Adaptive Numerical Simulation of Streamer Propagation in Atmospheric Air

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Abstract: In this paper, we report on a 2D axis-symmetric numerical model of the streamer-type discharge process in atmospheric air, as it can be implemented into COMSOL Multiphysics. The charge conserving Boltzmann drift-diffusion equations are solved in a logarithmic representation for improved numerical stability, whilst reducing the need for artificial diffusion terms. A charge density adaptive mesh is utilized and the calculation domain is reduced to the relevant dimensions for improved numerical performance. The calculation domain for the Poisson-equation is spatially extended as appropriate to the physical boundary conditions. Results of the so obtained numerical simulations of a nanosecond discharge are analyzed and compared with previously published experimental data.

Keywords: streamer propagation, numerical simulation, adaptive meshing, logarithmic formulation

1. Introduction

In many practical cases, an electrical breakdown of gaseous insulation is preceded by so called streamer discharges. These are self-sustained ionization avalanches travelling through an initially neutral gas at a sufficiently high electric field. Streamer development in a gas is mainly determined by the external field itself, by ionization properties of the gas, and by the dynamics caused by the space charges created in that field, including photo-ionization processes. At atmospheric pressure in air, streamers are typically detected as thin plasma channels between energized electrodes to which a voltage is applied. The various stochastic structures of such observed cathode or anode directed filaments, which are usually attributed to streamer discharge, though, cannot be obtained by a set of field-type equations, since such a model assumes smooth and continuous field properties for a gas in local electrodynamic equilibrium. No thermal effects are considered here. Thermal instabilities and magnetic forces may trigger filamentation. Nevertheless, knowledge on the conditions for growth of a streamer as an initial ionization avalanche, that can fully bridge the gap between electrodes, allows to identify the critical breakdown condition. Computer simulations are considered as a powerful tool complementing experimental investigations. Simulations of streamers are carried out via different approaches, like hydrodynamic approximation, particle in cell models, equivalent circuits, and others. Hybrid methods involving combinations of these models have been also reported in literature.

Most often the drift-diffusion method is employed, sometimes also called the hydrodynamic approach, which is derived as zero-moment of Boltzmann's equation for energy distribution function [1]. This model describes a collective motion of electric charge carriers, electrons and ions, driven by an electric field, in terms of their densities and fluxes and it is governed by partial differential equations (PDEs) of the continuity or drift-diffusion. Rates of individual physical processes like electron impact ionization, photoionization, electron attachment, or charge recombination are accounted for as source and sink terms. Solving such PDEs numerically is challenging. Each term is non-linear in field and/or charge density. Strong gradients of the charge carrier densities develop. The drift terms are dominant over the diffusion term. Thus, the Peclet number for electrons transport may reach a value of ~105, limiting the applicability of conventional numerical stabilization techniques, which in return can lead to non-physical numerical artifacts, like negative charge carriers densities.

In regions where steep charge density gradients exist (typically more than 10 orders of magnitude within ~50 mm distance), numerical instabilities arise and become magnified with simulation time. To deal with such problems, some numerical techniques originally developed in other fields, like flux-corrected transport and other flux-limiting methods [1], have been adopted for modeling gas streamer discharges.

In the present study, though, facilities readily provided with COMSOL Multiphysics are used to overcome the described difficulties.

2. Theory and Model Formulation

Streamer discharges are usually initiated by an electron avalanche in gas formed by free electrons accelerated in an applied electric field. These free electrons are produced by cosmic radiation, background radioactivity, etc. If the gas pressure (density) is sufficiently high, the electrons will collide with neutral gas molecules and if the energy acquired on a free path between collisions is high enough, they eventually cause ionization leading to an increase in their number. The electrons may also be lost, e.g., due to recombination with produced ions. A ration between the rates of processes of generation and losses decides if there will be exponential rise in the availability of free electrons, which is known as an electron avalanche. Under certain conditions, the number of charge carriers in an avalanche may become so high that the created space charge may screen out the external electric field and further development of the discharge is governed by its own electric field and is supported by photoionization in the vicinity of the plasma front. Such discharges are essentially ionization waves propagating in gas in form of plasma channels and are called streamers.

A set of PDEs describing the evolution of charge carriers densities in space and time typically consists of convection-diffusion equations (CDE) expressing conservation of mass for each type of charge species coupled with Poisson's equation for the electric potential. In case of air, three generic types of charge carriers are usually considered (electrons, positive and negative ions) if a detailed plasma chemistry is not of interest, and the following equations are applied

$$\frac{\partial N_e}{\partial t} + \nabla \cdot (-D_e \nabla N_e + \mu_e N_e \mathbf{E}) = S_e + S_{photo} \quad (1)$$

$$\frac{\partial N_p}{\partial t} + \nabla \cdot \left(-D_p \nabla N_p + \mu_p N_p \mathbf{E}\right) = S_p \quad (2)$$

$$\frac{\partial N_n}{\partial t} + \nabla \cdot (-D_n \nabla N_n + \mu_n N_n \mathbf{E}) = S_n \quad (3)$$

Here, subscripts e, p and n indicate electrons, positive and negative ions, respectively; N stands for the density; D is the diffusion coefficient; μ is the mobility of charged species; $\mathbf{E}$ is the vector of the electric field, and t stands for time. The net sources S are defined on the right hand sides of the equations. These terms includes the rates of impact ionization, recombination, electron attachment, detachment etc. The impact ionization is generation of electron and positive ion when a electron hits a neutral particle. The ionization rate is dependent on how energetic the striking electron was. This energy is provided by the acceleration of electron in the applied E field. The electron attachment is the capture of free electron by electronegative species like Oxygen etc. The ionization and attachment rates used are based on the work of Hagelaar and Pitchford [2].The electron cross section data is taken from experiments [3,4] and reaction rate is calculated as a function of E field. There are also some empirical relations available [5,6,7].The recombination rate is taken E field independent [5]. This is the rate at which positive and negative species combine to form neutral species.

The rate of photo-ionization in the equations for electrons require special consideration because this mechanism is essential for streamer development. The rate S_{photo} can be calculated either by utilizing an integral or a differential formulations. In the former case, integral models [8] are utilized, which involves operations with full matrixes that is computationally expensive. To avoid this, differential models have been proposed recently [9]. One of these is based on a solution of radiation transport problem and utilizes Helmholtz formulation by approximating the absorption coefficients of the gas. Within this model, the photo-ionization rate is obtained by solving Helmholtz equation

$$\nabla^2 U - \left(\lambda_j p_{O_2}\right)^2 U = -A_j \, p^2{}_{O_2} I(r) \quad (4)$$

where U is the intensity of photoionization per unit volume; λ_j and A_j are fitting parameters; p_{O_2} is partial pressure of oxygen; and $I(r)$ is the photon production rate proportional to the ionization rate. In the present study, three term approximation (j = 3) is adopted, i.e. three Helmholtz equations are added to the model to obtain the rate S_{photo}. Since streamer development is a space charge controlled process and most of the parameters in the PDEs are field dependent, a distribution of the electric field in space should be known at each instant. Therefore, the CDEs are to be coupled with Poisson's equation:

$$\varepsilon\nabla^2 V = e(N_p - N_e - N_n) \qquad (5)$$

Here V is the electric potential, ε is permittivity, and e is charge of electron. The electric field is obtained as $\mathbf{E} = -\nabla V$.

To form a consistent set of PDEs, all the equations above require boundary and initial conditions which are defined based on properties of a system to be analyzed, i.e. geometrical shapes of electrodes, materials involved, type of the applied voltage, etc.

3. Test Case and Model Implementation

To allow for verification, the simulations in the present study were conducted for the conditions used in the experimental investigations [10]. Recent experiments [11] with sub-millisecond resolution showed a first (straight) streamer followed by a second branched streamer to precede the breakdown. Nanosecond resolution imaging of a streamer [10] showed the growth in more detail, which finally allows for direct qualification of simulation models.

The experiments were carried out in atmospheric air. A stepped positive voltage of 28 kV with the rise time of 150 picoseconds was applied between point and plane electrodes separated by 15 mm gap. The anode was a conical needle with 0.1 mm tip that resulted in strongly inhomogeneous field distribution. At the given voltage, the electric field strength at the tip of the needle electrode was approximately in one order of magnitude stronger than the critical field in air, i.e. ~3 MV/m and the detected plasma region had a spherical shape expanding towards the plane.

The first step in solving the streamer propagation problem included defining field dependencies of source terms in the CDEs that involved in particular calculations of ionization and electron attachment coefficients. These were obtained from the solution of Boltzmann equation (two-term approximation) and the result is compared with that obtained from well-known BOLSIG solver in Figure 1.

Further, the problem formulated in section 2 was implemented for the experimental conditions [10] using the general mathematical interfaces of COMSOL Multiphysics. This provided a possibility of utilizing a logarithmic representation of the CDEs and to define their weak formulation. Such approach imposed the physical constraint of non-negative charge carriers' densities without a necessity of introducing corrections for the source terms or any artificial diffusion.

To resolve strong variations of the field near electrodes and at the tip of propagating streamer head, an unstructured mesh was utilized with non-uniform discretization of solution domain. To reduce the solution time, the transport domain and Poisson domain were separated. A field adaptive meshing along the direction of streamer propagation was employed, which allowed for application of a coarser mesh in the main domain but still resolving the steep gradients concentrations at streamer front. An example of the computational mesh is shown in Figure 2.

The set of the PDEs was solved with implicit time stepping using backward differentiation formula of first or second order. A damped newton solver iteratively solved the equations in each time step with continuous update of Jacobian. The geometric multigrid scheme with successive over-relaxation as pre-smoother and successive over-relation of upper triangle as post-smoother in V-cycle worked well for the mentioned problem.

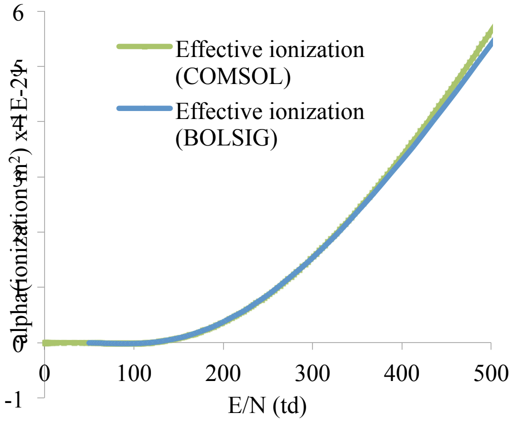

Figure 1. Effective ionization coefficient in air as a function of reduced field strength calculated using COMSOL and BOLSIG solvers.

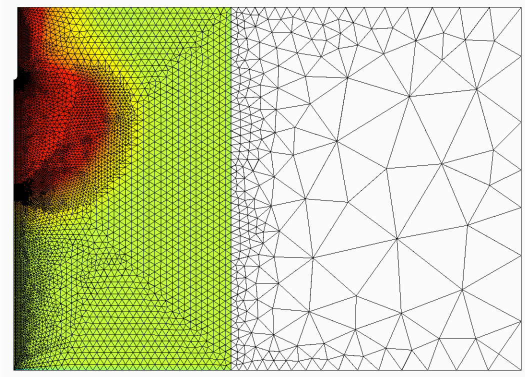

Figure 2. Adaptive mesh used in the charge transport domain (region on the left). Note that Poisson's equation was solved for the entire domain.

4. Results

The results of the simulations are compared with the experimental data in Figure 3 where the development of the streamer is shown with the time interval of 1 ns. As seen from the experiment, the discharge process starts by forming a spherical region near the anode tip due to the extremely strong electric field. After approximately 3 ns, the plasma front starts propagating that is observed as the elongation of the bright region towards the cathode. As seen, the simulations capture the time development quite well and there is good agreement in the shape of the ionizing wave with that recorded in the experiment.

5. Model Performance

The introduction of several techniques in the numerical model, such as logarithmic formulation of the CDEs, adaptive mesh refinement, separation of the domains for different physics, enhanced the efficiency of the model. Thus the solution time has reduced from few days in a workstation server to few hours on a present generation standard notebook.

6. Conclusions

The streamer propagation at high pressure is an important research topic in high voltage engineering. The developed approach allows for improving strongly the efficiency of the simulations and thus opens a possibility to model real life problems including complicated geometries, presence of solid insulating elements, complex gas mixtures, etc.

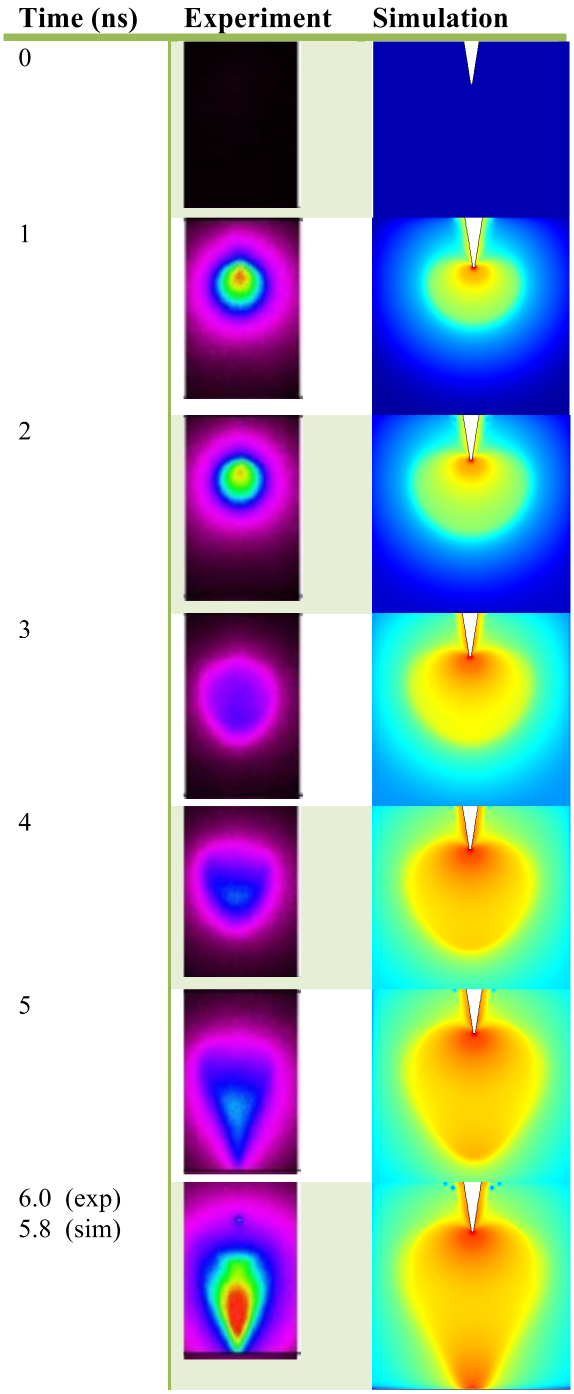

Figure 3. Comparison of experimentally recorded visual appearance of the streamer with the results obtained from the simulations.

7. References

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