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Homogeneous Heating of Milk

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Abstract:
When milk is taken out of a refrigerator it has to be heated up to 37 C. This should be done fast, without ever exceeding 40 C at any point on the milk bottle. The standard solution is to put the bottle in a bath of warm water and wait. The aim of this paper is to show how this process can be improved. The main idea is to use heating foils on the surface of the bottle and then find good spacial and temporal heating patterns to achieve a uniform, fast heating of the liquid. The external heating foils will put thermal energy into the boundary layers of the bottle. Due to diffusion the heat will spread and a free convection of the fluid in the bottle will start and thus the liquid will start to stir. With COMSOL Multiphysics a mathematical model is set up, using heat equations and weakly compressible Navier-Stokes equations. This allows to separate two effects leading to uniform temperature distributions: heat conduction and convection of the fluid. Building a physical setup with 6 temperature sensors the mathematical model is calibrated to real world measurements. Using this verified model a number of heating patterns are examined. We have a closer look at the effect of heating the bottom, lower, middle or upper part of the bottle. The applied heating power is modified as time evolves. Many simulations are performed and for a few good candidates the results are confirmed by measurement with the real world model. It is shown that the heating can be done faster, avoiding large temperature gradients and without exceeding the limiting temperature of 40 C.

Keywords: uniform heating, free convection.

1. The measurement setup

The bottles examined have a diameter of 5.5 cm and a height of 7.8 cm. The wall has a thickness of approximately 0.8 cm and consists of polypropylene. Half of a cross section through the bottle is shown in Figure 1, together with the positions of the temperature sensors. The heating foils are attached in three segments: bottom, lower part and higher part. Using the volume of liquid to be heated up by 40 C within the given minimal time (120 sec), we had to assure a maximal heating power of 300 W. To assure good thermal contact between the bottle and the foils thermal paste is used. At five points (see Figure 1) in the bottle we placed temperature sensors, consisting of thermocouples with wires of 0.8 mm thickness. To achieve better accuracy we calibrated the sensors using a temperature calibration device by Ametek. With a DAQ card the sensor signals were recorded by a LabVIEW program. The same program also controlled the attached heat foils. The device was put in a custom built insulating box of foamed polystyrene.

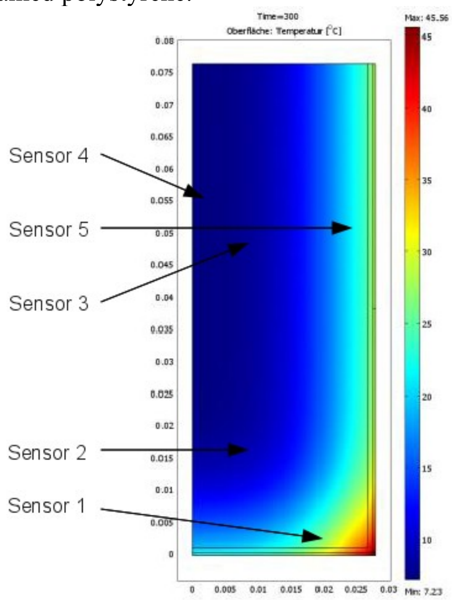

Figure 1. Half of a cross section through the bottle.

2. Simulation results with COMSOL Multiphysics

To setup a realistic simulation we used a 2D model with, assuming radial symmetry. Thus the domain to be examined is half of a cross section through the bottle. We used $0<r<R=5.5$ cm and $0<z<H = 8.5$ cm. The wall of the bottle and the heating coils are added as strips.

The following physical effects have to be taken into account:

- heat conduction within the liquid

- heat convection within the liquid, i.e. free convection caused by heating
- heat conduction within the walls of the bottle
- external heating by the heating foils
- heat losses on the boundary

For the above we used a heat conduction and convection module (cc), combined with the weakly compressible Navier-Stokes mode (chns).
Figure 1 shows the typical temperature distribution when the thermal effects on the fluid motion is ignored. The result is not realistic since:

- With increasing temperature the liquid will expand, using a coefficient of 2e-4/K.
- Due to the expansion the density will decrease, leading to a net upward buoyancy force.
- This force will drive the free convection in the liquid.

The top of the liquid is in contact with a closed container of air. A rough estimate showed that the loss of energy trough the top can safely be ignored. Since the external heat losses through bottom and walls are difficult to control, we used a calibration measurement and simulation to determine the percentage of energy actually arriving in the liquid and the container. The amount of thermal energy created in the heating foil layers was adjusted accordingly. Since the bottom heating foil showed little influence on the behavior but turned out to be difficult to control we used the side heating foils only.

In Figure 2 and 3 COMSOL results are shown with heating the sides only. In Figure 2 no buoyancy force is taken into account. For Figure 3 the free convection clearly shows the different temperature profile. The liquid is taking up thermal energy along the side of the bottle and the warmed water will move to the top. Thus the top layers will warm up considerably, while all cold liquid ends up at the bottom. In Figure 4 the typical stream lines on the moving liquid are shown. A similar computation with ANSYS generated a similar result. The heat to be generated by the foils for all the above examples was chosen to be a function of time. We start out with maximal heat generation, but then lower the power to not exceed the maximal temperature of 40 C. Find a typical, not identical, heating profile in Figure 8. At first the same pattern was used for the lower and upper heating foil. The fluid flow showed first sign of turbulence. This leads to local, fast variations in temperature.

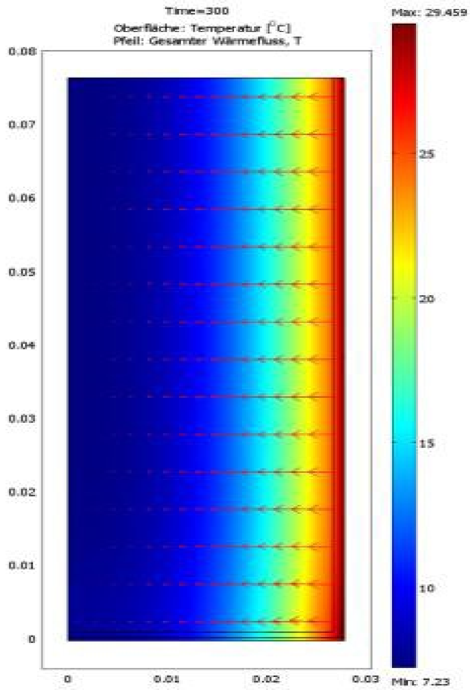

Figure 2. Temperature distribution, pure diffusion

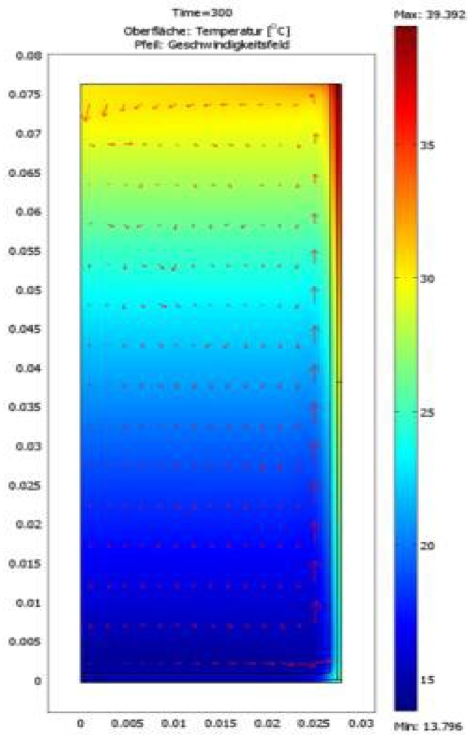

Figure 3. Temperature distribution, convection and conduction

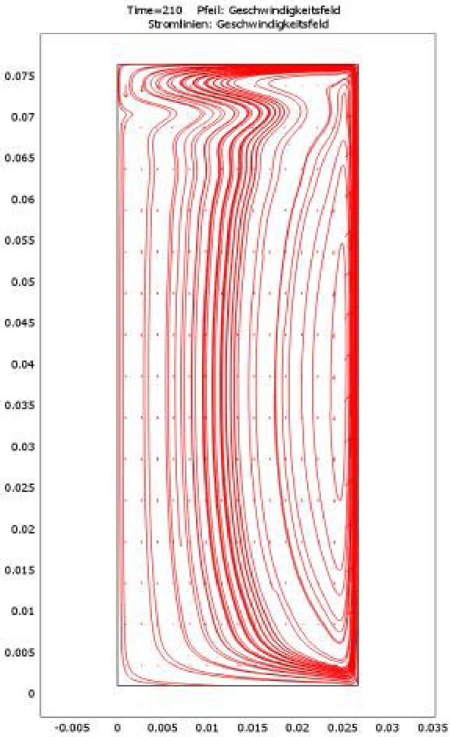

Figure 4 Stream lines for the free convection

3. Comparison of results of simulation and measurements

With the above setup of COMSOL the results of the simulation and measurements were compared, leading the satisfactory results in Figures 2 and 3. In Figure 2 on the left find the measurements, including the result of the additional sensor measuring the temperature of the environment. In Figure 5 find the results obtain by COMSOL. The wiggles are caused by localized domains with turbulence. The table below shows the temperatures at the final time of 300 sec.

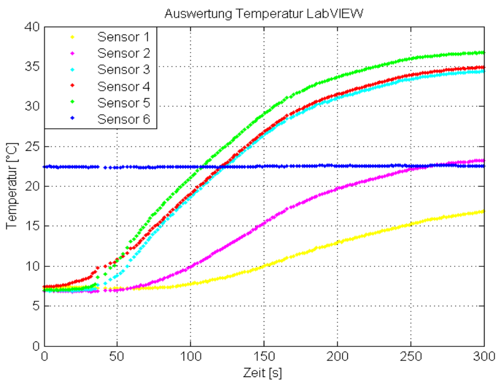

Figure 5. Temperature results for the measure-ments

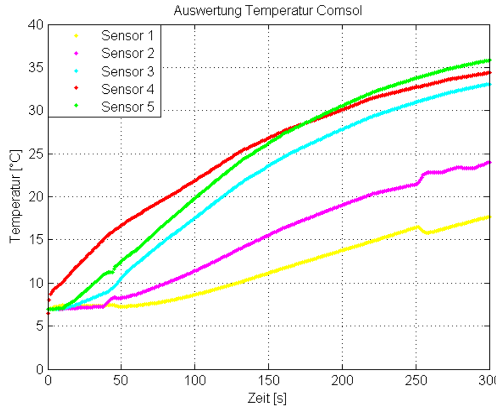

Figure 6. Temperature results for the COMSOL simulations

	Measured [C]	COMSOL [C}	Difference [C]
Sensor 1	16.90	17.66	+0.76
Sensor 2	23.29	24.03	+0.74
Sensor 3	34.45	33.08	-1.37
Sensor 4	34.93	34.41	-0.52
Sensor 5	36.78	35.83	-0.95

4. Diffusion and convection

For a fixed radius r=1.5 cm the horizontal energy flow through the vertical wall is examined. There are two contribution: the diffusion flux and the convection flux. Along a vertical axis the horizontal energy flux is computed by

$$FluxDiff = \int_0^H 2\pi rk \frac{dT}{dr} dz$$

and

$$FluxConv = \int_0^H 2\pi rcv_r T dz$$

In Figure 7 find the horizontal, convective energy flux as function of the height z at the final time t=300 sec. The graph clearly shows that most of the energy transport to the inner part of the bottle occurs in a small layer at the top. The values of the corresponding diffusion flux are very small, as shown in the table below.

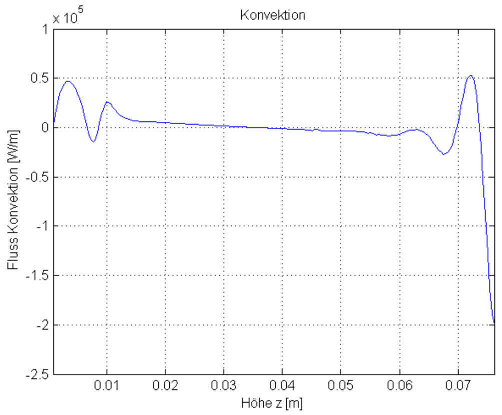

Figure 7 Convective flux, horizontal

Time [s]	Convection [W]	Diffusion [W]
100	-27.51	0.01
200	-26.82	0.02
300	-25.83	0.006

5. Obtaining a uniform temperature distributions

Based on the observations of the above section we aimed for a more uniform temperature distribution by heating in a nonuniform fashion. To apply more heating in lower sections the heating power was multiplied by a linear function depending on the height z. The factor equals 1 at the bottom and we use 0.2 at the top. For these simulations the bottom was heated too. On the innermost section of the bottom the heating was slightly lowered. The heating power was also varied with respect to time. The profiles for the bottom and side are slightly different, as shown in Figure 8.

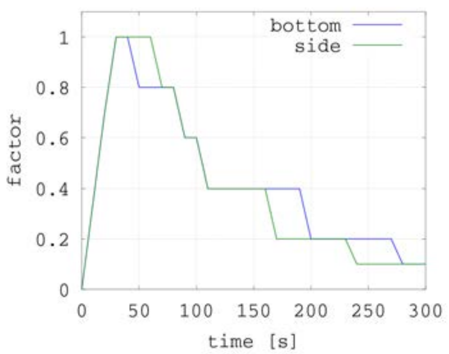

Figure 8 Heating profiles in time

The resulting temperatures at the 5 sensor spots are shown in Figure 9. At all times we obtain a more uniform temperature distribution, compare to Figure 6. Based on these simulation results a physical device can now be built to warm a bottle of a liquid in a very uniform fashion.

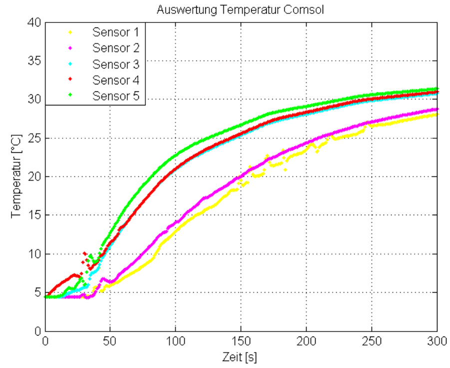

Figure 9 Temperature results, with nonuniform heating

By extending the heating time we obtained the rather uniform temperature profile in Figure 10 at a final time of t=420 sec.

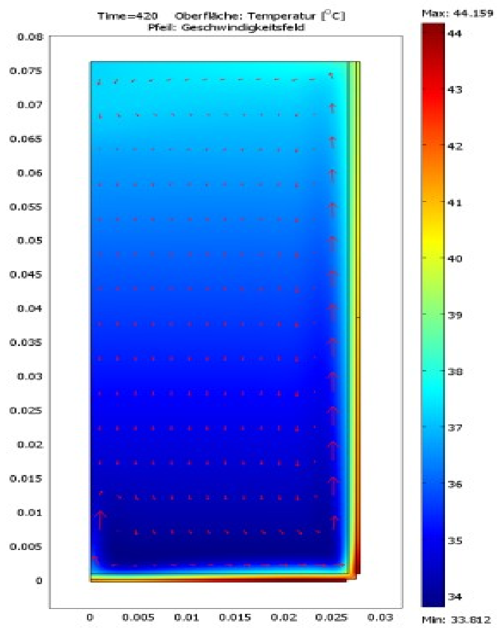

Figure 10 Temperature results, with longer and nonuniform heating

6. Acknowledgements

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