Design and Simulation of Unimorph Piezoelectric Energy Harvesting System

E.varadrajan[1],M.Bhanusri[2].
1. Research and Innovation Centre (RIC), IITM Research Park, Chennai, Tamil Nadu, India, evaradarajan@ric.drdo.in
2.Department of Physics,Indian Institute of Technology (IIT) Madras, Chennai, Tamil Nadu, India

***Abstract*—In this paper we made an attempt to maximize the power output in the different piezoelectric materials in a unimorph cantilever beam configuration. In this research, a macro -scale uni-morph piezoelectric power generator prototypes consists of an active piezoelectric layer, stainless steel substrate and titanium proof mass was designed for frequencies 60 Hz - 200 Hz[. An analytical model of a micro power generator is used to obtain displacement, voltage and generated power which are the figures of merit for energy harvesting. This model is presented for three different piezoelectric materials like, PbZrTiO3 (PZT), PVDF and PMN-PT. The designed unimorph piezo energy harvesting system was modeled using COMSOL multi physics and the observed parameters are compared with analytical results.**

I. Introduction:

Energy harvesting is used for capturing minute energy from surrounding sources, accumulating them and storing them.With recent advancements in wireless technology,energy harvesting is highlighted as alternative for conventional battery. While there are different ways through which energy is harvested, piezoelectric devices shows a great promise. Piezoelectric materials have the property of producing electrical charge when strained. This is called direct piezoelectric effect.On the other hand,these materials undergo deformation when an electric field is applied.This is called converse piezoelectric effect.This property of piezoelectric materials is used in converting vibrational energy to electrical energy which may be stored and used as an alternative power source for portable electronics.In recent advancements,energy harvesting have attracted considerable attention as an energy source for wireless sensor networks beacuse batteries cause a series of inconviences like limited operating life,size and contamination issues.Solar energy provides some solutions but it is limited in dark conditions. Piezoelectric devices are proved to be the potential source for power generation[1].Therfore they serve as a good alternative for conventional batteries.

A. The Piezoelectric cantilever configuration:

There are two types of piezoelectric materials, piezoceramics like Lead Zirconate Titanate(PZT) and piezopolymers like Polyvinylidene Fluoride(PVDF).When piezoelectric materials are deformed or stressed, voltage appears across the material. The mechanical and electrical behavior can be modeled by two constitutive equations[2]

$$S = \mathrm{s}^E T + \mathrm{d}_t E \tag{I.1}$$

$$D = \mathrm{d}_t T + \epsilon^T E \tag{I.2}$$

where S-mechanical strain, T-applied mechanical stress, E-Electric field, D-Electric displacement, s^E- matrix of elasticity under conditions of constant electric field,d-piezoelectric coefficient matrix, ϵ^T=permittivity matrix at constant mechanical strain.A cantilever type vibration energy harvesting has very simple structure and can produce large deformation under deformation. The cantilever model can be used in two different modes,33 mode and 31 mode. The 33 mode(compressive mode) means the voltage is obtained in the 3 direction parallel to the direction of applied force. The 31 mode(Transverse mode) means the voltage is obtained in 1 direction perpendicular to the direction of applied force(3). The most useful mode in harvesting applications is 31 mode, because an immense proof mass would be needed for 33 configuration[1].The vibration spectrum shows that the acceleration decreases[1] for higher modes of frequency compared to fundamental mode of frequency. Therfore, the design of the cantilever beam focusses on fundamental mode of frequency.

II. Governing equations and theory:

A. A simply supported cantilever beam:

The resonant frequency of an cantilever without a proof mass for a simply supported cantilever beam is given by

$$f_n = \frac{\nu_n^2}{2\pi}\sqrt{\frac{EI}{12AL^4}} \tag{II.1}$$

where, E-Young's modulus,I-Moment of inertia,A-Area,L-Length of the cantilever beam

ν_n=1.875 for fundamental mode, ν_n=4.694 for second mode.The simulation is done in comsol and both the frequencies are compared. Different modes are shown below:

B. Unimorph cantilever configuration

Cantilever beam piezoelectric generator has three types unimorph, bimorph series and parallel configurations.When the beam has only one piezolelectrical layer attached to the substrate,the device is known as unimorph.On the other hand,if a metal shim is sandwiched between two piezoelectric layers,the device is known as bimorph. For energy harvesting, an unimorph structure is chosen. One of the most important design parameter in designing a vibration energy harvesting device is resonant frequency. The power density would be maximum when the vibration frequency matches the resonant frequency of piezoelectric generator. It has been proved that power density decreases when resonant frequency deviates from the vibration frequency[1]. The frequency range of common environmental vibrations is between 60 Hz and 200 Hz[1].Moroever acceleration decreases with higher modes of frequencies[1].Therfore fundamental mode is considered in designing the cantilever.The unimorph cantilever configuration looks as in Fig0.2.2.

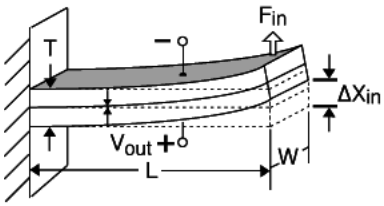

Figure II.1. Unimorph cantilever

The frequency of an unimorph cantilever is given by[3]

$$f = \frac{\nu_n^2}{2\pi}\sqrt{\frac{0.236D_p w}{(l-\frac{l_m}{2})^3(m_e+m_p)}} \quad \text{(II.2)}$$

$$\nu_n = 1.875$$

$$m = \rho_p t_p + \rho_s t_s \quad \text{(II.3)}$$

$$m_e = 0.236mw(l-\frac{l_m}{2}) + mw\frac{l_m}{2} \quad \text{(II.4)}$$

$$D_p = \frac{(E_p^2 t_p^4 + E_s^2 t_s^4 + 2E_pE_st_pt_s(2t_p^2+2t_s^2+3t_pt_s))}{12(E_pt_p+E_st_s)} \quad \text{(II.5)}$$

where,
E_p =Young's modulus of piezoelectric material,
E_s = Young's modulus os substrate,
l_m = Length of proof mass,
$l = l_b$= Length of the beam,
$w = w_b = w_m$ =Width of the beam,
t_p= Thickness of piezoelectric material,
t_s= Thickness of substrate,
m_p =Proof mass,
ρ_p= Densityof piezoelectric material,
ρ_s= Density of substrate material.:

The dimensions of a cantilever are chosen such that the frequency range is between 60Hz and 200Hz.The dimensions and parameters of cantilever are shown in table below:

Table I
DIMENSIONS OF CANTILEVER

$l_b(cm)$	$w_b(cm)$	$t_b(cm)$	$l_m(mm)$	$w_m(cm)$	$t_m(mm)$
6	3	0.1	12	3	3.5

The parameters of the cantilever are shown below:

Table II
PARAMETERS OF CANTILEVER

$E_p(Mpa)$	$E_s(Gpa)$	$t_p(mm)$	$t_s(mm)$	$\rho_p(Kg/m^3)$	$\rho_s(Kg/m^3)$
2450	205	0.11	1	1770	7850

C. Energy parameters of unimorph cantilever[6]:

$$Q = \frac{-3d_{31}s_ss_pt_s(t_s+t_p)l^2F}{B} \quad \text{(II.6)}$$

$$s_s = \frac{1}{E_s}, s_p = \frac{1}{E_p} \quad \text{(II.7)}$$

$$s_h = s_st_p + s_pt_s \quad \text{(II.8)}$$

$$B = s_s^2t_p^4 + 4s_ss_pt_st_p^3 + 6s_ss_pt_s^2t_p^2 + 4s_ss_pt_pt_s^3 + s_pt_s^4 \quad \text{(II.9)}$$

$$V = \frac{-3d_{31}s_ss_pt_st_p(t_s+t_p)lF}{\varepsilon_{33}^TwB(1+(\frac{3s_p^2s_st_pt_s^2(t_p+t_s)^2}{s_hB}-1)K_{31}^2)} \quad \text{(II.10)}$$

$$U = \frac{-9d_{31}^2s_ss_p^2t_s^2t_p(t_s+t_p)l^3F^2}{\varepsilon_{33}^TwB^2(1+(\frac{3s_p^2s_st_pt_s^2(t_p+t_s)^2}{s_hB}-1)K_{31}^2)} \quad \text{(II.11)}$$

III. DESIGN OF UNIMORPH CANTILEVER USING COMSOL

The different modes of a simply supported cantilever beam are shown below:

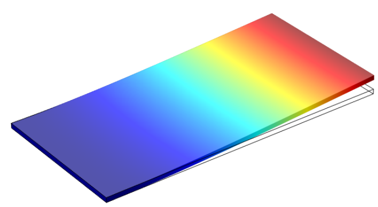

Figure III.1. First mode

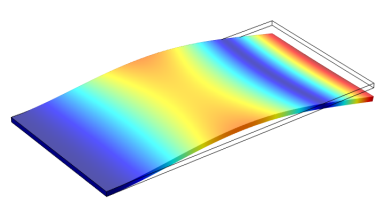

Figure III.2. Second mode

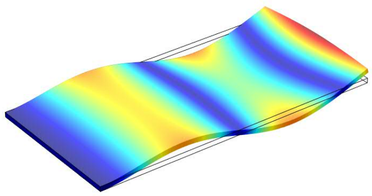

Figure III.3. Third mode

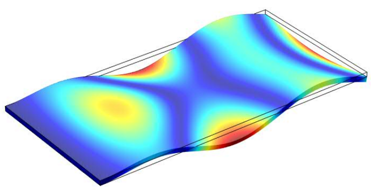

Figure III.4. Fourth mode

Table III
COMAPRISION OF SIMULATED AND ANALYTICAL FOR DIFFERENT MODES

	Analytical	Simulated	error(%)
First	1290.015	1328.695	2.9
Second	8084.96	8271.757	2.3

The model is designed in comsol.A 3 dimensional unimorph cantilever is used for the simulation in comsol. Piezopolymer material PVDF is used as a piezoelectric and stainless steel is used as substrate.It has been proved that cantilever beam with higher effective mass and less damping factor gives high output power.The proof mass not only increases effective mass but decreases damping.So the cantilever beam with proof mass has the power 10 times of the power of the cantilever beam without proof mass[4].Therfore, a proof mass made of titanium is used.The power is maximum when non-piezoelectric length and piezoelectric lengths are equal[5].So the lengths of substrate and piezomaterial are made equal.Using solid mechanics module ,one end of the model is fixed and the other end is made to move freely.The eigen frequency analysis is done.The frequency of 153.22Hz is designed using comsol.The analytical and simulated results vary by 1.82%.The designed model is shown below.The model consists of non piezoelectric material made of steel,piezoelectric material made of pvdf and proof mass made of titanium.

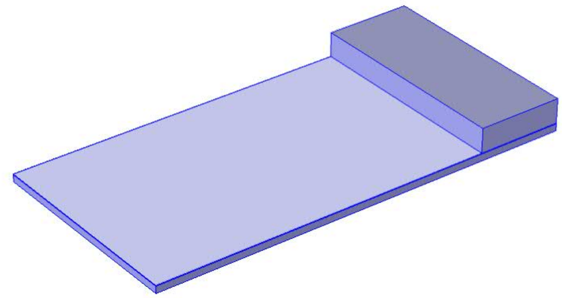

Figure III.5. Designed model in comsol

1) Meshing:: The model is meshed with physics contolled mesh amd element size fine.The meshed model looks as follows:

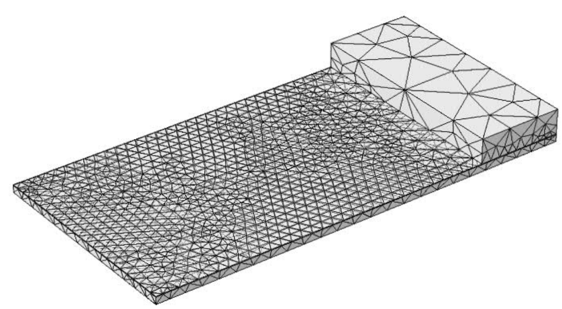

Figure III.6. Meshed model

2) Model shape:: The study of the model is carried out with the eigenfrequency step.The model shape looks as below:

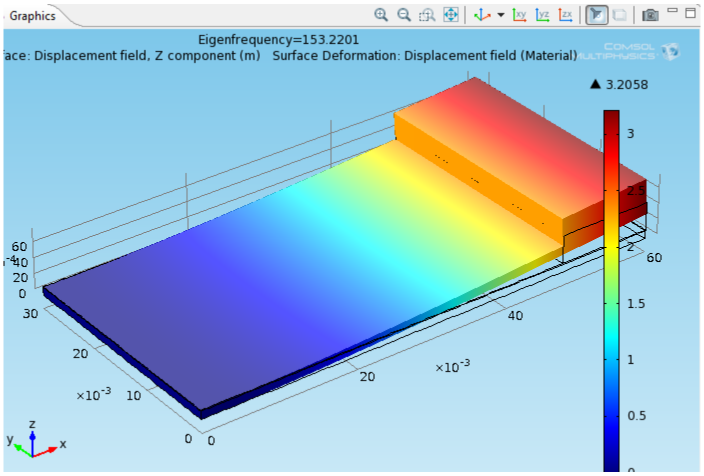

Figure III.7. Designed model of frequency 153.22 Hz

IV. Results and Discussions

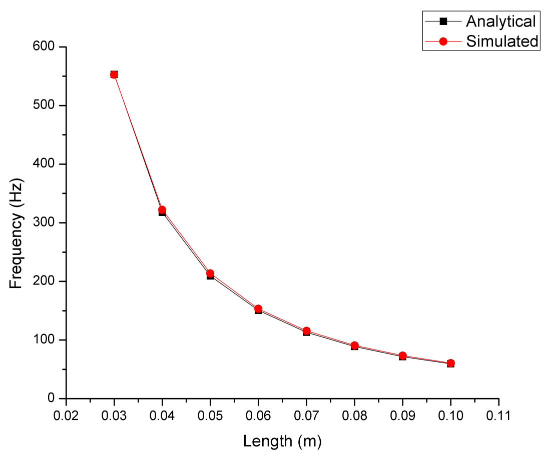

Figure IV.1. Variation of frequency with the length of the beam

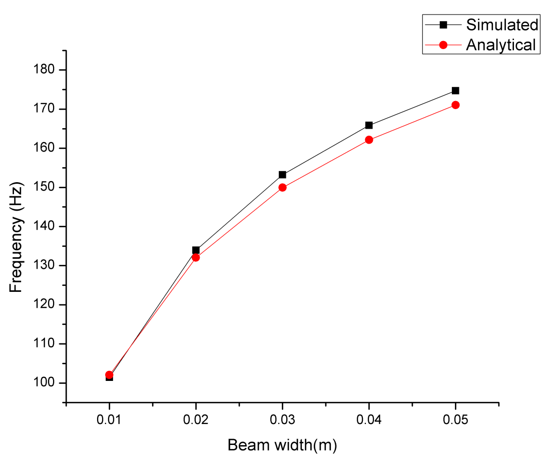

Figure IV.2. Variation of frequency with the width of the beam

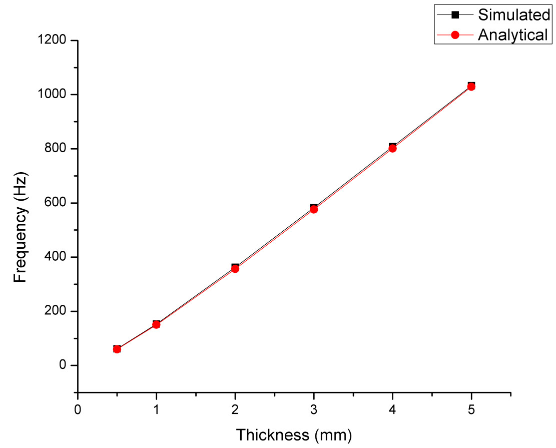

Figure IV.3. Variation of frequency with the thickness of the beam

The variation of frequency with length,width and thickness of beam are shown above.The thickness of the beam has great impact on the frequency of the cantilever.It is concluded from the graph that the frequency is directly proportional to the thickness of beam.As the thickness increases,stiffness increases which inturn increases the frequency.The width of the beam has no significant effect on the frequency compared to length and thickness of beam.The frequency increased from 100 to 180 Hz as width increased from 0.01 to 0.05 metre.The length is inversely proportional to the frequency.After some point, the change in frequency is reduced.The desired frequency can be obtained for the unimorph cantilever structure considering these variations in design parameters.

A. *Sensitivity of a unimorph cantilever:*

The design parameters of an cantilever would affect the charge,voltage and energy produced by an unimorph cantilever.The variations are shown below.

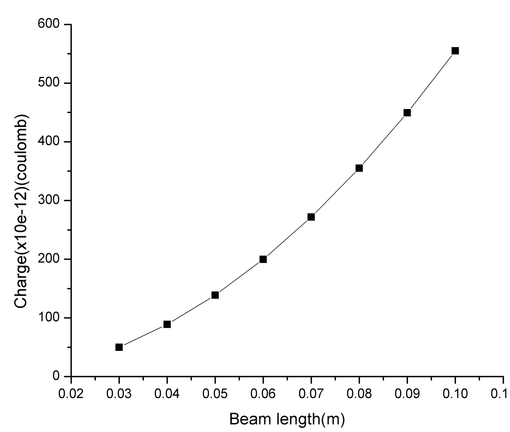

Figure IV.4. Variation of charge with length of beam

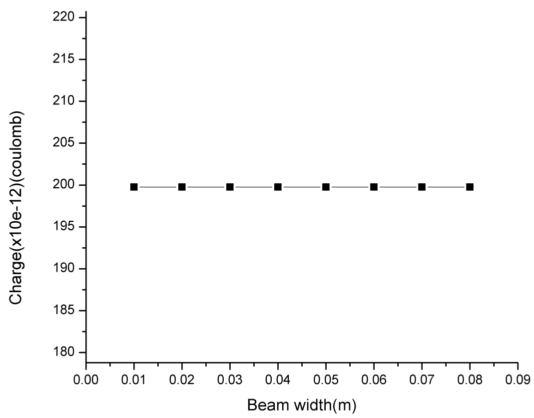

Figure IV.5. Variation of charge with width of the beam

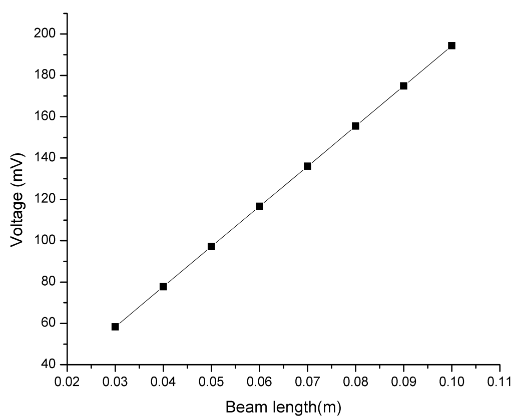

Figure IV.6. Variation of voltage with length of beam

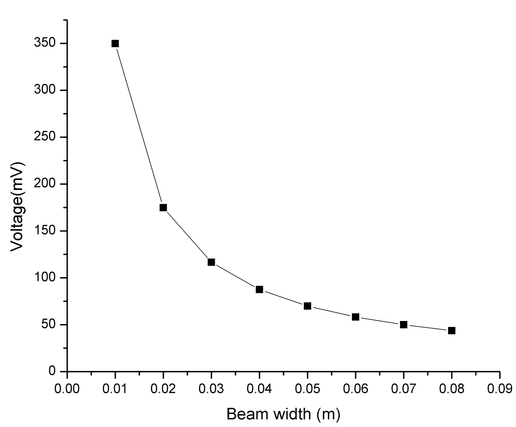

Figure IV.7. Variation of voltage with width of the beam

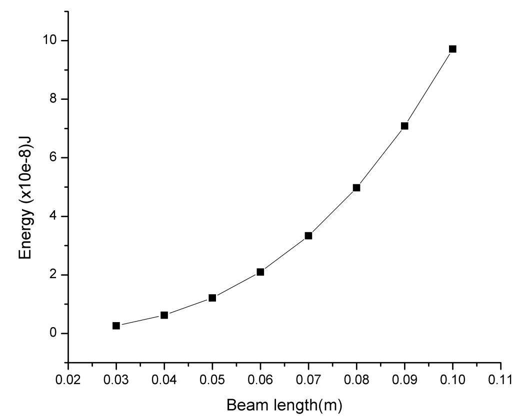

Figure IV.8. Variation of energy with length of beam

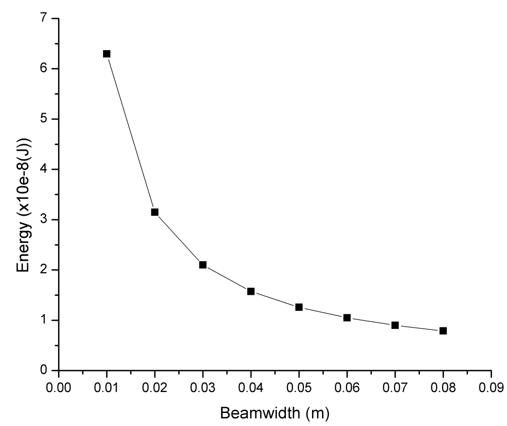

Figure IV.9. Variation of energy with width of the beam

The width of the beam does not effect the charge produced.As the length of the beam increases,the charge produced also increases.While length is directly proportional,width is inversely proportional to the voltage produced.The length of the beam increases the energy produced.The width of the beam decreases the energy produced.

B. Comparision of different piezoelectric materials

The comparision of sensitivities of different materials is done.The table below illustrates the comparision between these materials[7][8].

Table IV
COMPARISION OF DIFFERENT PARAMETERS FOR DIFFERENT MATERIALS

$\frac{Material}{Parameters}$	PVDF	PZT-5H	PMN-0.33Pt
Capacitance(Cp)	1.936nF	0.548μF	0.564nF
Charge(Q)	199.76x10^{-12}C	52.39x10^{-9}C	39.17x10^{-9}C
Voktage(V)	116.6mV	123.2mV	1209.7mV
Energy(J)	2.099x10^{-8}J	5.815x10^{-76}J	3.3x10^{-5}J

The following graph shows the comparision between analytical and simulated voltage at different values of acceleration

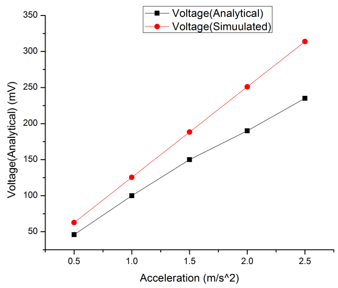

Figure IV.10. Comparisio of simualted and analytical voltage with different values of acceleration

V. Conclusions

From the above results,PVDF is chosen to be an appropriate material for unimorph energy harvesting system.

VI. References:

1) Roundy, S., Wright, P.K and Rabaey, J., "A study of low level vibrations as apower source for wireless nodes," Computer Communications, Vol.26, pp.1131-1144,2003
2) Roundy, S. and Wright, P.K., "A piezoelectric vibration based generator for wireless electronics," Smart Materials and structures,Vol 13, N0.%,pp.1131-1142,2004
3) Shen, D., Park,J.H., Noh,J.H., Choe,S.Y., Kim S.H., Wikle., H.C. and Kim,D.J., "Micromachined PZT cantilever based on SOI structure for low frequency vibration energy harvesting," Sensors and actuators A:physical,Vol 154,No.1,pp.103-108,2009
4) Choi,W.J ., Jeon, Y., Jeong, J.H., Sood, R. and Kim S.G.,"Energy harvesting MEMS device based on thin film piezoelectric cantilevers," Journal of Electroceramics,Vol.17,N0.2-4,pp.543-548,2006.
5) Xiaotong Gao,1 Wei-Heng Shih,1,a and Wan Y. Shih2 "Vibration energy harvesting using piezoelectric unimorph cantilevers with unequal piezoelectric and non-piezoelectric lengths Applied physics letters 97 233503,2010
6) He Qing a, Yan Zhen b, Fei Likai, Song Bo "Modelling and analysis of piezoelectric vibrating generator of cantilever, "Applied mechanics and materials vols", 148-149, pp 1327-1330 ,2012
7) R.Ambrosio,A.Jimenez,J.Mireles,M.Moreno,K.Monfil & H.Heredia "Study of piezoelectric energy harvesting based on PZT", "Integrated Ferroelectronics",126:1,77-86
8) Hu Cao ,V. Hugo Schmidt,Rui Zhang ,Wenwu Cao,Haosu Luo,"Elastic, piezoelectric, and dielectric properties of 0.58Pb Ñ Mg 1 ' 3 Nb 2 ' 3 Ö O 3 -0.42PbTiO 3 single crystal","Journal of applied physics",vol 96,No 1,July 2004.