Propagation of Cathode-Directed Streamer Discharges in Air

Yuriy V. Serdyuk

Chalmers University of Technology

SE-412 96, Gothenburg, Sweden, e-mail: yuriy.serdyuk@chalmers.se

Abstract: Implementation of a drift-diffusion model of an electrical streamer discharge in air is presented. Numerical challenges associated with physical features of the modeled phenomena are highlighted. Results of simulations for two study cases demonstrating propagation of short and long streamers in air under normal conditions in a needle-plane electrode system are analyzed. Improvements needed to be introduced in the numerical model are discussed.

Keywords: streamer discharge, hydrodynamic model, drift-diffusion equations, charge transport

1. Introduction

Cathode-directed (also called positive) streamer electrical discharges are considered as a main cause of a complete electrical breakdown in air at atmospheric pressure. Streamers are usually formed in regions with strong electric fields which may exist, e.g., at a surface of an electrode with high curvature (needle, wire, etc.). Being formed, a streamer is able to propagate for a long distance even in space where the field is relatively weak. Visual appearance of streamer discharges varies depending upon conditions, in particular, gas density, pressure, applied voltage, etc. Under normal conditions in air, they are usually registered as filamentary plasma channels crossing gaps between electrodes. Development of such discharges is supported by strong fields at propagating plasma fronts which are generated by produced space charges. Therefore, a streamer is considered as a self-sustained ionization wave propagating in neutral gas which is converted into low-temperature plasma behind the wave front. Typically, the radius of a single streamer channel is of order of hundreds of micrometers and propagation velocity is in the range 10^5-10^7 cm/s.

Numerical simulations of streamers in air have attracted significant attention during last two decades due to practical needs, e.g., ongoing development of new insulation technologies for voltage levels above 1 MV for both dc and ac power transmission systems. For such applications, knowledge about physical mechanisms of electrical breakdown is essential and computer simulations are considered as a powerful tool for analyzing and predicting streamers properties. However, numerical solution of streamer problem is challenging and requires special numerical techniques for, e.g., accurate treatment of discontinuities appearing at propagating plasma front, effective meshing (adaptive time dependent) for resolving strong variations of the field at moving streamer tip and electrodes, etc. By today, numerous studies on streamers simulations reported in the literature have been conducted utilizing homemade codes and no commercial software is available which would offer a user friendly computational environment and a choice of instruments allowing for efficient implementation and solution of the problem. Therefore, an attempt to simulate development of streamer discharges in air utilizing features provided in Comsol Multiphysics has been undertaken and the results are presented below.

2. Governing Equations

The most simple and popular formulation of a streamer propagation model in air is based on so-called drift-diffusion (or hydrodynamic) approach within which variations of densities of electrons and two generic types of ions (positive and negative) in space and time are considered, see e.g. [1, 2]. Such formulation results in three partial differential equations (PDEs) of convection-diffusion type, which account also for rates of physical processes leading to generation and loses of charged species. The resulting set of PDEs can be represented in the following manner:

$$\partial n_e / \partial t + \nabla \cdot (-n_e \, \mu_e \, \mathbf{E} - D_e \nabla n_e) = R_e$$
$$\partial n_p / \partial t + \nabla \cdot (n_p \, \mu_p \, \mathbf{E} - D_p \nabla n_p) = R_p \qquad (1)$$
$$\partial n_n / \partial t + \nabla \cdot (-n_n \, \mu_n \, \mathbf{E} - D_n \nabla n_n) = R_n$$

Here: subscripts e, p and n indicate electrons, positive and negative ions, respectively; n is the density, m^{-3}; μ is the mobility, m^2/Vs; D is the diffusion coefficient, m^2/s; R is the net rate of the generation and loss processes, m^{-3}s^{-1}; $\mathbf{E}$ is the

vector of the electric field strength, V/m; and t stands for time. The main processes usually taken into account in (1) are represented by their rates: electron impact ionization $R_{ion} = \alpha\, n_e\, \mu_e\, E$; attachment of electrons to electronegative molecules (CO_2, H_2O, O_2, etc.) present in air $R_{att} = \eta\, n_e\, \mu_e\, E$; detachment of electrons from negative ions $R_{det} = k_{det}\, n_e\, n_n$; electron-ion recombination $R_{ep} = \beta_{ep}\, n_e\, n_p$; recombination of positive and negative ions $R_{pn} = \beta_{pn}\, n_p\, n_n$; natural background ionization R_0 and photo-ionization R_{ph}. In the expressions above, α is Townsend's ionization coefficient, m^{-1}; η is the attachment coefficient, m^{-1}; k_{det} is detachment coefficient, m^3/s; β stands for respective recombination coefficient, m^3/s. Hence, the net rates for different charged species are

$$R_e = R_{ion} + R_{det} + R_0 + R_{ph} - R_{att} - R_{ep}$$
$$R_p = R_{ion} + R_0 + R_{ph} - R_{ep} - R_{pn} \qquad (2)$$
$$R_n = R_{att} - R_{det} - R_{pn}$$

The set of PDEs (1) needs to be complemented by Poisson's equation for electric potential ϕ. The solution provides electric field distributions affected by the space charge which is needed for obtaining the kinetic coefficients and the rates of individual processes

$$\nabla \cdot (\varepsilon_0\, \varepsilon_r\, \nabla\phi) = -q(n_p - n_e - n_n),\ \ \mathbf{E} = -\nabla\phi \qquad (3)$$

Here, q is the elementary charge; ε_0 stands for permittivity of vacuum and ε_r is the dielectric constant of the material (unity for air).

Equations (1) and (3) with proper boundary and initial conditions (problem dependent) as well as kinetic and rate coefficients (2) form a self-consistent model which needs to be solved numerically due to its strongly non-linear nature.

According to modern theories, propagation of positive streamers in air is supported by photo-ionization of oxygen molecules at streamer front (where the field is strongest) by photons emitted due to quenching of highly excites states of nitrogen molecules. The rate of this process can be calculated using integral radiation transfer models that is, however, computationally expensive. Alternatively, it can be obtained utilizing the differential approach proposed in [3], where it is found as a sum $R_{ph}(\mathbf{r}) = \sum_j R_{ph}^j(\mathbf{r})$ of solutions of Helmholtz

equations which are used to represent the actual problem of radiation transport in a gas:

$$\nabla^2 R_{ph}^j(\mathbf{r}) - (\lambda_j\, p_{O_2})^2\, R_{ph}^j(\mathbf{r}) = -A_j\, p_{O_2}^2\, I(\mathbf{r}) \qquad (4)$$

Here, p_{O2} is the partial pressure of oxygen in air; $\mathbf{r}$ is the position vector; λ_j and A_j are fitting parameters. In the present study, two exponential fit is used with the parameters λ_j and A_j from [3].

The rate and kinetic coefficients used in the model are provided in Table 1. Dependences of the ionization and attachment coefficients on the field strength were taken from [2] and are reproduced in Figure 1. The dependencies of the electrons drift velocity and diffusion coefficient were approximated as $w_e = 3.2\cdot10^3\cdot(E/N)^{0.8}$ m/s and $D_e = 7\cdot10^{-2} + 8\cdot(E/N)^{0.8}$ m^2/s, respectively. In the latter expressions, E/N is the reduced field in Td (1 Td $= 10^{-21}$ Vm^2), N is the gas density, m^{-3}. The parameters mentioned were selected based on the performed literature analysis.
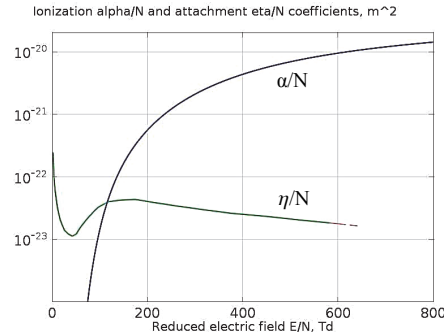
Figure 1. Ionization and electron attachment coefficients as functions of the reduced field E/N.

Table 1. Input parameters for the model

μ_p, m^2/Vs	$2.0\cdot10^{-4}$
D_p, m^2/s	$5.05\cdot10^{-6}$
μ_n, m^2/Vs	$2.2\cdot10^{-4}$
D_n, m^2/s	$5.56\cdot10^{-6}$
β_{ep}, m^3/s	$5.0\cdot10^{-14}$
β_{pn}, m^3/s	$2.07\cdot10^{-12}$
R_0, 1/m^{3}s	$1.7\cdot10^{9}$
k_{det}, m^3/s	10^{-18}

3. Model Implementation

The model implemented in Comsol Multiphysics 4.3a consisted of three convection-diffusion equations in conservative form together with Poisson's and two Helmholtz equations. To stabilize the solution process and to prevent appearance of numerical artifacts (e.g., negative concentrations) in the solution of (1), source term stabilization was utilized following the approach introduced in the Plasma module (see documentation).

The crucial part in the model implementation was meshing of the computational domain. As known, the thickness of the layer at the streamer head where separations of charges takes place is in the order of tens of micrometers. To be able to capture such a thin layer, an extremely fine mesh is required. Moreover, the mesh should follow the propagating plasma front. An ideal solution for this would be using the adaptive mesh refinement provided for time dependent solver. An attempt to incorporate this feature in the model was, however, unsuccessful at the time of conducting the simulations. Instead, the mesh refinement was done manually. For this, a frame with an extremely fine mesh (typical size of elements ~10 μm) was placed in front of the moving streamer head and the simulations run for a time interval long enough for the streamer to cross this region. Further, the calculations were interrupted, the window was moved to the front position, a mesh with elements 50-70 μm was generated to resolve the plasma channel behind the front, and the simulations were continued. In addition to this, static boundary layers were created around surfaces of electrodes to capture strong variations of the field.

Segregated time-dependent solver with continuous Jacobian update was used. Two segregated steps were assembled, on one step equations (1) and (3) were solved while on the second step equations (4) were resolved. Direct solvers were employed for both steps. For time stepping, BDF solver was utilized.

4. Study Cases

Two situations implementing different conditions for propagating streamer were modeled. In both cases, a hyperbolic needle-plate electrode system immersed in air at pressure 760 Torr and temperature 293 K was considered. A dc positive step voltage with the rise time 0.1 ns was applied between the needle and grounded plate. To avoid long formations stage, a discharge was initiated from a seeding charge spot of Gaussian shape ($n_e = n_p = 10^{20}$ m^{-3}, $\sigma = 30$ μm) placed at 0.1 mm from the needle tip. Under given conditions, a streamer propagated from the high field region at the needle (anode) towards the plane (cathode). The difference in the considered cases was in the length of the air gap between electrodes which was set to 5 mm and 30 mm (the applied voltage was +15 kV and +40 kV, respectively). In the first situation, the electrostatic coupling between the propagating streamer and the needle was retained during the entire process while for the longer distance this coupling was practically negligible and the discharge was self-sustained.

In the simulations, the geometry was reduces to 2D utilizing axial symmetry. Drift of ions was neglected due to short durations of the discharges under given conditions (~2 ns for 5 mm inter-electrode distance and ~30 ns for 30 mm gap). The boundary conditions for equations (1) on the electrodes were set to zero flux and outward flux for charge carriers on surfaces with the same and opposite signs of the potential, respectively.

The development of a streamer in the short gap is demonstrated in Figure 2. On the surface plots, one can observe concentration profiles of electrons progressing towards the plane electrode (bottom). The tip of the channel and the side boundaries are satisfactory resolved. However, some instability (see graph for 2 ns) can be noticed on radial boundaries of the channel. Nevertheless, the distributions of the densities along symmetry axis show that steep gradients of the densities of electrons were captured successfully and the absolute magnitudes of the concentrations are in the range 10^{19}–10^{21} m^{-3} that corresponds to the ionization degree 10^{-5}–10^{-4} typical for streamer discharges. The electric field distributions corresponding to the electron density profiles are presented in Figure 3. One may observe the enhanced field at the streamer tip and strongly reduced field strength in the plasma channel behind the front where the electric conductivity is high. Note the increase in the magnitude of the peak of the field occurring while the streamer tip approaches the cathode (plane) electrode. That reflects electrostatic interaction between the plasma channel having certain potential and the grounded electrode.

In the case of long streamer, the instabilities in the density profiles mentioned above are more pronounced as seen in Figure 4 (surface plots). The analysis of the simulation procedure showed that they occurred during several time steps after the manual adjustment of the mesh around the


| 0.5 ns | 1.5 ns | 2 ns |
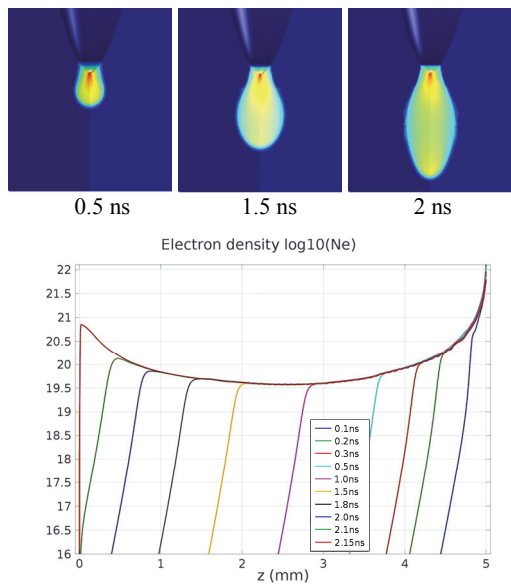
Figure 2. Electrons density profiles of a streamer propagating in 5 mm air gap. Line graph shows distributions along symmetry axis (the co-ordinate on the horizontal axis is accounted from the plane).

| 0.5 ns | 1.5 ns | 2 ns |
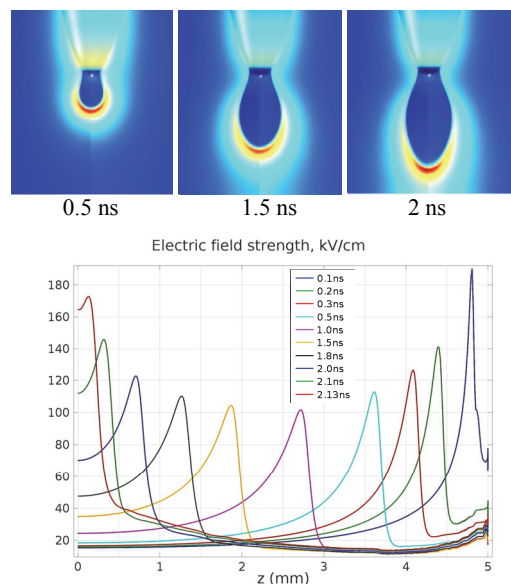
Figure 3. Electric field distributions corresponding to the electrons densities shown in Figure 2.

| 0.5 ns | 5 ns | 15 ns | 25 ns | 29 ns |
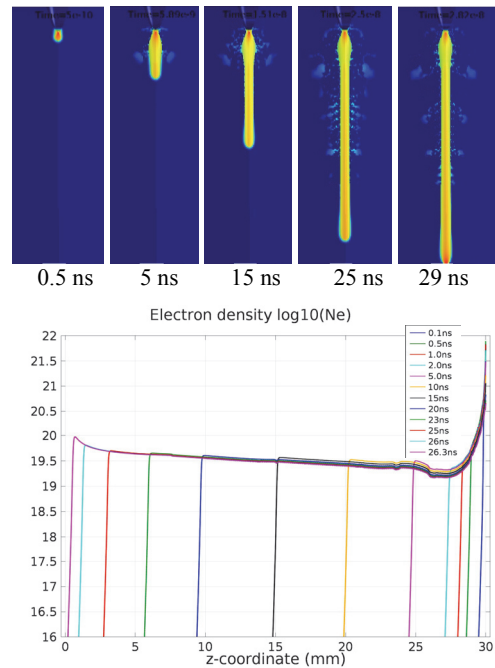
Figure 4. Electrons density profiles of a streamer propagating in 30 mm air gap. Line graph shows distributions along symmetry axis (the co-ordinate on the horizontal axis is accounted from the plane).

| 0.5 ns | 5 ns | 15 ns | 25 ns | 29 ns |
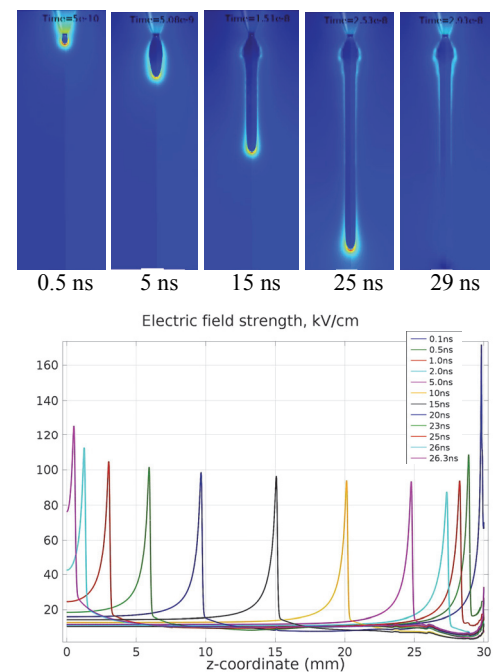
Figure 5. Electric field distributions corresponding to the electrons densities shown in Figure 4.

streamer channel and were associated with electron fluxes compensating the unbalance of charge carriers obtained on different meshes. This numerical effect was observed only on the radial boundaries of the channel and did not affect the propagating front where the density profiles remained stable during entire process as seen in Figure 4 (line graphs). The electric fields associated with the developing plasma channel are shown in Figure 5. As compared with the previous case, the field peak at the streamer head remains practically constant while the discharge expands into the gap. This indicates weak electrostatic coupling between the streamer and the electrodes during propagation.

The calculated macroscopic parameters of the streamers agree well with other simulations [4] and experimental results [5]. Thus, the estimated front velocity at stable propagation is $\sim 1.8 \cdot 10^6$ m/s for the short streamer and $\sim 1.0 \cdot 10^6$ m/s for the long one. The channel radiuses at stable propagation computed based on the rate of photo-ionization (assuming radiation in visible frequency range) are 400-500 μm and 200-300 μm for 5 mm and 30 mm streamers, respectively.

5. Conclusions

The drift-diffusion (hydrodynamic) model of non-thermal streamer discharges in air has been successfully implemented in the commercial software. The simulations carried out for two study cases resulted in streamers parameters which are in agreement with those obtained in other simulations and from experiments. This confirms that the developed model yields reasonable results. The simulation experience showed that proper meshing and numerical stability are crucial for the considered problem. For dealing with these issues, a moving frame with a fine mesh can be utilized, which would allow for capturing steep gradients at propagating plasma fronts. Implementation of a technique for flux corrections/compensations would strongly facilitate the simulations and enhance stability.

8. References

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