# Optical Forces and Torques Acting on Non-Spherical Metallic Particles 

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## Summary

Metallic nanoparticles are nowadays used in various areas of biology chemistry or physics, e.g. they are used as a heat source causing hybridization of DNA or probes for surface enhanced Raman spectroscopy. Laser manipulation with such metal nanoparticles enables their precise delivery to the targets and their confinement for a given time. A single tightly focused laser beam - optical tweezers - was also employed to three-dimensional trapping of gold and silver nanoparticles with diameters between 20 to 250 nm [1, 2]. However, theoretical models assuming the spherical shape of a nanoparticle predict spatial confinement only for particles with diameter lower than 100 nm .

We show that the shape of nanoparticles is a key parameter that enables their trapping even by the low NA objective optical tweezers. Especially larger flat particles tend to reorient themselves perpendicularly to the beam polarization which strongly decreases their scattering and enables the optical trapping [3, 4].

## Golden Nanoparticles

Noble metals form variously shaped nanoparticles (NPs) during their growth because they crystallize in a face-centered cubic lattice. Various shapes may be prepared by the different chemical conditions of their growth, i.e. sphere, cube, cuboctahedron, octahedron, decahedron or various plates [5]. Typically mixture of particles is prepared. We studied golden NPs having nominal diameters 50 nm up to 250 nm produced by the British Biocell company. Figure 1 shows Scanning Electron Microscopy image of particles having nominal diameter 100 nm and its right part shows various particle shapes present. Note that no spherical particle is present.


Figure 1: Gold NPs (British Biocell, nominal diameter 100 nm ) observed via scanning electron microscope (Magelan, FEI). Right-hand column shows detailed images of various particle shapes: decahedral bi-pyramid, icosahedron, triangular and hexagonal plates.

## Optical tweezers

Optical tweezers (originally called "single-beam gradient force trap") are scientific instruments that use a highly focused laser beam to provide an attractive or repulsive force (typically on the order of piconewtons) to physically hold and move microscopic dielectric objects. Small objects are pushed towards the places of high electromagnetic field intensity due to the "gradient force" and accelerated along the field propagation direction by the "scattering force," see figure 2 a . If the resulting force changes sign from positive to negative stable trapping position occurs. This condition is hard to meet namely for metallic absorbing particles.


Figure 2: (a) "Gradient" and "scattering" forces occuring near fo
focused laser beam. (b) Experimental setup, for details see [6].
We used a low numerical aperture optical tweezers (NA $\sim 0.37-0.17$, corresponding beam waist radii $w_{0}=0.91-1.7 \mu \mathrm{~m}, \lambda \mathrm{vac}=1064 \mathrm{~nm}$ ), see figure 2 a , to manipulate and trap these golden NPs.

## Computational Methods

The calculation of optical force $\mathbf{F}$ and torque $\mathbf{T}$ is based on integrating Maxwell stress tensor $\hat{T}$ over a surface enclosing the studied object as follows

$$
\begin{aligned}
\langle\mathbf{F}\rangle & =\left\langle\oint_{S} \mathbf{n} \hat{T} \mathrm{C} S\right\rangle, \text { and }\langle\mathbf{T}\rangle=-\left\langle\oint_{S} \mathbf{n}(\hat{T} \times \mathbf{r}) \mathrm{d} S\right\rangle, \\
\text { where } \hat{T}_{i j} & =\left[\varepsilon E_{i} E_{j}+\mu_{0} H_{i} H_{j}-\frac{1}{2}\left(\varepsilon E^{2}+\mu_{0} H^{2}\right) \delta_{i j}\right]
\end{aligned}
$$

Problem of scattering
In order to evaluate the optical force and torque the full electromagnetic field has to be known. We describe the highly focused incident (background) field in analytically using the angular spectrum description [7] in cylindrical coordinates $r, \theta, z$
$E_{x}=-\frac{\imath}{2} k\left(I_{0}+I_{2} \cos 2 \theta\right), \quad E_{y}=-\imath k I_{2} \sin 2 \theta, \quad E z=-k I_{1} \cos \theta$, where
$I_{0}=\int_{0}^{a_{a}} A_{0} \sqrt{\cos \alpha} \sin \alpha(1+\cos \alpha) J_{0}(k r \sin \alpha) \exp (k z z \cos \alpha) d \alpha$, $I_{1}=\int_{0}^{\alpha_{0}} A A_{0} \sqrt{\cos \alpha} \alpha \sin ^{2} \alpha J_{1}(k r \sin \alpha) \exp (k z z \cos \alpha) d \alpha$, $I_{2}=\int_{0}^{\alpha_{0}} A_{0} \sqrt{\cos \alpha} \alpha \sin \alpha(1-\cos \alpha) J_{2}(k r \sin \alpha) \exp (k z z \cos \alpha) d \alpha$, where $J_{n}$ denotes the Bessel function of the first kind and $n$-th order. The amplitude $A_{0}$ can be related to the electric field intensity at the beam focus $E_{x}(0,0,0)=\frac{k}{15} A_{0}\left(8-3 \cos ^{5 / 2} \theta-5 \cos ^{3 / 2} \theta\right)$

## Comsol Model

- We use Radio Frequency Module to solve problem of electromagnetic scattering on gold nanoparticle of triangular decahedral shape - Background electric field is given by the equations above, integration is replaced by the explicit summation ( 11 terms)
- Numerical aperture of focusing lens is $0.5\left(\alpha_{a}=21.5 \mathrm{deg}\right)$
- Studied geometry consists of metallic object of refracive index $n=0.2851-7.36 \imath$ (variously oriented) which is enclosed by a spherical medium (refractive index $n_{m}=1.33$ ) and by a spherical PML layer, see figure 3


Figure 3: The model geometry and the shapes of golden nanoparticles - decaheral and triangular plates of various aspect ratios. Note, that all triangular plates have the same volume.

- up to $\sim 75000$ mesh elements, $>500000$ degrees of freedom - Optical forces and torques are evaluated as Boundary probes. Maxwell stress tensor defined by Comsol Multiphysics is employed - Power absorbed by the particle is calculated as a volume integral of resistive loses. Temperature increase may be calculated by approximation or using Heat Transfer Module
- Various particle locations with respect to incident beam are evaluated using Auxiliary sweep. Particle orientations and sizes are studied using Parametric sweep


## Optical Forces and Torques: Results

We calculated optical forces and torques acting on a decahedron and triangular plate. Both particles have the same volume as the sphere having diameters $100,150,200$ and 250 nm ; we considered the aspect ratios of the object height and circumscribed circle diameter 0.3 or decahedron and 0.15 for triangular plate.
The optical torque orienting particle (tilted by 45 degrees with respect to beam polarization) is plotted in first row of figure 4. Negative $T_{z}$ orients decahedral particle parallel to polarization (left par of figure 4) while positive $T_{z}$ orients triangular plate perpendicular to the beam polarization Middle row of figure 4 shows the optical force along beam propagation axis for the particle oriented to its stable orientation. We can see that decahedrons with $d_{\text {eff }}=100$ and 150 nm are stably trapped and that all sizes of triangular plates are trapped. Bottom row shows electromagnetic power $P_{\text {abs }}$ absorbed by the particle in different $z$.


Figure 4: (top) Optical torques acting on a decahedron (left) or a triangular plate (right) placed in a distance $z$ behind the focal point. The object is $45^{\circ}$ tilted with (right) placed in a distance $z$ behind the focal point. The object is $45^{\circ}$ tilted with respect to beam polarization. Different colors of curves correspond to $d_{\text {eff }}=100$ 250 nm . (middle) Optical force $F_{z}$ acting on a particle in stable orientation along beam propagation axis. Inset magnifies the negative forces indicating
ping. (bottom) Power absorbed by the particles in various positions.

Further, we calculated the effect of varying the aspect ratio of triangular plate height and the circumscribed circle diameter in order to obtain object stable orientation. Figure 5 shows the optical torque that rotates particle parallel to beam polarization $\left(T_{z}<0\right)$ or perpendicular to beam polarization ( $T_{z}>0$ ). Only very flat small ob ject are rotated perpendicular to polarization. Similarly, even thicker and larger objects are still oriented perpendicular to beam polarization. This suggests, that such larger and thicker objects may still be trapped in optical tweezers.
 Figure 5: Optical torques acting on a triangular plate having different aspect ra-
tios of plate height and circumscribed circle diameter. The object is $45^{\circ}$ tilted with respect to beam polarization placed in a distance $z=5 \mu \mathrm{~m}$ behind the fo cal point. Different colors of curves correspond to effective diameters $100-250 \mathrm{~nm}$ a spheres having the same volume.

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