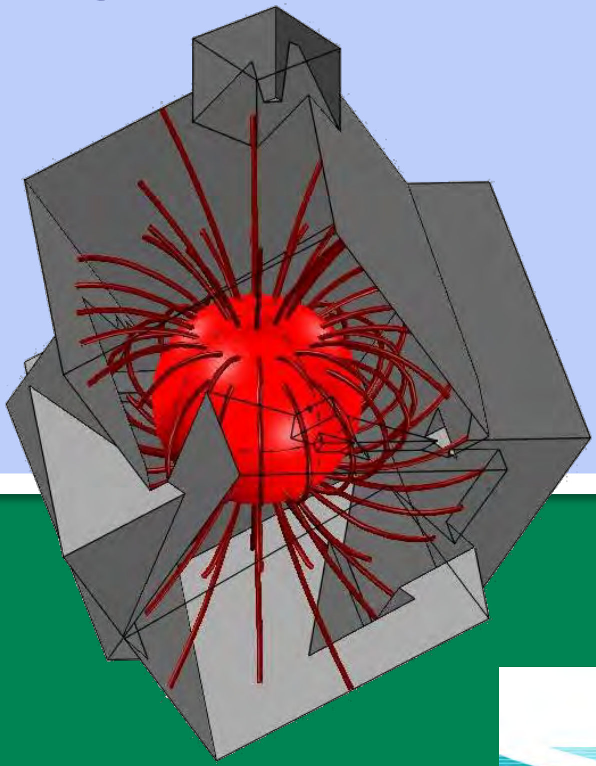
Hybrid FEM-BEM approach for two- and three-dimensional open boundary magnetostatic problems



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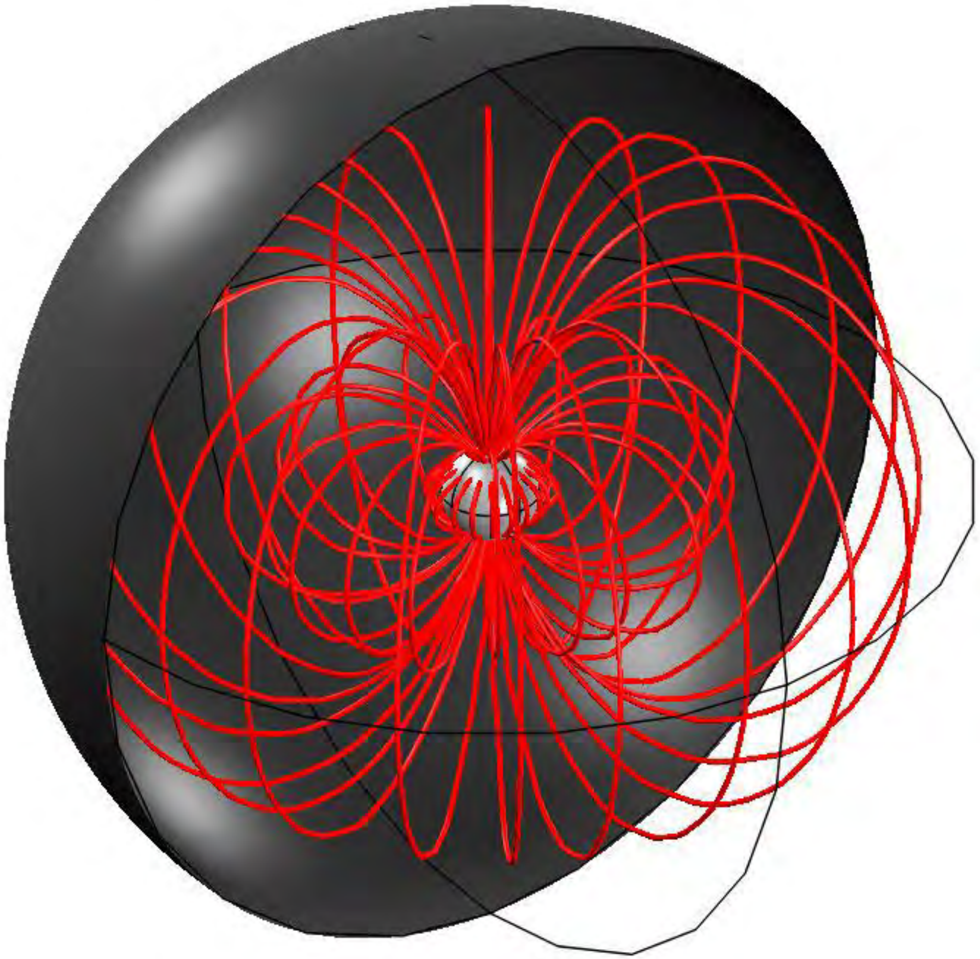
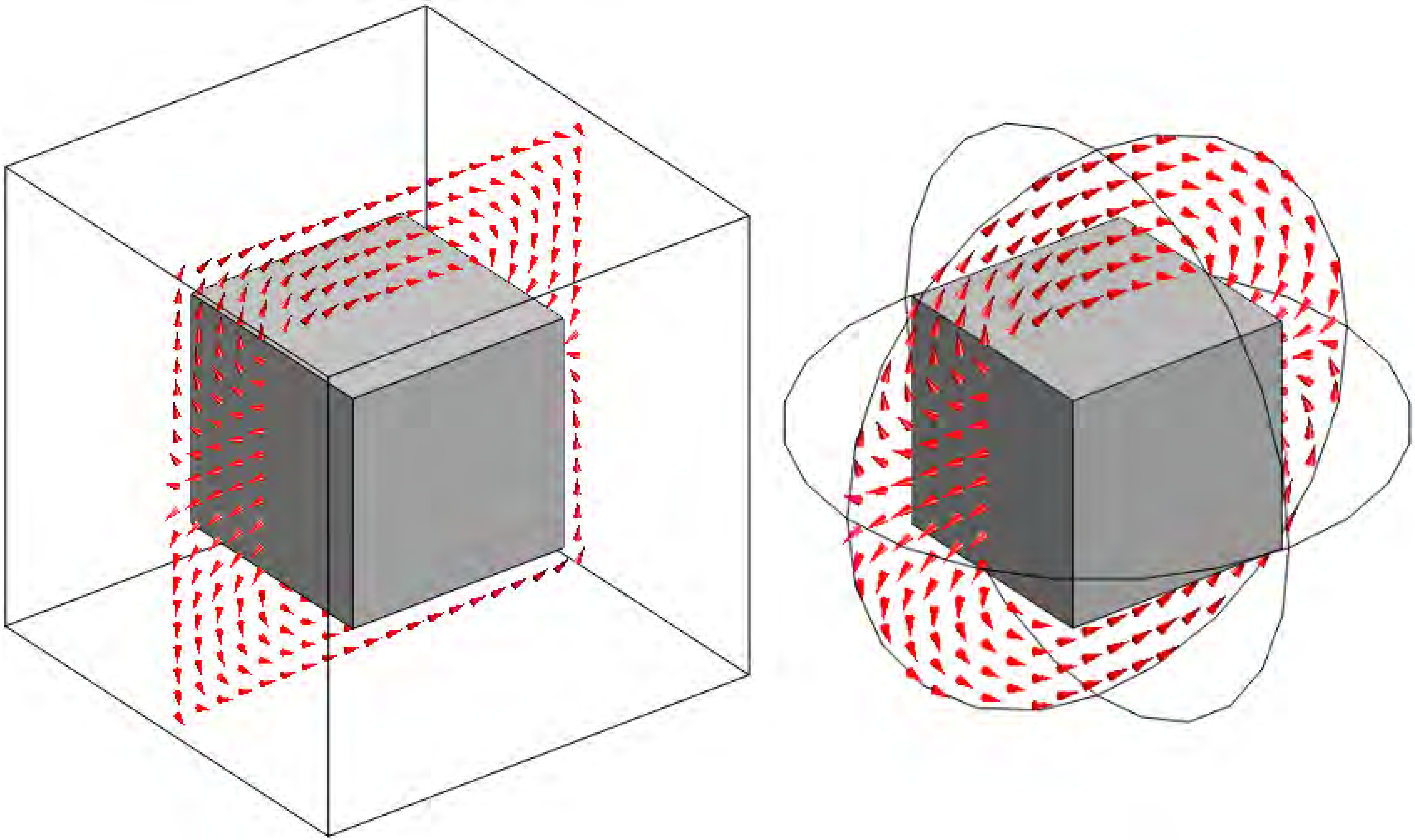
COMSOL CONFERENCE 2011

Motivation

Magnetostatics

- Behavior and properties of magnetic objects
- Described by Maxwell equations
    - Depends on non-local properties
- Need of an outer domain in FEM simulations
- Unphysical cutoff leads to errors





Hybrid FEM-BEM approach

Basic Idea

- Split potential
- Using potential theory
  - ➢ Eliminate outer domain with an integral expression
- FEM to reduce calculation time

Advantages

- No outer domain needed
- Calculation can be restricted to important domains
- Analytical approach, errors are numerical ones

Equations to solve

- Split : $\varphi = \varphi_1 + \varphi_2$
- FEM : $\Delta\varphi_1 = -\nabla \cdot \mathrm{M} \quad \forall r \in V_{mag}$

  $\partial_{\mathrm{n}}\varphi_1 = \mathrm{n} \cdot \mathrm{M} \quad \forall r \in S$
- BEM : $\varphi_{2,\mathrm{r}} = \int_S \varphi_1(r')\, \partial_{\mathrm{n}} \left| r - r' \right|^{-1} \mathrm{d}^2 r'$
- FEM : $\Delta\varphi_2 = 0$

  $\varphi_2 = \varphi_{2,\mathrm{r}} + \left(\frac{\Omega}{4\pi} - 1 \right) \cdot \varphi_1$

Implementation in COMSOL

- Two simulation steps
  - FEM solution for φ_1
  - FEM solution for φ_2 with boundary values from integration variables

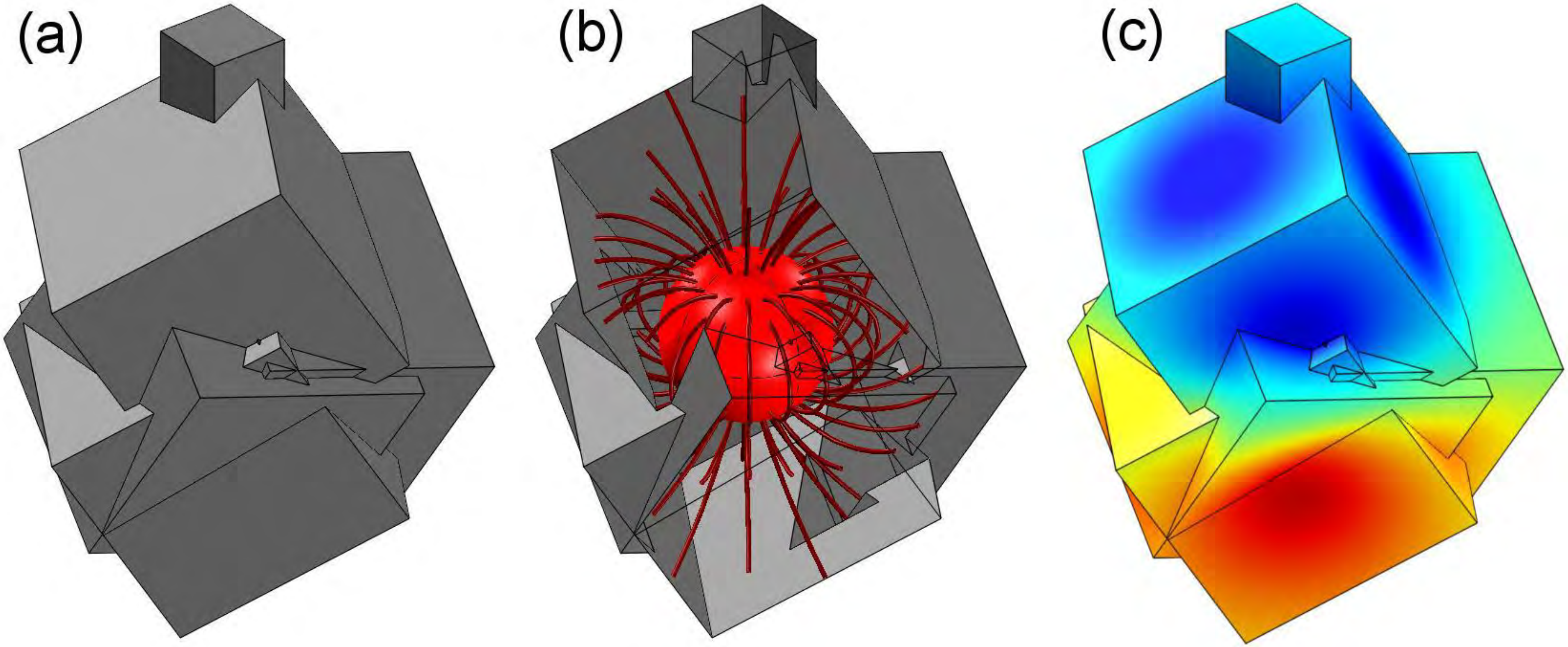
In COMSOL 4.x

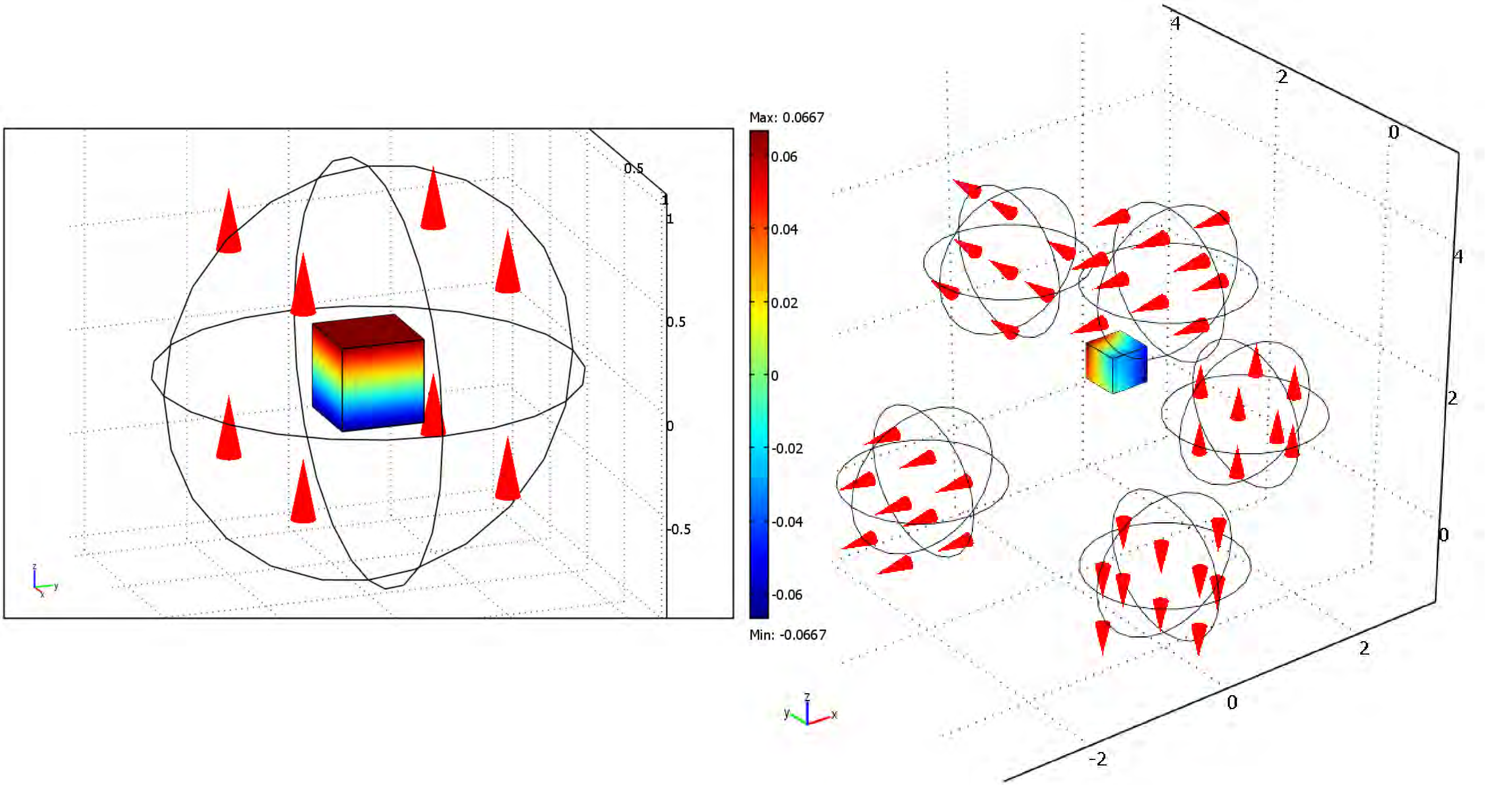
- Study (PDE Mode)
- Introduce Boundary variable with integration
- Study (PDE Mode) with Dirichlet boundary condition

In COMSOL 3.x

- Solve equation in PDE Mode
- Store solution and introduce another PDE Mode for φ_2
- Use Boundary Coupling Variables
- Solve equation for φ_2

In case of non smooth surfaces need solid angle


(a)
(b)
(c)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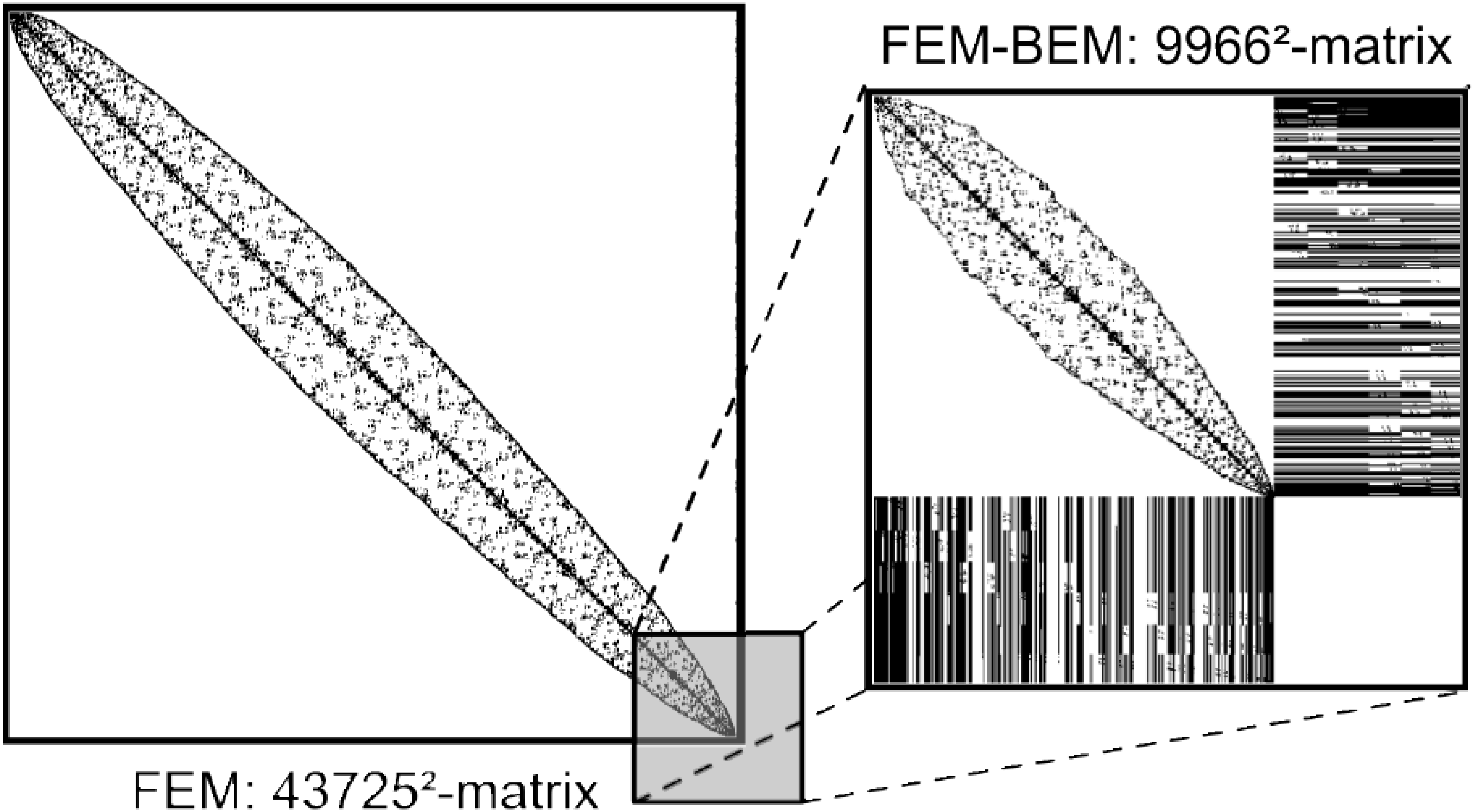
FEM-BEM: 9966²-matrix
FEM: 43725²-matrix

Application for simulations with thin films

Basic Idea

- Magnetization mainly in-plane
  - ➢ Magnetic field in-plane
  - ➢ Problem can be reduced to two dimensions

Advantages

- No dependence on thickness
- Reduced dimension
  - ➢ Faster calculations, huge reduction in DoFs