Computer-aided design of the heating section of a continuous kheer (rice pudding)-making machine

Swati Kadam¹, Tushar Gulati² and Ashis Datta¹

¹Department of Agriculture and Food Engineering, Indian Institute of Technology Kharagpur, India ²Department of Biological and Environmental Engineering, Cornell University, Ithaca, NY

Introduction

- Kheer is a popular Indian dairy dessert prepared from concentrating milk with simultaneous cooking of rice grains. Conventional methods of preparing kheer have limited its mechanized production. Therefore, a conceptual design of continuous *kheer*-making machine has been prepared which among other components consists of a heating section for cooking *kheer*
- The present study investigates the CFD analysis of fluid flow, heat transfer and fouling characteristics in the cooking section comprising of a helical coil heat exchanger heated with saturated steam on the outside.
- Development of such comprehensive models is appreciated to aid in efficient design of the continuous *kheer* making machine and to help overcome the adverse effects of milk fouling to obtain a product of desired quality

Problem Formulation Steam condensation (150°C) $U_o A_o (T_s - T_f) \delta t$ Inlet Outlet $A_{f}\rho_{f}C_{pf}\frac{\partial T_{f}}{\partial t}\delta x\delta t$ $A_{f}\rho_{f}C_{pf}\frac{\partial T_{f}}{\partial t}\delta x\delta t$ (127°C) $U_o A_o (T_s - T_f) \delta t$. The Heating Section **Assumptions: Steam condensation** rice and milk form an overlying continua for 3D modeling, flow is fully developed – T_f / T_d the transient period is assumed small and ignored Milk + Rice thermal equilibrium between rice and milk

Governing Equations

Fluid flow: k-ε turbulence model

Momentum balance

$$\rho(\vec{u} \cdot \nabla)\vec{u} = \nabla \cdot \left[-p\vec{I} + (\mu + \mu_T)(\nabla \vec{u} + (\nabla \vec{u})^T) - \frac{2}{3}(\mu + \mu_T)(\nabla \cdot \vec{u}) - \frac{2}{3}\rho k\vec{I} \right]$$
Turbulent kinetic energy equation:
$$\mu_T = \rho C_\mu \frac{k^2}{\varepsilon}$$

$$\rho(\vec{u} \cdot \nabla)k = \nabla \cdot \left[\left(\mu + \frac{\mu_T}{\sigma_k} \right) \nabla k \right] + P_k - \rho \varepsilon$$
----(1)

Turbulent dissipation energy equation:

$$\rho(\vec{u} \cdot \nabla)\varepsilon = \nabla \cdot \left[\left(\mu + \frac{\mu_T}{\sigma_{\varepsilon}} \right) \nabla \varepsilon \right] + C_{\varepsilon 1} \frac{\varepsilon}{k} P_k - C_{\varepsilon 2} \frac{\varepsilon^2}{k} \rho_k \qquad ----(3)$$

$$P_k = \mu_T \left[\nabla \vec{u} : (\nabla \vec{u} + (\nabla \vec{u})^T) - \frac{2}{3} (\nabla \cdot \vec{u})^2 \right] - \frac{2}{3} \rho k \nabla \cdot \vec{u}$$

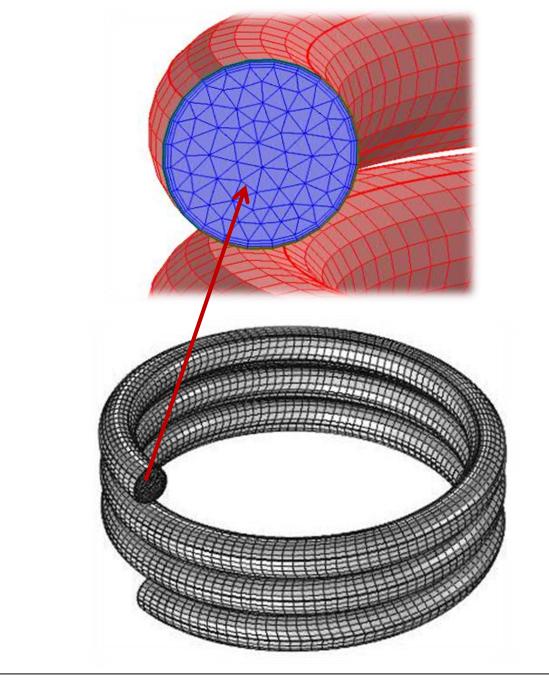
Heat transfer in fluids

$$\rho C_p \vec{u} \cdot \nabla T = \nabla \left(\left[k + \frac{\mu_T C_p}{Pr} \right] \Delta T \right) + Q_{vh}$$
 ----(4)

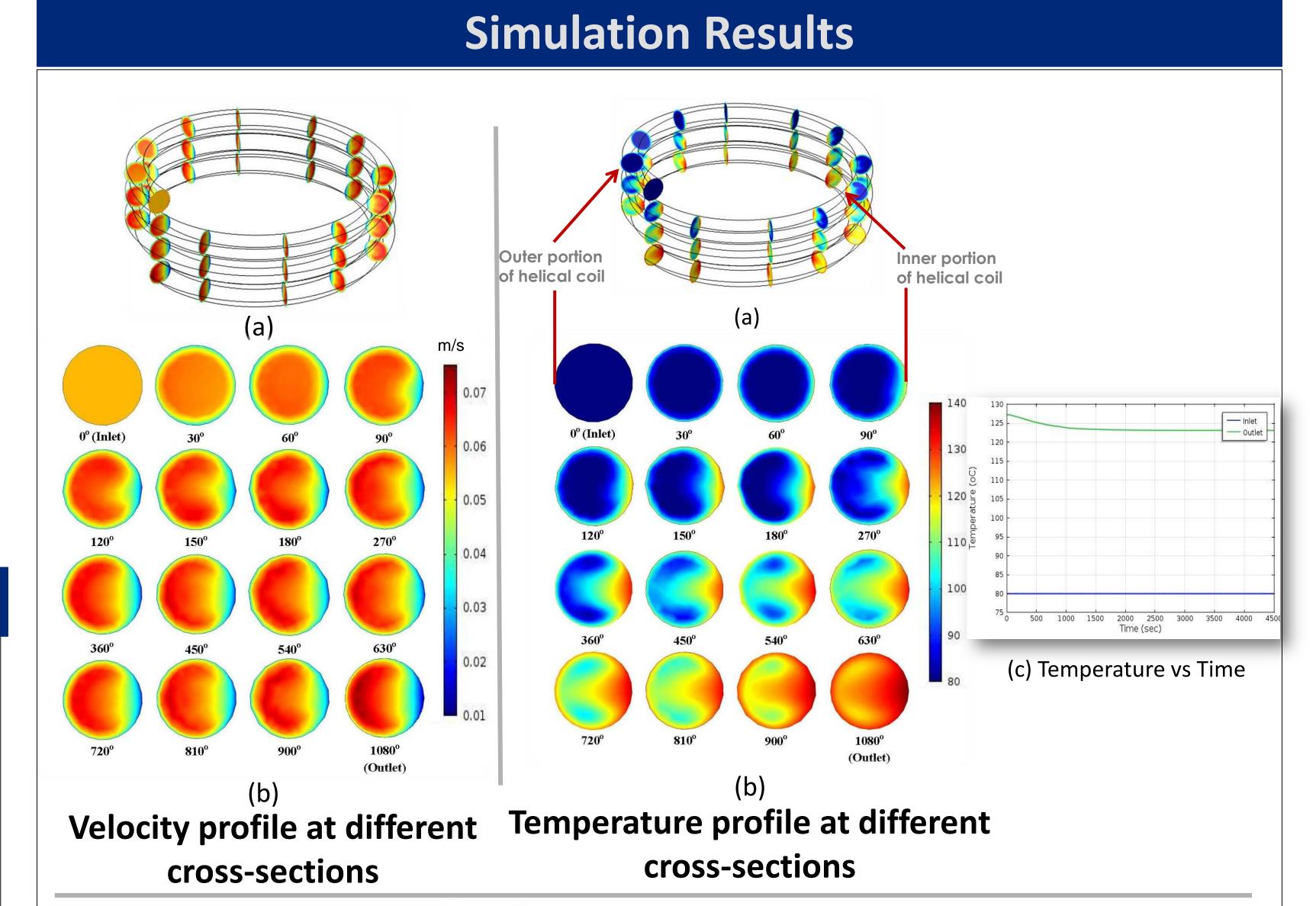
Local Fouling Rate

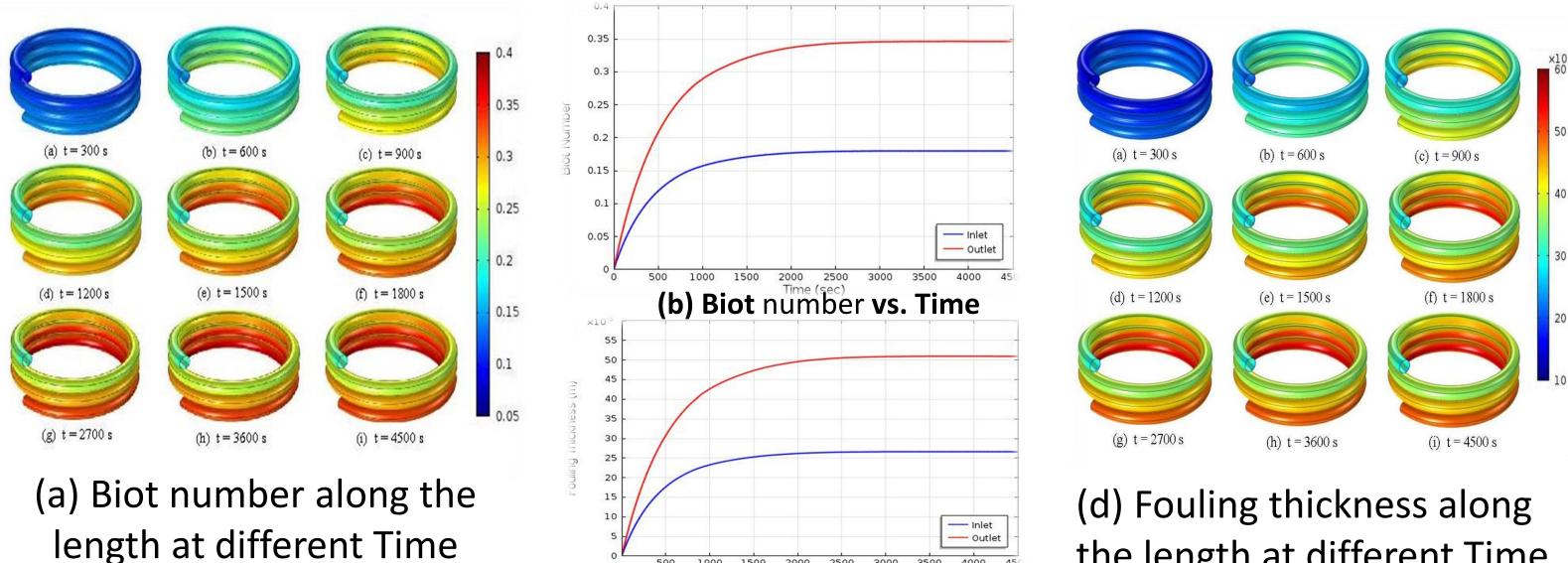
$$\frac{dBi_{m}}{dt} = k_{d}exp \left[-\frac{E_{a}}{R} \frac{\phi_{m} + f(Bi_{m}) + \frac{r_{o}}{r_{i}}}{\left(T_{s} + 273\right) \left(\frac{h_{f}^{o}r_{o}}{h_{f}r_{d}}\right) + (T_{f} + 273) \left[\phi_{m} + \frac{r_{o}}{r_{i}}Bi_{m}\right]} \right] - k_{r}Bi_{m} \qquad ----(5)$$

Initial and Boundary Conditions



	Rice-milk	Unit
Fluid flow		
Inlet velocity	0.055	m s ⁻¹
Wall	No slip	-
Heat transfer		
Inlet temperature	80	°C
Steam	150	°C
Heat flux	$U_oA_o(T_s-T)$	W
Fouling		
Inlet Biot number	0	-





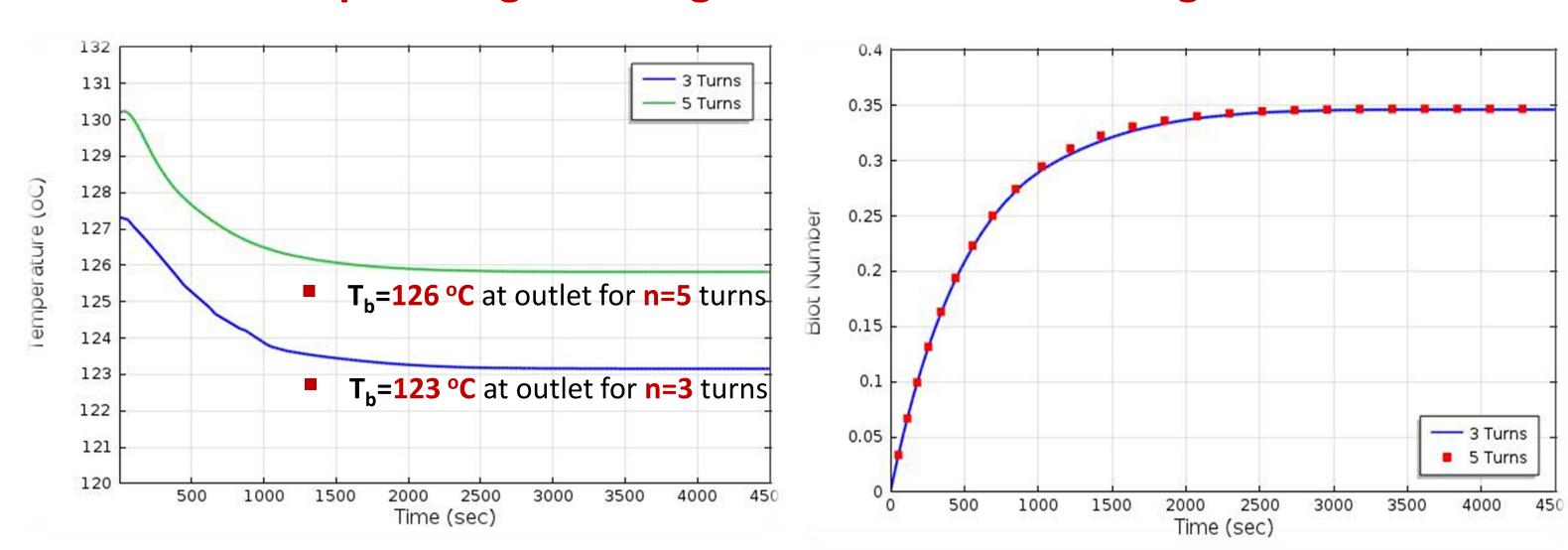
Biot number and fouling thickness profile at different cross-sections

Optimizing the design features of the heating section

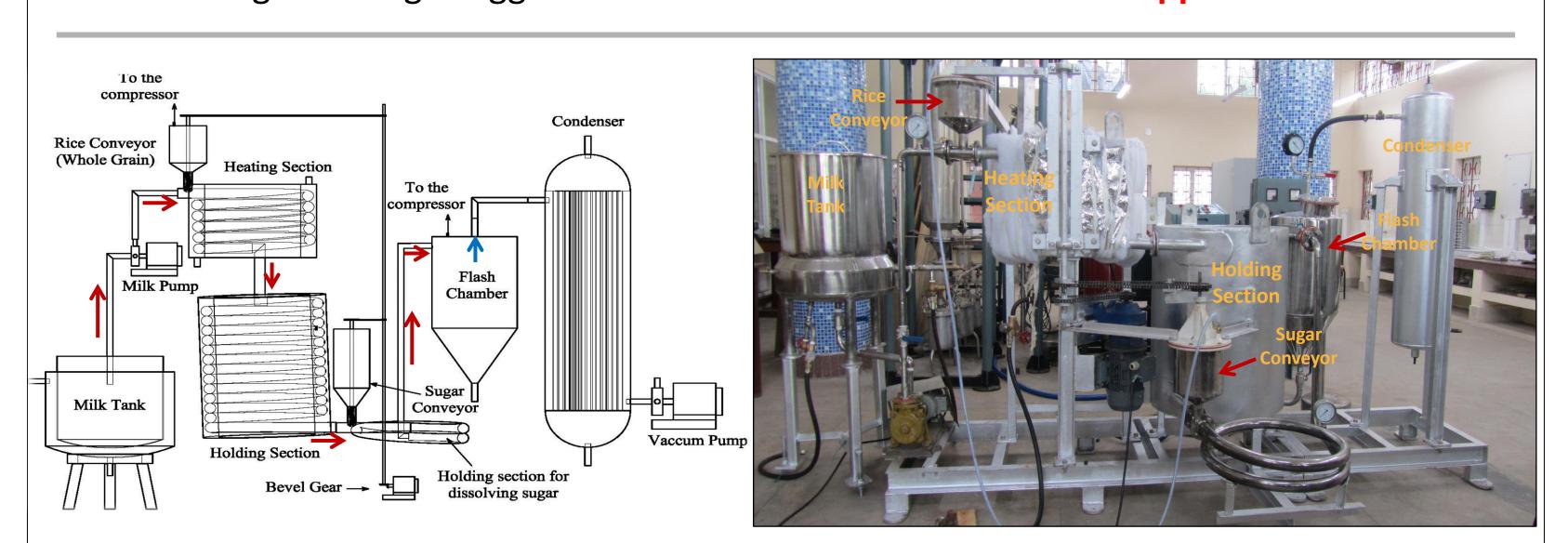
(c) Fouling thickness vs. Time

the length at different Time

(c) Biot number vs. Time



- (a) Temperature vs. Time
- Change in design suggested to include 5 turns in helical coil opposed to 3



Front view of continuous *kheer* making machine

Conclusions

- The 3D model is comprehensively built that includes the coupling of fluid flow, heat transfer and fouling kinetics with temperature dependent thermophysical properties
- Inner portion of the heating section is found more prone to fouling compared to the outer one due to curvature effect of the helical coil causing higher temperature and lower velocity
- Based on bulk mean temperature at the outlet of the helical pipe, change in design of the heating section is suggested to include 5 turns instead of 3