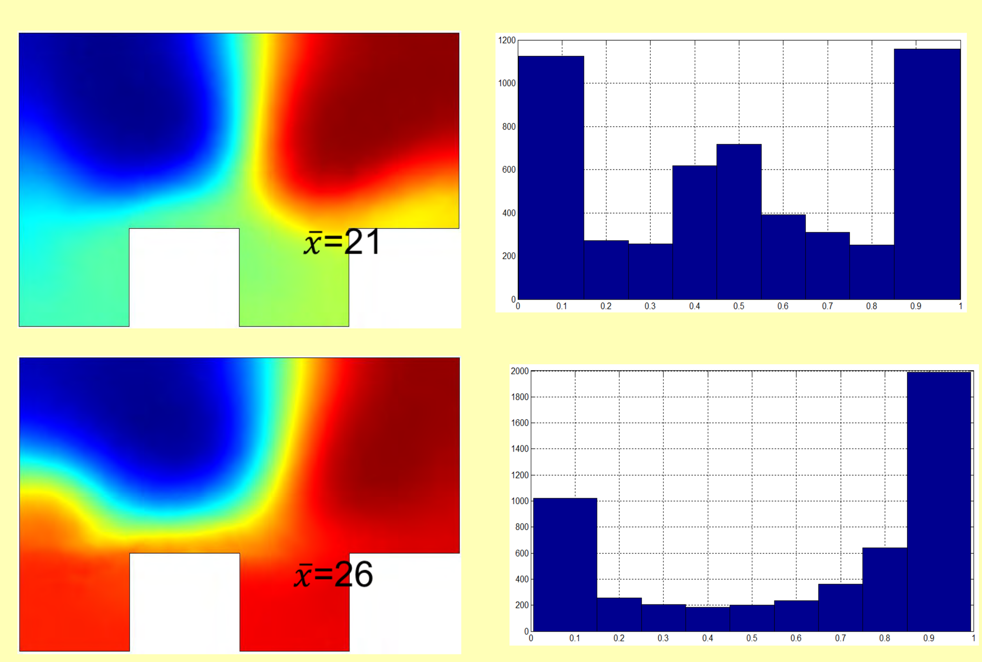


Effect of Flow Reversibility



Staggered arrangement for slanted grooves.





The flow reversibility induced by the staggered slanted groove structure deteriorates the mixing performance after each alternate groove cycle.

Contour plots and histograms are shown for cross-sectional concentration data.

Conclusions

- **Binary optimization is utilized to identify the optimal heterogeneous charge pattern for improved electrokinetic micromixing.**
- **Bezier curve based approach is employed for groove shape optimization for pressure driven flow based groove micromixers.**
- **The proposed approach could be utilized for micro-device design optimization.**

References

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