Computation of Electrical Parameters for Different Conducting Bodies Using Finite Element Method

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Abstract: Accurate and efficient computation of electrical parameters for different conducting bodies represents an essential part of spacecraft modern integrated circuits. In this paper, we will illustrate modeling of inhomogeneous quasi-TEM shielded rectangular, cylindrical, and triangular transmission lines using the finite element method. We specifically determine the capacitance per unit length, inductance per unit length, and characteristic impedance of the shielded transmission lines. Excellent agreement with some results obtained previously is demonstrated.

Keywords: Finite element method, shielded transmission lines, capacitance per unit length, inductance per unit length, characteristic impedance.

1. Introduction

Today, space development and production have rapidly increased. This has led to significant interest in the evaluation of the electrical parameters such as capacitance, inductance, and characteristic impedance of different conducting structures located in free space due to their use in spacecraft. For example, the capacitance has been used in determination of spacecraft equivalent circuit models for the predication electrostatic discharge. Therefore, the improvement of accurate and efficient computational method to analyze the modeling of various shapes transmission lines such as rectangular, cylindrical, and triangular becomes an important area of interest in spacecraft technology.

Previous attempts at the problem include using method of moments [1-4], analytical formulas and methods [5-6], surface charge method [7], method of subareas [8], and boundary element method [9]. We illustrate that our approach using finite element method (FEM) is suitable and effective as other methods for modeling of inhomogeneous quasi-TEM shielded rectangular, cylindrical, and triangular transmission lines.

In this work, we design shielded rectangular, cylindrical, and triangular transmission lines using FEM with COMSOL to calculate the capacitance per unit length, inductance per unit length, and characteristic impedance of the shielded transmission lines and then compare the results of our modeling with some previous works and methods. We use FEM in modeling the transmission line structures, because FEM is especially suitable for the computation of electric and electromagnetic fields in strongly inhomogeneous media. Also, it has high computation accuracy and fast computation speed.

2. Discussion and Results

Using COMSOL for modeling and simulation of the transmission lines involves taking the following steps. We develop the geometry of the line and take the difference between the conductor and dielectric material. We select the relative permittivity as 1. For the boundary, we select the outer conductor as ground and inner conductor as port. We generate the finite element mesh, and then solve for the potential. As postprocessing, we select Point Evaluation and choose capacitance element to find the capacitance per unit length of the line. We use the results of our computation to find the inductance per unit length and the characteristic impedance.

The inductance per unit length L and capacitance per unit length C of a singe transmission line are related as

$$L = \frac{\mu_o \varepsilon_o}{C} \quad , \tag{1}$$

$\varepsilon_o =$ permittivity of free space or vacuum ($\frac{10^{-9}}{36\pi}$, approximate value for problem work).

μ_o = permeability of free space or vacuum (12.6×10^{-7}, approximate value for problem work).

Also, the characteristic impedance Z and the capacitance per unit length are related as

$$Z = \sqrt{\frac{L}{C}} = \frac{1}{uC} \quad , \tag{2}$$

where
u = 3 X 10^8 m/s (the speed of light in vacuum).

We use FEM with COMSOL, a multiphysics software, to compute the capacitance per unit length, inductance per unit length, and characteristic impedance of the shielded rectangular, cylindrical, and triangular transmission lines.

2.1 Shielded Rectangular Transmission Line

In this section, we illustrate the modeling of the shielded rectangular transmission line by focusing on the calculation of the capacitance per unit length, inductance per unit length, and characteristic impedance. Figure 1 shows the geometry of the model with the parameter values. From the model, we plot the contour plot of the potential distribution as shown in Figure 2.

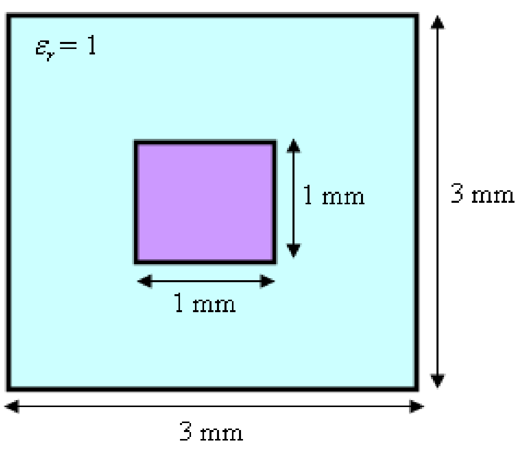

Figure 1. Cross-section of shielded rectangular transmission line.

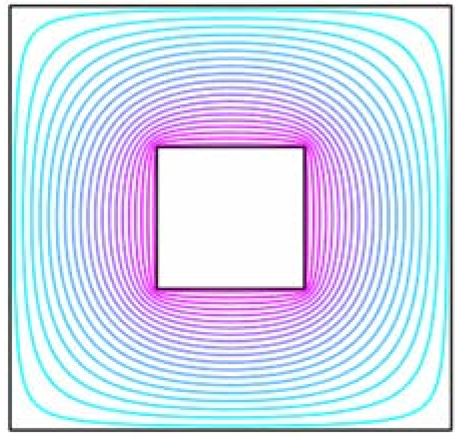

Figure 2. Contour plot of the shielded rectangular transmission line.

Table 1 shows the finite element method results for the capacitance per unit length of the shielded rectangular transmission line compared with the pervious work using the eq. (3) of the capacitance, C as in [5]. We extend our computation by using eqs. (1) and (2) to find the inductance and characteristic impedance as shown in the table. They are in good agreement.

$$C = 2\varepsilon_o \frac{K(x')}{K(x)} \tag{3}$$

where
C = capacitance per unit length of the rectangular transmission line
ε_o = the permittivity of free space or vacuum
$K(x')$ = complete elliptic integrals of the first kind to the modulus x'
$K(x)$ = complete elliptic integrals of the first kind to the modulus x
x' = complementary modulus for x = 1- x .

Table 1: Values of capacitance (in pF/m), inductance (in μ H/m), and Characteristic Impedance Z (Ω) coefficients for the shielded rectangular transmission line

Method	Capacitance $(C_{ii}) = C_{11}$	Inductance L_{11}	Characteristic Impedance Z
Reference [5]	54.970	0.2027	60.72
Our Work	55.057	0.2024	60.63

2.2 Shielded Cylindrical Transmission Line

In this section, we demonstrate the modeling of the shielded cylindrical transmission line by finding the capacitance per unit length, inductance per unit length, and characteristic impedance. Figure 3 shows the geometry of the model with the parameters values, while Figure 4 shows the 2D surface potential distribution of the shielded cylindrical transmission line.

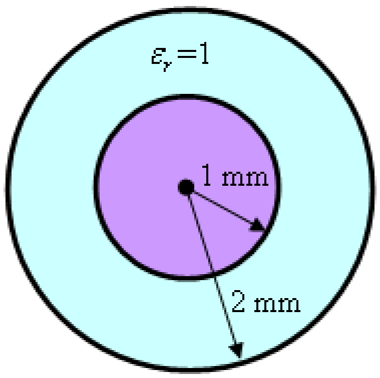

Figure 3. Cross-section of shielded cylindrical transmission line.

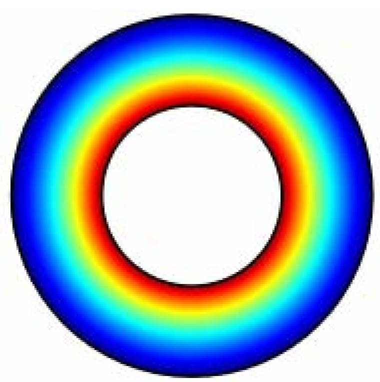

Figure 4. 2D surface potential distribution of the shielded cylindrical transmission line.

Table 2 shows the finite element method results for the capacitance per unit length of the shielded cylindrical transmission line compared with the pervious work using the eq. (4) of the capacitance, C as in [6]. We extend our computation by using eqs. (1) and (2) to find the inductance and characteristic impedance as presented in the table. They are in good agreement.

$$C = \frac{2\pi\varepsilon_o}{\ln\left(\frac{b}{a}\right)} \qquad (4)$$

where
C = capacitance per unit length of the cylindrical transmission line
ε_o = the permittivity of free space or vacuum
b = outer radius of the shield
a = inner radius of the conductor

Table 2: Values of capacitance (in pF/m), inductance (in μ H/m), and Characteristic Impedance Z (Ω) coefficients for the shielded cylindrical transmission line

Method	Capacitance $(C_{ii}) = C_{11}$	Inductance L_{11}	Characteristic Impedance Z
Reference [6]	80.150	0.1390	41.64
Our Work	80.261	0.1388	41.59

2.3 Shielded Triangular Transmission Line

In this section, we illustrate the modeling of the shielded triangular transmission line (annular triangular plate) by calculating the capacitance per unit length, inductance per unit length, and characteristic impedance. Figure 5 shows the geometry of the model with sides of 1mm for the outer triangle and 0.25 mm for the inner triangle sides, while Figure 6 shows the contour plot of the potential distribution of the annular triangular plate.

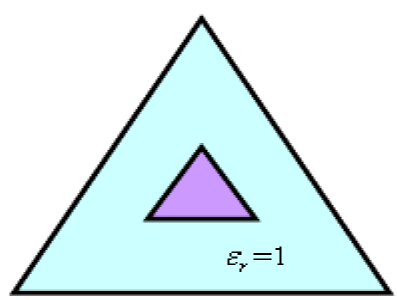

Figure 5. Cross-section of shielded triangular transmission line.

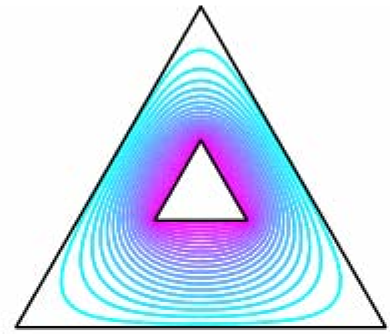

Figure 6. Contour plot of the shielded triangular transmission line.

Table 3 shows the FEM results for the capacitance per unit length of the shielded triangular transmission line compared with the pervious work using eq. (5) of the capacitance, C as in [1]. We extend our computation by using eqs. (1) and (2) to find the inductance and characteristic impedance as shown in the table. They are not in good agreement.

$$C = \frac{1}{V}\sum_{n=1}^{N}\sigma_n A_n \, , \qquad (5)$$

where

V = the potential of a perfect conducting surface

σ_n = unknown charge density in subdomain n

A_n = area of the source rectangle in subdoamin n

N = the total number of rectangular subsections

Table 3: Values of capacitance (in pF/m), inductance (in μ H/m), and Characteristic Impedance Z (Ω) coefficients for the shielded triangular transmission line

Method	Capacitance $(C_{ii}) = C_{11}$	Inductance L_{11}	Characteristic Impedance Z
Reference [1]	26.97	0.4131	123.76
Our Work	50.76	0.2195	65.76

Figures 2, 4, and 6 present the potential distributions of inhomogeneous quasi-TEM shielded rectangular, cylindrical, and triangular transmission lines, respectively. These potentials are not mentioned or studied by other researchers; hence they can not be verified with other approaches. The potential distributions show how the potential become smaller when we get farther from the selected input port/conductor.

Tables 1, 2, and 3 present the comparison of the computation of capacitance, inductance, and characteristics impedance of the shielded rectangular, cylindrical, and triangular transmission lines, respectively. These tables can help the improvement of accurate and efficient computational method of the electrical parameters(C, L, and Z) in designing of modern integrated circuits of spacecraft. All methods considered and referred to in this paper are numerical solutions and there is no exact, analytical solution to compare with.

3. Conclusions

In this paper, we have presented the modeling of inhomogeneous quasi-TEM shielded rectangular, cylindrical, and triangular transmission lines. We computed the capacitance per unit length, inductance per unit length, and characteristic impedance of the transmission lines and identified their potential distribution. Some of results obtained efficiently using finite element method (FEM) for the electrical parameters agree well with those found in the literature.

4. References

1. A. Ghosh and A. Chakrabarry, Estimation of capacitance of different conducting bodies by the method of rectangular subareas, *Journal of Electrostatics*, **66**, 142-146 (2008)

2. A. E. Ruehli and P. A. Brennan, Efficient capacitance calculations for three-dimensional multiconductor systems, *IEEE Transactions on Microwave Theory and Techniques*, **21** (2), 76-82 (1973)

3. C. Chakraborty, D. R. Poddar, A. Chakraborty, and B. N. Das, Electrostatic charge distribution and capacitance of isolated cylinders and truncated cones in free space, *IEEE Transactions on Electromagnetic Compatibility*, **35** (1), 98-102 (1993)

4. S. M. Rao, A. W. Glisson, D. R. Wilton, and B. S. Vidula, A simple numerical solution procedure for statics problems involving arbitrary-shaped surface, *IEEE Transactions on Antennas and Propagation*, **27** (5), 604-608 (1979)

5. G. M. Anderson, The calculation of the capacitance of coaxial cylinders of rectangular cross-section, *AIEE Transactions*, **69**, 728-731 (1950)
6. M. N. O. Sadiku, *Elements of Electromagnetics*, p.237, Oxford University Press, 4th ed. (2007)
7. E. Goto, Y. Shi, and N. Yoshida, Extrapolated surface charge method for capacity calculation of polygons and polyhedra, *Journal of Computational Physics*, **100**, 105-115 (1992)
8. T. J. Higgins, D. K. Reitan, Calculation of the capacitance of a circular annuals by the method of subareas, *AIEE Transactions*, **70** (1951)
9. N. Nishiyama, M. Nakamura, Capacitance of disk capacitors by the boundary element method, *Proceedings of the First European Conference on Numerical Methods in engineering, (*1992)