time the thermal strain is used. This will have a negative impact on the performance, when compared to using a secant coefficient of thermal expansion.

- Precompute the expression in Equation 2-29 externally for the intended range of temperatures. This can for example be done in a spreadsheet program. Enter the computed result as a function, which is then used as any other secant temperature dependent thermal expansion coefficient.

When using **Tangent coefficient of thermal expansion**, the integrate operator is used. It is called with the two integration limits being the reference temperature `<phys>.Tref` and the current temperature `<phys>.T`, where `<phys>` is the tag of the physics interface. If you define the expression for the coefficient of thermal expansion yourself, you must ensure that it depends on a 'free' variable, and not use the same temperature variable as you use to prescribe the current temperature `<phys>.T`.

Thermal Expansion Coefficient Dependence on Reference Temperature

Let $\alpha_m(T)$ be the temperature-dependent function that represents the measured values of the secant thermal expansion coefficient. The change in length of a sample at a given temperature T with respect to the sample's original length at a temperature T_m is called *dilation*.

Note that by definition, the dilation at $T = T_m$ is zero, so T_m denotes the strain-free state of the material as far as the measured values of $\alpha_m(T)$ is concerned. Denote the length of the sample at a temperature T as $L(T)$ and the strain-free length as $L_0 = L(T_m)$. The dilation can be then expressed as $L(T) - L(T_m)$. Using the definition of the secant coefficient of thermal expansion, $L(T)$ can be written as:

$$L(T) = [1 + \alpha_m(T)(T - T_m)]L(T_m) \tag{2-32}$$

When using the measured data, it is possible that the strain-free state occurs at a temperature T_{ref} which differs from T_m. The dilation at any temperature T would then be defined as $L(T) - L(T_{ref})$, where $L(T_{ref})$ can be written as.

$$L(T_{ref}) = [1 + \alpha_m(T_{ref})(T_{ref} - T_m)]L(T_m) \tag{2-33}$$

As a result of this shift in the strain-free temperature, it is necessary to redefine the thermal expansion coefficient so that $L(T)$ and $L(T_{ref})$ can be related using Equation 2-32 but with T_m replaced by T_{ref}.

$$L(T) = [1 + \alpha_r(T)(T - T_{ref})]L(T_{ref}) \tag{2-34}$$

Here $\alpha_r(T)$ is the redefined thermal expansion coefficient, based on T_{ref}. It can be derived from the relations above. Using Equation 2-32 and Equation 2-34 there are two ways of writing the current length $L(T)$, so that

$$[1 + \alpha_r(T)(T - T_{ref})]L(T_{ref}) = [1 + \alpha_m(T)(T - T_m)]L(T_m) \tag{2-35}$$

Equation 2-33 makes it is possible to eliminate $L(T_{ref})$ and $L(T_m)$ from Equation 2-35:

$$[1 + \alpha_r(T)(T - T_{ref})][1 + \alpha_m(T_{ref})(T_{ref} - T_m)] = 1 + \alpha_m(T)(T - T_m) \tag{2-36}$$

It is now possible find $\alpha_r(T)$, expressed in known quantities. After some algebra, the final expression is

$$\alpha_r(T) = \frac{\alpha_m(T) + (T_{ref} - T_m)\dfrac{\alpha_m(T) - \alpha_m(T_{ref})}{T - T_{ref}}}{1 + \alpha_m(T_{ref})(T_{ref} - T_m)} \tag{2-37}$$

In order to arrive at this form of $\alpha_r(T)$, the numerator has been rewritten, using

$$\begin{aligned}
&\alpha_m(T)(T - T_m) - \alpha_m(T_{ref})(T_{ref} - T_m) = \\
&\alpha_m(T)(T - T_{ref}) + \alpha_m(T)(T_{ref} - T_m) - \alpha_m(T_{ref})(T_{ref} - T_m) = \\
&\alpha_m(T)(T - T_{ref}) + (T_{ref} - T_m)(\alpha_m(T) - \alpha_m(T_{ref}))
\end{aligned} \tag{2-38}$$

Representation in COMSOL Multiphysics

Most materials listed in the material libraries and databases available with COMSOL Multiphysics and its add-on products contain a function for the measured temperature-dependent thermal expansion coefficient curve. You can find this from the Materials branch, as shown in Figure 2-19. The Piecewise function named `alpha_solid_1` is the measured thermal expansion coefficient $\alpha_m(T)$.

Using Functions in Materials in the *COMSOL Multiphysics Reference Manual*

The Material Contents section in Figure 2-19 shows the material property `alpha`, which is the redefined thermal expansion coefficient $\alpha_r(T)$. The complete expression for `alpha` is as follows: