Computational and Experimental Study on Electrostatic Properties of Fractal Capacitors

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Abstract

The dielectric properties of heterogeneous media are of increasing interest as many applications exploit the usage of dielectric property detection in bio-imaging, scanning, microwave tomography and other electromagnetic applications. Using Laplace's equation for non-integer dimensions, we model electrostatic behaviour in heterogeneous fractal media. An equation for capacitance is then derived. Dielectric constant for media with multiple interfaces is universally governed based on the effective medium theory. For fractal geometry, it is also a function of the fractal dimension. Both the effective medium theory and the employment of fractal theory allows us to calculate the fill factor and hence the effective permittivity of the medium through the parallel plate capacitance. Conversely, by measuring the parallel plate capacitance through COMSOL simulation and actual LCR measurements, we can derive the effective permittivity of the medium. By working backwards, we plot capacitance against the physical dimensions to validate that fractal theory adequately models electrostatic problems in heterogeneous media. We perform electrostatic simulations on COMSOL and perform actual LCR measurements for simpler 3D printed fractal geometries to prove that our model is correct.

We begin the experiment by analysing cantor sets (a simple enough fractal shape), in which the fractal nature is exhibited only along the axis of the parallel plate separation distance. We first perform practical measurements of capacitance using an ISOTECH-1701 LCR meter for the cantor sets and analyse if the effects of fractal nature is observed when capacitance 'C' varies with plate separation distance 'd' according to our mathematical model. We then perform simulations on COMSOL that validate our experimental results. We obtain a good agreement among our theoretical model, COMSOL simulation, and the actual experimental data. Once we have validated our model in both COMSOL simulation and real practical experiments on simple cantor sets, we proceed with more complicated fractal shapes that can be simulated on COMSOL. We choose three more fractal shapes: the Cantor Bar, the Sierpinski Carpet, and the Menger Sponge. We obtain the scatter plot of the capacitance 'C' against geometric parameters of these fractal structures, and compare it with our theoretical model. This verifies that the solution to Laplace's equations in non-integer dimensions can model heterogeneous dielectric media with fractal geometries, while accommodating classical electrostatics in integer-dimensions.

Figures used in the abstract

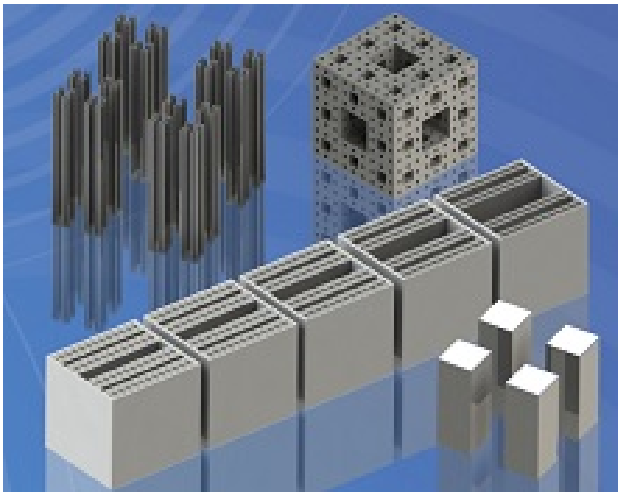

Figure 1: "Does electrostatic behaviour of fractal geometries also exhibit fractal nature?"