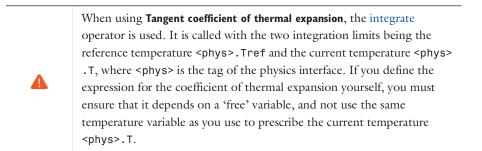
time the thermal strain is used. This will have a negative impact on the performance, when compared to using a secant coefficient of thermal expansion.

• Precompute the expression in Equation 2-29 externally for the intended range of temperatures. This can for example be done in a spreadsheet program. Enter the computed result as a function, which is then used as any other secant temperature dependent thermal expansion coefficient.



Thermal Expansion Coefficient Dependence on Reference Temperature Let $\alpha_{\rm m}(T)$ be the temperature-dependent function that represents the measured values of the secant thermal expansion coefficient. The change in length of a sample at a given temperature T with respect to the sample's original length at a temperature $T_{\rm m}$ is called *dilation*.

Note that by definition, the dilation at $T = T_m$ is zero, so T_m denotes the strain-free state of the material as far as the measured values of $\alpha_m(T)$ is concerned. Denote the length of the sample at a temperature T as L(T) and the strain-free length as $L_0 = L(T_m)$. The dilation can be then expressed as $L(T) - L(T_m)$. Using the definition of the secant coefficient of thermal expansion, L(T) can be written as:

$$L(T) = [1 + \alpha_{\rm m}(T)(T - T_{\rm m})]L(T_{\rm m})$$
(2-32)

When using the measured data, it is possible that the strain-free state occurs at a temperature T_{ref} which differs from T_m . The dilation at any temperature T would then be defined as $L(T) - L(T_{ref})$, where $L(T_{ref})$ can be written as.

$$L(T_{\rm ref}) = [1 + \alpha_{\rm m}(T_{\rm ref})(T_{\rm ref} - T_{\rm m})]L(T_{\rm m})$$
(2-33)

As a result of this shift in the strain-free temperature, it is necessary to redefine the thermal expansion coefficient so that L(T) and $L(T_{ref})$ can be related using Equation 2-32 but with $T_{\rm m}$ replaced by $T_{\rm ref}$.

$$L(T) = [1 + \alpha_{\rm r}(T)(T - T_{\rm ref})]L(T_{\rm ref})$$
(2-34)

Here $\alpha_{\rm r}(T)$ is the redefined thermal expansion coefficient, based on $T_{\rm ref}$. It can be derived from the relations above. Using Equation 2-32 and Equation 2-34 there are two ways of writing the current length L(T), so that

$$[1 + \alpha_{\rm r}(T)(T - T_{\rm ref})]L(T_{\rm ref}) = [1 + \alpha_{\rm m}(T)(T - T_{\rm m})]L(T_{\rm m}) \tag{2-35}$$

Equation 2-33 makes it is possible to eliminate $L(T_{ref})$ and $L(T_m)$ from Equation 2-35:

$$[1 + \alpha_{\rm r}(T)(T - T_{\rm ref})][1 + \alpha_{\rm m}(T_{\rm ref})(T_{\rm ref} - T_{\rm m})] = 1 + \alpha_{\rm m}(T)(T - T_{\rm m})$$
(2-36)

It is now possible find $\alpha_r(T)$, expressed in known quantities. After some algebra, the final expression is

$$\alpha_{\rm r}(T) = \frac{\alpha_{\rm m}(T) + (T_{\rm ref} - T_{\rm m}) \frac{\alpha_{\rm m}(T) - \alpha_{\rm m}(T_{\rm ref})}{T - T_{\rm ref}}}{1 + \alpha_{\rm m}(T_{\rm ref})(T_{\rm ref} - T_{\rm m})}$$
(2-37)

In order to arrive at this form of $\alpha_r(T)$, the numerator has been rewritten, using

$$\begin{aligned} &\alpha_{\rm m}(T)(T-T_{\rm m}) - \alpha_{\rm m}(T_{\rm ref})(T_{\rm ref} - T_{\rm m}) = \\ &\alpha_{\rm m}(T)(T-T_{\rm ref}) + \alpha_{\rm m}(T)(T_{\rm ref} - T_{\rm m}) - \alpha_{\rm m}(T_{\rm ref})(T_{\rm ref} - T_{\rm m}) = \\ &\alpha_{\rm m}(T)(T-T_{\rm ref}) + (T_{\rm ref} - T_{\rm m})(\alpha_{\rm m}(T) - \alpha_{\rm m}(T_{\rm ref})) \end{aligned}$$

$$\end{aligned}$$

$$(2-38)$$

Representation in COMSOL Multiphysics

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Most materials listed in the material libraries and databases available with COMSOL Multiphysics and its add-on products contain a function for the measured temperature-dependent thermal expansion coefficient curve. You can find this from the Materials branch, as shown in Figure 2-19. The Piecewise function named alpha_solid_1 is the measured thermal expansion coefficient $\alpha_m(T)$.

Using Functions in Materials in the COMSOL Multiphysics Reference Manual

The Material Contents section in Figure 2-19 shows the material property alpha, which is the redefined thermal expansion coefficient $\alpha_{\rm T}(T)$. The complete expression for alpha is as follows: